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GIFT OF THE
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A
PRACTICAL
BUSINESS ARITHMETIC;

DESIGNED AS

A TEXT-BOOK FOR COMMERCIAL COLLEGES,
ACADEMIES, AND HIGH-SCHOOLS,

AND FOR THE

USE OF BUSINESS MEN, ACCOUNTANTS, CLERKS, AND PRIVATE STUDENTS.

BY

LORENZO FAIRBANKS, A.M.,

Author of a Treatise on Bookkeeping by Double and Single Entry.

UNIVERSITY PUBLISHING COMPANY,
NEW YORK AND BALTIMORE.
1875.

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PREFACE.

THIS work is designed as a text-book for commercial colleges, normal and high schools, private schools, and academies, and as a practical manual for the counting-house.

With these objects in view in its preparation, the author has endeavored to present the science of arithmetic and its applications in a clear and philosophical manner, and to make the development of the shortest and best practical methods of computation, as employed by experts in various branches of business, a prominent feature.

Special attention has been devoted to subjects relating particularly to commercial and financial transactions, such as Percentage, Interest and Discount, Exchange, Partnership Average, Equation of Payments, Stocks and Bonds, Operations in the Stock Exchange, Custom-House Business, Measurement of Lumber, etc., all of which, in their treatment, will be found unusually extensive and complete. The article on the Stock Exchange is illustrated by practical examples obtained from Wall-Street Brokers, and that on Custom-House Business, by examples of real transactions at the New York Custom-House, showing the exact method of making the calculations.

Besides these, all the other topics usually presented in common arithmetics are fully treated, with the exception of Numeration and Notation of simple numbers, and the four fundamental rules, which it is presumed will have been already mastered by those who use this book.

An effort has been made to arrange the whole upon a systematic plan, and a progressive order of development, to meet the requirements of teachers, learners, and business men. A clear elucidation of principles has been attempted in the explanation of every subject, upon the analytic system; special care has been taken to give concise and accurate definitions; to use simple, yet scientific and comprehensive language; and, incidentally, throughout the work, to give a very considerable amount of useful information on business affairs.

It is hoped that the work will be found well adapted for the purposes for which it is intended. Whatever may be its imperfections, it is the result of patient labor and investigation. Every sentence has been carefully weighed, with an earnest wish to avoid errors, either in statements of facts or principles, to the end that the book may be accepted as a reliable authority on all the subjects discussed.

THE AUTHOR.

NEW YORK, August 14, 1871.

NOTE TO THE REVISED EDITION.

The passage of an Act of Congress of March 3, 1873 (see page 329), relative to foreign currencies, and its practical application in business during the past year, have rendered obsolete the methods formerly used in computations of Foreign Exchange, and in some departments of Custom House business. The changes thus effected have greatly simplified the whole subject. The calculation of German Exchange is also modified by the issue of a new German coinage.

All the portions of this work affected by these changes have been thoroughly revised, and now present and explain current usage.

L. F

NEW YORK, Jan. 1, 1875.

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PRACTICAL
BUSINESS ARITHMETIC.

ADDITION.

1. **Skill** in **Addition** is the foundation of readiness and accuracy in all arithmetical computations; and this skill is to be acquired only by practice, and careful attention to the principles and combinations of numbers. A clear head, quick perception, and proper cultivation, are the essential qualifications to insure success. With these any one may place himself in the rank of experts.

Those who have already learned the rudiments of arithmetic will need, under this head, nothing more than a few practical hints. First, it is important to observe that every operation should be simplified as much as possible. The mind should be disencumbered of the clogs imposed by school-boy rules, and trained to arrive at results in the most direct and rapid manner. Even the thought of *words* ought to be banished. Thus, instead of carrying on a mental operation as follows,—8 and 6 are 14, and 14 and 9 are 23, and 23 and 7 are 30,—glance over the figures, and read, or *think*, 8, 14, 23, 30. Besides this, the following directions may be of service:

1. Observe combinations, and make use of them: thus, in adding a column consisting of the figures 8, 9, 7, 6, 5, 5, 6, 4, 3, 2, 9, 3, connect 8 and 9, 7 and 6, 5 and 5, 6 and 4, etc., and you will have 17, 13, 10, 10, etc., and the column can be footed very rapidly, the different combinations accumulating in this manner,—17, 30, 50, 55, 67. When the same figure occurs several times, multiply it, instead of adding.

2. In adding long columns of figures, commence at the bottom and add upward, or *vice versa,* retain the number to be carried, if desired, as hereafter directed, and add it on commencing to add the next column. To insure correctness, add the columns both upward and downward. To avoid the necessity of making erasures or alterations, when the footings are to be written, as in a ledger, place a slip of paper under the column, add, and then transfer the amounts.

Examples for Practice.

Add the following columns upward and downward as rapidly as possible:—

(1.)	(2.)	(3.)	(4.)	(5.)
18	219	9876	356.01	92564
23	314	6789	375.18	36
34	428	5405	420.25	23508
46	537	3000	381.98	20001
58	675	3450	723.64	19000
67	895	1628	111.00	88
78	923	4721	100.97	91111
89	989	9999	400.00	99
98	375	2006	233.37	77777

2. The adding of numbers from right to left will often save labor. In this manner perform the following

Examples.

(1.) Add, as they stand,
15, 20, 33, 42, 58, 65, 73, 84, 97, 24.

(2.) Add
136, 242, 356, 983, 617, 538, 671, 708, 121.

(3.) Add
4256, 75, 365, 891, 46, 623, 50, 413.

3. To save the trouble of going over the whole work, in case of mistakes or interruption while footing long columns of figures, the number to be carried from each column may be retained by

writing it in pencil underneath, or by writing the sums of the several columns in the same order as they are added, as shown below. By the second method, the whole sum consists of the figures below the horizontal line and those above on the right of the perpendicular line. Those on the left of the perpendicular line are the numbers carried.

EXAMPLE.

1. Find the sum of

	Sums of the columns.	
8163		
2581	1	4
5437	2	4
9863	2	0
—	—	—
26044	2	6
221		

Accountants frequently divide long columns into sections by horizontal pencil lines, and add the several sections separately, and then their sums.

EXAMPLE.

1. Add the following numbers:—

1257.85	
2318.19	
657.17	4233.21
4102.50	
6309.42	
32673.89	43085.81
902.25	
182.30	
1428.60	2513.15
	49832.17, Total amount.

This practice is not recommended, as it is better to acquire expertness in "running up" columns of any ordinary length without this extra labor.

METHODS OF PROOF.

4. Of all the methods of proving addition, none is better or more satisfactory than the common practice of adding both upward and downward. By producing different combinations, this affords an accurate test. If the results agree, the work may be considered correct almost to an absolute certainty, and the agreement may be accepted as ample proof of the fact.

Another method is to compare the excesses of nines. This is done by adding all the digits of the several amounts, and "casting out," or dropping the nines. If the excess of the several numbers added equals the excess of their sum, the work is supposed to be right.

EXAMPLE.

1. What is the sum of

98654
37868
9046
3457
89642 . . . 5, excess of 9's in all the numbers,

238667 . . . 5, " " in their sum.

Explanation.—Commencing with the first number, at the left hand, drop 9. Then from the sum of 8 and 6 cast out 9, add the excess, 5, to the next digit, and so on; or any number of figures may be added, and the 9's cast out at once.

The practice of casting the 9's out of each separate number, and then out of the sum of the excesses, is convenient.

EXAMPLE.

1. Add the following numbers:—

36425 . . 2 excess.
18629 . . 8 "
7321 . . 4 "
8910 . . 0 " 5, excess in the sum of the excesses.

71285 . . 5, excess in the total amount.

5. To add several columns at one operation.

There are two methods: one is to commence with the lower number, and add the units to the units of the number above, then tens to tens, hundreds to hundreds, and so on. The other is, to begin with the highest order of figures in the lower number, and add each to the number above in the reverse order. The latter is regarded as the better method. The following examples will illustrate both:

EXAMPLES.

1. Add the following at one operation, commencing with the units of the lower number:—

243
362
521
657
——
1783

OPERATION.

657 + 1 = 658 + 20 = 678 + 500 = 1178 + 2 = 1180 + 60 = 1240 + 300 = 1540 + 3 = 1543 + 40 = 1583 + 200 = 1783.

2. Commencing with the hundreds of the lower number, find the sum of

624
836
347
423
——
2230

OPERATION.

423 + 300 = 723 + 40 = 763 + 7 = 770 + 800 = 1570 + 30 = 1600 + 6 = 1606 + 600 = 2206 + 20 = 2226 + 4 = 2230.

These operations may seem tedious, but in practice they are greatly abridged by combining and adding whole amounts at once. Thus, glancing at the two lower numbers, we see that their sum is 770; then, noticing that 7 tens plus 3 tens will require one to be carried, we perceive at once that the next number added will give 1606, and that the next will make the required sum, 2230. Sufficient practice of this kind will insure great facility and accuracy in the operation.

Add,	2875	695	3472	762
	321	321	6810	918
	487	482	327	329

SUBTRACTION.

6. Beyond the elementary principles of SUBTRACTION, there is little to be taught. A few practical suggestions may, however, be useful.

In one sense, subtraction is the reverse of addition; in another, it is the same thing. To increase a number by adding to it, is the reverse of diminishing it by subtracting from it; but the process of finding the difference between two numbers may be regarded either as addition or subtraction. Thus, to find the difference between eight and twelve separate things, we may take eight, and add enough to make twelve, or from twelve we may take eight, and *count* the number left. Hence, by adding to the subtrahend we may find the remainder. This principle may often be applied to advantage, especially when it is required to take the sum of two or more numbers, or their product, from another. See Art. 38, page 38.

7. To subtract two or more numbers from another at a single operation.

RULE.—*Add the given numbers in the usual manner, and for the remainder write the figures required to be added to form the units of each order of the minuend.*

EXAMPLE.

(1.) A merchant had 1550 barrels of flour, and sold 425 barrels to A, 225 barrels to B, and 314 barrels to C. How many had he left?

OPERATION.

```
1550
————
 425
 225
 314
————
 586, Ans.
```

Adding the first column of the numbers to be subtracted, we find that 6 must be added to produce a cipher in the units' place in the minuend, and that the sum with 6 added is 20; hence 2 must be carried. Carrying 2, and adding the next column, 8 is required to form the tens of the minuend, and 1 is to be carried. Carrying 1, and adding, we obtain, in the same way, 5, the last figure of the remainder.

Note.—The same may, of course, be performed by the usual process of subtraction. But it must be borne in mind, that the number of the next higher order above the sum of each column, must be borrowed and carried. As this must constantly vary, the *form* of addition is deemed less liable to cause confusion.

8. This method is especially convenient in balancing accounts. The balance may be found and inserted at the same time, without the trouble of writing the separate amounts and subtracting.

Example.

Dr.				Cash.			Cr.
1868.				1868.			
May 1	To Stock	2000	00	May 2	By Expense	85	00
" 4	" U. S. Bonds	2000	00	" 3	" Mdse	900	00
" 5	" Mdse	800	00	" 8	" Sundries	404	00
" 7	" M. Mathews	199	50	" 9	" N. Bk. Commerce	1500	00
" 10	" Sundries	150	00	" 10	" Mdse	216	00
" 12	" Mdse	165	00	" 13	" Sundries	2002	67
" 16	" "	75	00	" 15	" Bills Payable	247	58
" 21	" Smith & Kirkham	200	00	" 20	" Profit and Loss	22	50
				" 21	" *Balance*	*211*	*75*
		5589	50			5589	50

First, rule for the footings in the proper manner; then add the larger side, and complete the work, by adding the smaller side, inserting the balance, and footing the column.

In the same manner balance the following columns of figures:—

(2.)		(3.)		(4.)	
8763.10	1247.00	197.62	2472.09	467.10	342.20
1451.85	865.25	384.79	250.50	735.05	168.18
1376.08	2478.35	805.50	376.91	812.78	454.88
1921.17			2500.00	911.26	

When the smaller number stands over the larger, the subtraction should be performed without re-writing to change the order.

EXAMPLES.

Subtract, as they stand, the upper from the lower numbers, in the following cases:—

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
1937	423	1634	3001	275.16	2841.15
2346	1874	2648	4002	856.27	4268.91

PROOF.

9. The following are the principal methods of proving subtraction:—

1. Add the remainder to the subtrahend; and if the sum is equal to the minuend, the work may be accounted correct.

2. Subtract the remainder from the minuend; if the work is right, the difference will be equal to the subtrahend.

3. Find the excess of 9's in the remainder and subtrahend together, and it will be equal to the excess of 9's in the minuend, if the work is correct.

SHORT PRACTICAL METHODS OF MULTIPLICATION.

10. The operations of MULTIPLICATION admit of the employment of numerous methods of contraction, of which the following will be found the most useful in practice.

CASE I.

11. To multiply together any two numbers between which there is a mean number of tens.

RULE.—*Square the tens of the larger number, and subtract the square of the unit-figure of the same number*

NOTE.—This rule is based upon the principle that the product of any two numbers is equal to the square of the mean minus the square of half the difference.

EXAMPLES.

1. Multiply 76 by 84.

$$\begin{array}{r} 76\\ 84\\ \hline 6384 \end{array} \qquad \begin{array}{r} 80 \times 80 = 6400\\ 4 \times 4 = 16\\ \hline 6384, \textit{ Ans.} \end{array}$$

Practically, square 8, subtract 1, and annex $100 - 4^2 = 84$.

2. Multiply 98 by 82.

$$\begin{array}{r} 98\\ 82\\ \hline 8036 \end{array} \qquad 9 \times 9 - 1, \text{ with } 100 - 8^2 \text{ annexed, } = 8036, \textit{ Ans.}$$

NOTE.—It will be observed that in these numbers the difference of the tens is 1, and the sum of the units is ten. Their sum is equal to twice the number of tens of the larger number, which is, consequently, an arithmetical mean between them.

3. Multiply 103 by 97.

$$\begin{array}{r} 103\\ 97\\ \hline 9991 \end{array} \qquad 10^2 - 1, \text{ with } 100 - 3^2 \text{ annexed, } = 9991, \textit{ Ans.}$$

4. Multiply 69 by 71
5. " 68 " 72
6. " 67 " 73
7. " 66 " 74
8. " 65 " 75
9. " 59 " 41

10. Multiply 58 by 42
11. " 57 " 43
12. " 56 " 44
13. " 55 " 45
14. " 111 " 109
15. " 118 " 102

CASE II.

12. To multiply together any two numbers of two figures each, when the tens are alike, and the sum of the units is ten.

RULE.—*Multiply one of the tens by the other plus* 1, *and annex the product of the units.*

EXAMPLES.

1. Multiply 66 by 64.

$$\begin{array}{r} 66 \\ 64 \\ \hline 4224 \end{array}$$

$6 \times \overline{6+1} = 42$, and annexing 6×4, we have 4224, *Ans.*

2. Multiply 77 by 73.

$7 \times 8 = 56$, and $7 \times 3 = 21$. *Ans.* 5621.

3. Multiply 81 by 89
4. " 82 " 88
5. " 83 " 87
6. " 59 " 51
7. " 198 " 192
8. " 1207 " 1203
9. Multiply 72 by 78
10. " 47 " 43
11. " 22 " 28
12. " 98 " 92
13. " 36 " 34
14. " 16 " 18

15. What will 38 yards of print cost at 32 cents per yard?

Hence,—

13. To square any number of two places whose unit-figure is 5.

RULE.—*Multiply the number expressing the tens by itself plus* 1, *and annex* 25.

1. Find the square of 35.

$\overline{3+1} \times 3 = 12$, and annexing 25 we have 1225, *Ans.*

2. What is the square of 65?

$\overline{7 \times 6} = 42$, and 42 with 25 annexed = 4225, *Ans.*

3. Find the square of 95. 4. Of 85. 5. Of 75. 6. Of 45.
7. Of 55. 8. Of 15. 9. Of 25.

This rule, as well as the last, may be extended to numbers of more than two places.

EXAMPLES.

1. What is the square of 125?

$13 \times 12 = 156$, and annexing 25 we have 15625, *Ans.*

2. What is the square of 115?

CASE III.

14. To multiply any number of two figures whose sum is less than ten, by 11.

RULE.—*Place the sum of the digits between them, and the result will be the product.*

EXAMPLES.

1. Multiply 27 by 11.

$2 + 7 = 9$, which, placed between 2 and 7, gives 297 for the product. *Ans.* 297.

2. Multiply 44 by 11
3. " 43 " 11
4. " 36 " 11
5. Multiply 63 by 11
6. " 54 " 11
7. " 72 " 11

15. To multiply any number by 11.

RULE.—*Write as the first figure of the product the unit-figure of the multiplicand; then add successively, from right to left, units to tens, tens to hundreds, and so on, carrying, and writing the results in the product;* or,

Write the multiplicand under itself one place to the right, and add.

EXAMPLES.

FIRST METHOD.

1. Multiply 6971 by 11. 76681, *Ans.*

Explanation.—Write 1, and, for the remaining figures of the product, add 1 to 7, 7 to 9, 9 to 6, carrying as required, and, finally, write 7, which is the last figure plus 1 carried.

SECOND METHOD.

2. Multiply 2785 by 11.

$$\begin{array}{r} 2785 \\ 2785 \\ \hline 30635 \end{array}$$

30635, *Ans.*

3. Multiply 4645 by 11
4. " 125 " 11
5. " 1008 " 11
6. Multiply 3162 by 11
7. " 162 " 11
8. " 968 " 11

CASE IV.

16. **To multiply by any number between 11 and 20.**

RULE.—*Multiply by the units of the multiplier, write the product under the multiplicand, one place to the right, and add.*

EXAMPLES.

1. Multiply 36857 by 16.

36857
221142

589712, *Ans.*

2. Multiply 1864 by 14
3. “ 2784 “ 13
4. “ 1975 “ 12
5. Multiply 2918 by 19
6. “ 361 “ 17
7. “ 13675 “ 15

This rule is easily adapted to cases where the multiplier is a little over 100 1000 etc.

EXAMPLE.

1. Multiply 723 by 105.

723
3615

75915, *Ans.*

17. **The operation may be condensed.**

RULE.—*Multiply by the units of the multiplier, carry, and add continually the figure to the right of the one multiplied; finally, to the left-hand figure of the multiplicand add the number to be carried, if any, and write the result.*

EXAMPLE.

1. Multiply 23758 by 18.

23758
18

427644, *Ans.*

Explanation.—$8 \times 8 = 64$. Write 4, and carry 6. $\overline{8 \times 5} + 6 + 8 = 54$. Write 4, and proceed as before. Finally, $2 + 2$ carried completes the product. Great facility can be acquired by this method, with a little practice.

CASE V.

18. **To multiply by any number of two figures ending with 1.**

RULE.—*Multiply by the tens of the multiplier, write the product under the multiplicand, one place to the left, and add.*

EXAMPLE.

1. Multiply 4751 by 21.

$$\begin{array}{r} 4751 \\ 9502 \\ \hline 99771 \end{array}$$

99771, *Ans.*

19. **Or, condensing the operation,**

RULE.—*Write as the first figure of the product the unit-figure of the multiplicand; then multiply the whole multiplicand by the tens of the multiplier, and to the product of the figures successively add the number to be carried and the figure to the left of the one multiplied, writing the results in order in the product.*

EXAMPLE.

1. Multiply 7162 by 41.

$$\begin{array}{r} 7162 \\ 41 \\ \hline 293642 \end{array}$$

Explanation.—Write 2 in the product. $\overline{4 \times 2} + 6 = 14$. Write 4, and carry 1. $\overline{4 \times 6} + 1 + 1 = 26$. Write 6, and carry 2. $\overline{4 \times 1} + 2 + 7 = 13$. Write 3, and carry 1. $\overline{4 \times 7} + 1 = 29$, which, written to the left, finishes the work.

20. When the multiplier is any digit, any number of ciphers, and 1, the principle of the former rule is easily applied.

EXAMPLE.

1. Multiply 3682 by 501.

$$\begin{array}{r} 3682 \\ 18410 \\ \hline 1844682 \end{array}$$

1844682, *Ans.*

CASE VI.

21. **To multiply by any number which is a little more or less than any number of tens.**

RULE.—*Multiply by the nearest number of tens, and then add, or subtract, as many times the multiplicand as the number is units over, or under, the tens.*

EXAMPLES.

1. Multiply 326 by 101.

$$\begin{array}{r} 32600 \\ 326 \\ \hline 32926, \textit{ Ans.} \end{array}$$

2. Multiply 575 by 99.

$$\begin{array}{r} 57500 \\ 575 \\ \hline 56925, \textit{ Ans.} \end{array}$$

3. Multiply 251 by 102.

25100 + 502 = 25602, *Ans.*

4. Multiply 26 by 98.

2600 − 52 = 2548, *Ans.*

5. Multiply 837 by 101
6. " 279 " 99
7. " 347 " 102
8. Multiply 751 by 81
9. " 622 " 79
10. " 35 " 42

11. What is the value of 12 shares of railroad stock, of $100 each, at 97½%?

1200 − 30 (2½% of 1200) = 1170. *Ans.* $1170.

12. What is the value of $2400 U. S. Bonds at 105¾?

$$\begin{array}{rl} 2400 & \\ 120 & 5\% \text{ of } 2400. \\ 18 & \tfrac{3}{4}\% \text{ " \quad "} \\ \hline 2538 & \end{array}$$

Ans. $2538.

CASE VII.

22. **To multiply by any number one part of which is a factor of another part.**

RULE.—*Multiply by the factor; then multiply the product so obtained by the component factor or factors of the other part or parts of the multiplier, writing the first figure of each product* *under the first figure of the multiplier used. The sum of the partial products will be the required product.*

EXAMPLES.

1. Multiply 3276 by 124.

12 = 3 × 4

3276	
124	
13104	Product by 4.
39312	3 times the product by 4.
406224,	Product by 124, *Ans.*

2. Multiply 47852 by 24318.

In this case, 3 is a factor of two parts of the multiplier. 3 × 8 = 24, and 3 × 6 = 18.

47852	
24318	
143556	Product by 3 hundred.
861336	6 times the product by 3 = the product by 18.
1148448	8 times the product by 3 = the product by 24.
1163664936, *Ans.*	

23. The same principle is applicable when the multiplier may be resolved into two parts, one of which is a factor of the other.

EXAMPLES.

1. Multiply 2862 by 15.

15 = 10 + 5

2)28620	Product by 10.
14310	½ of itself added.
42930, *Ans.*	

2. Multiply 256 by 75.

4)25600	Product by 100.
6400	Less ¼ of itself.
19200, *Ans.*	

Or, 2)25600
2)12800
6400
19200, *Ans.*

3. Multiply 464 by 7½.

2)4640
2)2320
1160
3480, *Ans.*

Or,
4640
1160 Less ¼.
3480

CASE VIII.

24. **To multiply together two numbers of the same number of places, under and near 100, 1000, 10000, etc.**

RULE.—*To their sum, with the unit-figure at the left rejected, annex as many ciphers as there are places in each, and add the product of their complements.*

NOTE.—The arithmetical complement of a number is the difference between it and a unit of the next higher order.

EXAMPLES.

1. Multiply 97 by 98.

97 . . . . 3, complement.
98 . . . . 2, " $3 \times 2 = 6$.
9506

2. Multiply 96 by 85.

96 4
85 15
8160, *Ans.*

3. Multiply 91 by 88.

91
88
79
108
8008, *Ans.*

4. Multiply 997 by 989.

997
989
986033, *Ans.*

5. Multiply 975 by 985.

975
985
960375, *Ans.*

6. Multiply 995 by 985
7. “ 99 “ 92
8. “ 86 “ 94

9. Multiply 986 by 991
10. “ 987 “ 987
11. “ 9996 “ 9896

By this rule we can square any number of 9's at a glance.

EXAMPLE.

1. What is the square of 9999?

9999 1
9999 1

99980001, *Ans.*

CASE IX.

25. To multiply together any two numbers of the same number of places, over and near 100, 1000, 10000, etc.

RULE.—*Cancel one of the left-hand figures, add the remaining numbers, annex as many ciphers, less one, as there are places in each number, and add the product of their excesses, respectively, over* 100, 1000, *etc.*

EXAMPLES.

1. Multiply 106 by 105.

$\cancel{1}06$ Cancel the left-hand figure.
105 Add the remaining numbers, and annex

11130, *Ans.* 5 × 6 = 30.

2. Multiply 112 by 114.

$\cancel{1}12$
114

12600 Sum after cancelling 1, with two ciphers annexed.
168 Product of the excesses.

12768, *Ans.*

3. Multiply 1016 by 1007.

$\cancel{1}016$
1007

1023112, *Ans.*

4. Multiply 110 by 125
5. " 106 " 106
6. " 115 " 105
7. Multiply 1014 by 1004
8. " 1008 " 1009
9. " 1012 " 1007

CASE X.

26. **To multiply together any two numbers, one of which is more and the other less by a convenient amount than 100, 1000, 10000, etc.**

RULE.—*Cancel the left-hand figure of the larger number, add the remaining numbers, annex as many ciphers as there are places in the smaller number, and subtract the product of the differences between them, respectively, and* 100, 1000, *etc.*

EXAMPLES.

1. Multiply 109 by 97.

$\cancel{1}09$ . . . 9, excess.
97 . . . 3, complement.

10600
27 Product of excess and complement subtracted.

10573, *Ans.*

2. Multiply 126 by 96.

$\cancel{1}26$ . . . 26
96 . . . 4

12096, *Ans.*

3. Multiply 112 by 95.

$\cancel{1}12$
95

10640, *Ans.*

4. Multiply 118 by 95
5. " 115 " 85
6. " 109 " 92
7. Multiply 1018 by 996
8. " 1004 " 985
9. " 1009 " 992

CASE XI.

27. **To square any number of two or more places.**

RULE.—*Square the units; then multiply twice the tens (or twice the units and tens, if the number is over one hundred) by the units; finally, square the tens (or units and tens), carrying as usual.*

EXAMPLES.

1. What is the square of 64?

64
64
——
4096, *Ans.*

Explanation. $4^2 = 16$.
4 × 12 (6 + 6) = 48, and 1 to carry = 49.
6^2 + 4 carried = 40.

2. Square 123.

123
123
——
15129, *Ans.*

3 × 3 = 9. 3 × 24 = 72.
12 × 12 + 7 carried = 151.

3. Find the square of 18. 4. Of 23. 5. Of 63. 6. Of 92. 7. Of 37. 8. Of 33. 9. Of 55. 10. Of 66. 11. Of 109. 12. Of 112. 13. Of 113.

CASE XII.

28. Upon a similar principle, any two numbers whose unit-figures are alike may be multiplied together.

EXAMPLES.

1. Multiply 72 by 42.

72
42
——
3024, *Ans.*

2 × 2 = 4. 2 × 11 (the sum of the tens) = 22.
4 × 7 (product of the tens) + 2 (carried) = 30.

2. Multiply 126 by 36.

126
36
——
4536, *Ans.*

6 × 6 = 36. 6 × 15 (sum of all the figures to the left of units) + 3 (carried) = 93.

3 × 12 (product of all the figures to the left of units + 9 (carried) = 45.

CASE XIII.

29. And, with a slight variation, the rule may be applied to cases in which the tens, hundreds, etc., of the multiplicand and multiplier are alike.

EXAMPLES.

1. Multiply 73 by 75.

73
75
———
5475, *Ans.*

5 × 3 = 15. 7 × 8 (the sum of the units) + 1 (carried) = 57. 7 × 7 (product of the tens) + 5 (carried) = 54.

2. Multiply 124 by 123.

124
123
———
15252, *Ans.*

3 × 4 = 12. $\overline{7 \times 12} + 1 = 85$. $\overline{12 \times 12} + 8 = 152$.

3. What will 127 yards of cloth cost, at $1.25 per yard?

NOTE.—Write the answer directly from the example as it stands, or give it mentally.

CROSS MULTIPLICATION.

CASE XIV.

30. To multiply by any number of figures at once.

RULE.—*First, multiply units by units; then each figure of the multiplicand in order by the units of the multiplier, carrying as usual, and adding to each product the product, or products, obtained by multiplying, successively, the preceding figures of the multiplicand by the succeeding figures of the multiplier, up to the order of the figure multiplied, till all have been multiplied by the units of the multiplier; then drop the unit-figures, and proceed with the others in the same manner. Finally, multiply together the left-hand figures of the multiplicand and multiplier, add the number to be carried, if any, and write the result.*

Note.—This process is not so difficult as is generally imagined, and if once clearly comprehended, it can readily be applied in practice, especially in multiplying numbers of two and three places.

31. The elucidations which follow will give a better idea of the process than the rule, which may appear complicated to a novice.

Examples.

1. Multiply 231 by 465.

$$\begin{array}{r}231\\465\\\hline 107415,\ \textit{Ans.}\end{array}$$

Elucidation.

$$\begin{array}{r|l}
5 \times 1 = & 5 \\
\overline{5 \times 3} + \overline{6 \times 1} = 2 & 1 \\
2\text{ (carried)} + \overline{5 \times 2} + \overline{6 \times 3} + \overline{4 \times 1} = 3 & 4 \\
3\text{ (carried)} + \overline{6 \times 2} + \overline{4 \times 3} = 2 & 7 \\
\multicolumn{2}{r}{2\text{ (carried)} + \overline{2 \times 4} = \overline{1\quad 0}}
\end{array}$$

10, the last result, and the figures to the right of the perpendicular line, form the product. The figures on the left of the line represent the numbers carried.

Note.—It will be seen that there is a perfect system in the operation, which can be followed with facility and accuracy by any skillful person, with sufficient practice.

2. Multiply 4738 by 6523.

$$\begin{array}{r}4738\\6523\\\hline 30905974,\ \textit{Ans.}\end{array}$$

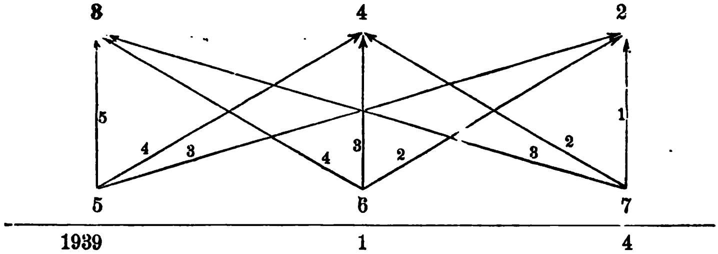
Elucidation.

$$\begin{array}{r|l}
3 \times 8 = 2 & 4 \\
\overline{3 \times 3} + 2 + \overline{2 \times 8} = 2 & 7 \\
\overline{3 \times 7} + 2 + \overline{2 \times 3} + \overline{5 \times 8} = 6 & 9 \\
\overline{3 \times 4} + 6 + \overline{2 \times 7} + \overline{5 \times 3} + \overline{6 \times 8} = 9 & 5 \\
\overline{2 \times 4} + 9 + \overline{5 \times 7} + \overline{6 \times 3} = 7 & 0 \\
\overline{5 \times 4} + 7 + \overline{6 \times 7} = 6 & 9 \\
\multicolumn{2}{r}{6 + \overline{6 \times 4} = \overline{3\quad 0}}
\end{array}$$

Note.—To become familiar with the operation, impress upon the mind the order of procedure. Thus, observe that we multiply units by units; tens by units, units by tens; hundreds by units, tens by tens, units by hundreds; thousands by units, hundreds by tens, tens by hundreds, units by thousands; and so on. The figures of the multiplicand are taken in order from left to right, and those of the multiplier from right to left.

DIAGRAM.

Illustrating Cross Multiplication.



The above diagram illustrates the operation of multiplying three figures by three. The different steps are indicated by the arrows, numbered in order, 1, 2, 3, 4, and 5. This, with the previous elucidations, will, it is believed, give a clear idea of the process, and enable any one of ordinary comprehension to understand it.

CASE XV.

32. To multiply by aliquot parts of 10, 100, 1000, etc.

RULE.—*Multiply the multiplicand by the number of which the multiplier is an aliquot part, by annexing ciphers; then take such part of the product as the multiplier is of that number.*

EXAMPLES.

1. Multiply 1824 by 25.

$25 = \frac{1}{4}$ of 100.

182400 ÷ 4 = 45600, *Ans.*

2. Multiply 66 by $33\frac{1}{3}$.

$33\frac{1}{3} = \frac{1}{3}$ of 100.

6600 ÷ 3 = 2200, *Ans.*

3. What is $2\frac{1}{2}$ % of $462?

$2\frac{1}{2} = \frac{1}{4}$ of 10.

4620 ÷ 4 = 1155, and, pointing off two figures, we have, 11.55. $11.55, *Ans.*

Table of Aliquot Parts of 10, 100, *and* 1000.

$5 = \frac{1}{2}$ of 10	$6\frac{1}{4} = \frac{1}{16}$ of 100	$33\frac{1}{3} = \frac{1}{3}$ of 100
$2\frac{1}{2} = \frac{1}{4}$ " "	$8\frac{1}{3} = \frac{1}{12}$ " "	$50 = \frac{1}{2}$ " "
$3\frac{1}{3} = \frac{1}{3}$ " "	$12\frac{1}{2} = \frac{1}{8}$ " "	$125 = \frac{1}{8}$ of 1000.
$1\frac{2}{3} = \frac{1}{6}$ " "	$16\frac{2}{3} = \frac{1}{6}$ " "	$166\frac{2}{3} = \frac{1}{6}$ " "
$1\frac{1}{4} = \frac{1}{8}$ " "	$20 = \frac{1}{5}$ " "	$250 = \frac{1}{4}$ " "
$1\frac{1}{9} = \frac{1}{9}$ " "	$25 = \frac{1}{4}$ " "	$333\frac{1}{3} = \frac{1}{3}$ " "

4. Multiply 98 by $33\frac{1}{3}$
5. " 146 " $12\frac{1}{2}$
6. " 224 " $66\frac{2}{3}$
7. " 540 " $16\frac{2}{3}$
8. " 219 " $333\frac{1}{3}$
9. " 248 " 250
10. Multiply 3642 by $1\frac{1}{4}$
11. " 216 " 20
12. " 406 " $1\frac{2}{3}$
13. " 864 " 50
14. " 275 " $37\frac{1}{2}$
15. " 902 " $8\frac{1}{3}$

33. This rule, with a slight variation, may also be used when a part of the multiplier is an aliquot part of 10, 100, etc.

EXAMPLES.

1. Multiply 456 by 2125.

$2125 = 2\frac{1}{8} \times 1000$; hence, $456 \times 2125 = 456000 \times 2\frac{1}{8}$,

$$\begin{array}{r} 456000 \\ 2\frac{1}{8} \\ \hline 912 \\ 57 \\ \hline 969000 \end{array}$$
969000, *Ans.*

2. Multiply 237 by $133\frac{1}{3}$.

$$\begin{array}{r} 3)23700 \\ 7900 \\ \hline 31600 \end{array}$$

3. Multiply 48 by $333\frac{1}{3}$
4. " 186 " $116\frac{2}{3}$
5. " 222 " 3250
6. Multiply 248 by $137\frac{1}{2}$
7. " 24 " $218\frac{3}{4}$
8. " 36 " $102\frac{1}{2}$

34. When the multiplier is $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, etc., it is easier to subtract $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$, etc., of the product from itself than to multiply by the fraction.

EXAMPLES.

1. Multiply 4617 by $66\frac{2}{3}$.

3)461700	Product by 100.
153900	$\frac{1}{3}$ of itself subtracted
307800, *Ans.*	

2. Multiply 3856 by $87\frac{1}{2}$.

8)385600
48200

337400, *Ans.*

35. To multiply by 5.

RULE.—*Annex a cipher to the multiplicand, and divide by* 2. Or,

If the multiplicand be an even number, annex a cipher to one-half of it; if odd, annex 5 *to one-half of the next lower number.*

EXAMPLES.

1. Multiply 18 by 5.

2)180	Or, 2)18	Annex a cipher, and divide by 2; or, divide by 2, and annex a cipher to the quotient.
90	90, *Ans.*	

2. Multiply 19 by 5.

2)190	Or, 2)18	Annex a cipher, and divide by 2: or, divide 18, the next lower number by 2, and annex 5.
95	95, *Ans.*	

MULTIPLICATION OF MIXED NUMBERS.

CASE XVI.

36. To multiply a mixed number by a mixed number.

RULE.—*Multiply by the whole number of the multiplier, and then by its fraction, or by the fraction first.*

EXAMPLES.

1. Multiply $14\frac{1}{4}$ by $16\frac{1}{2}$.

$$\begin{array}{rcrcr} & & & & 14\frac{1}{4} \\ & & & & 16\frac{1}{2} \\ \hline 14 & \times & 16 & = & 224 \\ \frac{1}{4} & \times & 16 & = & 4 \\ 14\frac{1}{4} & \times & \frac{1}{2} & = & 7\frac{1}{8} \\ \hline & & & & 235\frac{1}{8},\ \textit{Ans.} \end{array}$$

NOTE.—After finding the product of the whole numbers, either whole number and its fraction may be multiplied by the fraction of the other, according to convenience.

2. Multiply $37\frac{3}{4}$ by $16\frac{1}{8}$.

$$\begin{array}{rcrcrl} & & & & 37\frac{3}{4} & \\ & & & & 16\frac{1}{8} & \\ \hline 37 & \times & 16 & = & 592 & \\ \frac{3}{4} & \times & 16\frac{1}{8} & = & 12\frac{3}{32} & \\ 37 & \times & \frac{1}{8} & = & 4\frac{5}{8} & \text{or } \frac{20}{32} \\ \hline & & & & 608\frac{23}{32} & \end{array}$$

Or,

$$\begin{array}{rcrcr} & & & & 37\frac{3}{4} \\ & & & & 16\frac{1}{8} \\ \hline 37 & \times & 16 & = & 592 \\ \frac{3}{4} & \times & 16 & = & 12 \\ 37\frac{3}{4} & \times & \frac{1}{8} & = & 4\frac{23}{32} \\ \hline & & & & 608\frac{23}{32} \end{array}$$

NOTE.—Fractions of quantities and prices occur so frequently in business, that this method of multiplication is almost indispensable to the requisite rapidity of calculation.

Perfect exactness, however, is not always required in commercial transactions. Thus, suppose in the above example we were reckoning the cost of $37\frac{3}{4}$ yards of cloth at $16\frac{1}{8}$ cents per yard, we would call $\frac{1}{8}$ of $37\frac{3}{4}$ 5, and $\frac{3}{4}$ of $16\frac{1}{8}$ 12, and the result would be $6.09, which is accurate enough for all practical purposes.

3. Multiply $33\frac{1}{2}$ by $12\frac{1}{2}$
4. " $37\frac{1}{2}$ " $18\frac{3}{4}$
5. " $42\frac{1}{4}$ " $8\frac{2}{3}$
6. " $35\frac{3}{4}$ " $14\frac{1}{2}$
7. Multiply $26\frac{1}{8}$ by $27\frac{1}{2}$
8. " $45\frac{1}{4}$ " $31\frac{1}{4}$
9. " $56\frac{1}{4}$ " $11\frac{1}{8}$
10. " $49\frac{3}{4}$ " $24\frac{1}{4}$

CASE XVII.

37. To multiply a mixed number by a mixed number, when both have the same fraction.

RULE.—*To the product of the whole numbers add the product of their sum by the fraction, and the square of the fraction.*

EXAMPLES.

1. Multiply $18\frac{1}{2}$ by $6\frac{1}{2}$.

$$\begin{array}{rl} 18\frac{1}{2} & \\ 6\frac{1}{2} & \\ \hline 108 & = 18 \times 6 \\ 12\frac{1}{4} & = \overline{18 + 6} \times \frac{1}{2} + \overline{\frac{1}{2} \times \frac{1}{2}} \\ \hline 120\frac{1}{4}, & \textit{Ans.} \end{array}$$

2. Multiply $10\frac{1}{4}$ by $6\frac{1}{4}$
3. " $14\frac{1}{2}$ " $10\frac{1}{2}$
4. " $24\frac{1}{3}$ " $12\frac{1}{3}$
5. Square $9\frac{1}{4}$.
6. Multiply $16\frac{7}{8}$ by $8\frac{7}{8}$
7. " $22\frac{5}{6}$ " $8\frac{5}{6}$
8. " $36\frac{1}{4}$ " $4\frac{1}{4}$
9. Square $12\frac{3}{4}$.

NOTE.—$\frac{1}{4}$ of $\overline{9 + 9} = \frac{1}{2}$ of 9, and $\frac{3}{4}$ of $\overline{12 + 12} = \frac{3}{2}$ of 12. Therefore, in squaring a mixed number, multiply the whole number by twice the fraction, instead of twice the number by the fraction, if it is more convenient.

38. This rule can be used in squaring any number ending in 25, 50, and 75, by regarding these figures as $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of 100; also in multiplying together any two numbers ending in 5, considering 5 as $\frac{1}{2}$ of 10.

EXAMPLES.

1. What is the square of 1225 ?

The square of $12\frac{1}{4}$, found by the above rule, is $150\frac{1}{16}$, and reducing the fraction, we have, 1500625, *Ans.*

2. Find the square of 1675 ?

$(16\frac{3}{4})^2 = 280\frac{9}{16}$, which, reduced, $= 2805625$, *Ans.*

NOTE.—This can be done without changing to the fractional form, by using the rule for Case II., page 17. Thus, in the last example, square 16, add 24 ($\frac{3}{4}$ of 16 + 16), and annex the square of 75, namely, 5625.

CASE XVIII.

39. **When the whole numbers are alike and the sum of the fractions is 1.**

RULE.—*Multiply one of the whole numbers by the other plus 1, and annex the product of the fractions.*

EXAMPLES.

1 Multiply $8\frac{1}{4}$ by $8\frac{3}{4}$.

$8 \times 9 = 72$, and $72 + \frac{3}{4} \times \frac{1}{4} = 72\frac{3}{16}$, *Ans.*

2. What is the square of $12\frac{1}{2}$?

13×12 (or, $\overline{12 \times 12} + 12$) $+ \frac{1}{4} = 156\frac{1}{4}$, *Ans.*

3. Multiply $14\frac{1}{8}$ by $14\frac{7}{8}$
4. " $9\frac{1}{4}$ " $9\frac{3}{4}$
5. Find the square of $25\frac{1}{2}$
6. " " " $13\frac{1}{2}$

CASE XIX.

40. **When the difference of the whole numbers is 1, and the sum of the fractions is 1.**

RULE.—*From the square of the larger number subtract the square of its fraction.*

EXAMPLES.

1. Multiply $9\frac{1}{4}$ by $8\frac{3}{4}$.

$9 \times 9 = 81$, and $81 - \frac{1}{16} = 80\frac{15}{16}$, *Ans.*

2. Multiply $15\frac{3}{5}$ by $16\frac{2}{5}$.

$16^2 = 256$, and $256 - (\frac{2}{5})^2 = 255\frac{21}{25}$, *Ans.*

3. Multiply $9\frac{1}{2}$ by $8\frac{1}{2}$
4. " $10\frac{3}{4}$ " $9\frac{1}{4}$
5. " $7\frac{3}{4}$ " $6\frac{1}{4}$
6. Multiply $14\frac{1}{2}$ by $13\frac{1}{2}$
7. " $18\frac{3}{4}$ " $19\frac{1}{4}$
8. " $17\frac{1}{4}$ " $16\frac{3}{4}$

CONTRACTIONS IN DIVISION.

41. The following methods of contracting the operations of DIVISION include all in common use of practical value, and some that are new and original, which, in certain branches of business, may be employed with great advantage.

CASE I.

42. When the divisor is 1 with one or more ciphers at the right.

RULE.—*Point off as many figures from the right of the dividend as there are ciphers in the divisor.*

EXAMPLE.

1. Divide 73285 by 100.

732.85, or, $732\frac{85}{100}$, *Ans.*

CASE II.

43. **When the divisor has one or more ciphers at the right.**

RULE.—I. *Cut off the ciphers from the right of the divisor, and the same number of figures from the right of the dividend.*

II. *Then divide the remaining figures of the dividend by the remaining figures of the divisor.*

III. *Prefix the remainder to the figures cut off from the right of the dividend, for the true remainder.*

EXAMPLE.

1. Divide 96523 by 2800.

$$\begin{array}{l} 28.00)965.23(34\frac{1323}{2800}, \text{ quotient.} \\ 84 \\ \overline{125} \\ \underline{112} \\ 13 \end{array}$$

CASE III.

44. **When the divisor is an aliquot part of 10, 100, 1000, etc.**

RULE.—*Divide the dividend by the number of which the divisor is an aliquot part, by pointing off figures from the right; then take as many times the quotient as the divisor is contained in that number.*

EXAMPLES.

1. Divide 8675 by 25.

$100 \div 25 = 4.$

86.75	Pointing off two figures.
4	Multiplying by 4.
347.00	347, *Ans.*

2. Divide 17892 by $62\frac{1}{2}$.

$$100 \div 62\tfrac{1}{2} = \tfrac{8}{5}.$$

$$\overline{178.92 \times 8} \div 5 = \qquad 286.272, \textit{ Ans.}$$

3. Divide 1868 by $16\frac{2}{2}$
4. " 98723 " $333\frac{1}{3}$
5. " 758 " $37\frac{1}{2}$
6. Divide 1890 by $2\frac{1}{2}$
7. " 30396 " $87\frac{1}{2}$
8. " 7775 " $12\frac{1}{2}$

Case IV.

45. When a part of the divisor is an aliquot part of 10, 100, etc.

Rule.—*Multiply both dividend and divisor by that number which will change the divisor to a number ending in a cipher or ciphers, and then divide.*

Examples.

1. Divide 6852 by $233\frac{1}{3}$.

$$\begin{array}{r} 233\tfrac{1}{3})6582 \\ 3 \qquad 3 \\ \hline 7.00)197.46 \\ \hline 28\tfrac{146}{700}, \textit{ Ans.} \end{array}$$

2. Divide 42383 by 8125.

$$\begin{array}{l} \quad 8125) \quad 42386 \\ \qquad\quad 8 \qquad\quad 8 \\ \hline 65.000)339.088(5\tfrac{14088}{65000} = 5\tfrac{1761}{8125}, \textit{ Ans.} \\ \qquad\qquad 325 \\ \qquad\qquad \overline{14} \end{array}$$

3. Divide 9346 by 875
4. " 8451 " $162\frac{1}{2}$
5. Divide 3896 by $433\frac{1}{3}$
6. " 8179 " $166\frac{2}{3}$

7. How many bushels of corn can be bought for $4000, at $1.25 per bushel?

CASE V.

46. **When the divisor is a composite number.**

RULE.—I. *Divide the dividend by one of the factors, and the quotient thus found by another, and so on, until every factor has been used as a divisor.*

II. *Multiply each remainder by all the preceding divisors, except the one which produced it.*

III. *Add the several products and the first remainder: the amount will be the true remainder.*

EXAMPLES.

1. Divide 2835 by 25.

$$\begin{array}{ll} 25 = 5 \times 5. & 5)\underline{2835} \\ & \;\;5)\underline{567} \\ & \;\;\;\;113 \,..\, 2 \text{ Rem.} \quad 2 \times 5, \text{ True Rem.} \end{array}$$

Ans. $113\frac{10}{25}$.

In this example, 567 is only one-fifth of the dividend: consequently, after dividing by the second factor, the remainder is only one-fifth of the true remainder.

2. Divide 93652 by 56.

$$\begin{array}{llr} 56 = 8 \times 7. & 8)\underline{93652} & \\ & 7)\underline{11706} \;...\; 4, \text{1st Rem.} & 4 \\ & \;\;\;1672 \;...\; 2, \text{2d Rem.} \quad 2 \times 8 = & \underline{16} \\ & \text{True Rem.} & 20 \end{array}$$

Ans. $1672\frac{20}{56}$.

Here, 4 is a part of the true remainder, and 2 is $\frac{1}{8}$ of another part. Hence, $4 + \overline{2 \times 8} = 20$, the true remainder. And so for any number of divisors; as each remainder is divided by the preceding divisor or divisors, so, to produce the true remainder, we must multiply by the same.

ORDINARY METHOD, CONTRACTED.

47. The ordinary process of Long Division may be contracted by subtracting the product of the divisor by each quotient-figure from the partial dividend, without writing it. This can easily be done, especially by the *form of addition*, according to the rule on page 14.

EXAMPLES.

1. Divide 42952 by 182.

182)42952(236, quotient.
65
109

2. Divide 37562 by 92.

92)37562($408\frac{26}{92}$, quotient.
7
26

3. Divide 1448 by 62
4. " 92756 " 125
5. " 16161 " 19
6. Divide 2000 by 16
7. " 3376 " 27
8. " 4219 " 86

PECULIAR METHODS.

48. Increasing the divisor diminishes the quotient, and diminishing it increases the quotient in like proportion. Hence,

To divide by a number which may be increased or diminished by an aliquot part of itself, to form a more convenient divisor.

RULE.—*Divide by the new divisor so formed, and then increase or diminish the quotient in the same proportion as the divisor has been increased or diminished.*

EXAMPLES.

1. Divide 4290 by 15.

$15 - \frac{1}{3}$ of itself $= 10$.

3)429.0 quotient by 10.
143 $\frac{1}{3}$ of itself subtracted.

286, *Ans.*

2. Divide 147150 by 45.

$45 + \frac{1}{9}$ of itself $= 50$.

$$\begin{array}{rl} 5.0)14715.0 & \\ \hline 9)2943 & \text{quotient by 50.} \\ 327 & \frac{1}{9} \text{ of itself added.} \\ \hline 3270, & \textit{Ans.} \end{array}$$

A remainder produced by the first divisor, is so many parts of the true divisor, and the second remainder is such a part of the true remainder as the second divisor is of the true divisor.

3. Divide 78360 by 75.

$75 + \frac{1}{3}$ of itself $= 100$.

$$\begin{array}{rl} 3)783.60 & \text{quotient by 100.} \\ 261 & \frac{1}{3} \text{ of the whole number added.} \\ \hline 1044\frac{60}{75}, & \textit{Ans.} \end{array}$$

Here, the remainder found by dividing by 100, is 60, which, as will be seen, is written in the quotient as so many *seventy-fifths* instead of *hundredths*. This is the true result; for the remainder, as well as the whole number, must be increased by $\frac{1}{3}$ of itself; and when so increased, it will be equal to as many seventy-fifths as it was hundredths: $\frac{60}{100} + \frac{20}{100} = \frac{80}{100}$, which reduced $= \frac{4}{5}$. The rule will hold good in all cases; the required increase will make the first remainder the numerator of a fraction whose denominator is the true divisor. Therefore it may be taken as such without change.

But, by dividing the decimal also by 3, and adding, we shall obtain the true quotient, with a decimal remainder. Thus,

$$\begin{array}{r} 3)783.60 \\ 261.20 \\ \hline 1044.80, \end{array}$$

true quotient, which is the same in value as $1044\frac{60}{75}$, as before shown.

4. Divide 278917 by 75.

$$\begin{array}{rlr} 3)2789.17, & \text{quotient by 100; 1st rem.} & 17 \\ 929.2, & \frac{1}{3} \text{ added; 2d rem. 2.} \quad 2 \times 25 = & 50 \\ \hline 3718\frac{67}{75} & \text{True remainder,} & 67 \end{array}$$

$75 \div 3 = 25$, and hence 2, the second remainder, is $\frac{1}{25}$ of the true amount. $\overline{2 \times 25} + 17 = 67$, is the true remainder.

Or, continuing the division by 3, we have

$3)2789.17$
$929.72\frac{1}{3}$

$3718.89\frac{1}{3}$, True quotient.

$\frac{89\frac{1}{3}}{100}$ reduced will be found to be equal to $\frac{67}{75}$, as before.

5. Divide 3864 by $7\frac{1}{2}$.

$7\frac{1}{2} + \frac{1}{3}$ of itself $= 10$.

$3)386.4$
128.8

515.2, *Ans.*

6. Divide 7552 by 36.

$36 + \frac{1}{9}$ of itself $= 40$.

$4.0)755.2$	Or, $4.0)755.2$	1st rem.	32
$9)188.8$	$9)188.32$		
$20.9\frac{7}{9}$	$20.\ 8$	2d rem. $8 \times 4 =$	32
$209.7\frac{7}{9}$	$209\frac{28}{36}$		64

In the latter solution, the remainder is greater than the divisor. $\frac{64}{36} = 1\frac{28}{36}$, which added to 208 gives the true quotient.

7. Divide 362892 by $87\frac{1}{2}$.

$87\frac{1}{2} + \frac{1}{7}$ of itself $= 100$.

$7)3628.92$
$518.41\frac{5}{7}$

$4147.33\frac{5}{7}$ *Ans.*

8. Divide 4756 by $66\frac{2}{3}$.

$2)47.56$
23.78

71.34, *Ans.*

49. To divide by any number under and near 100, 1000, etc.

Rule.—I. *Divide by a unit of the order next higher than the divisor, by cutting off figures from the right of the dividend.*

II. *Multiply the whole number of the quotient so obtained by the complement of the divisor, cut off the same number of figures as before, and write the result under the first quotient.*

III. *Proceed in like manner with the whole number of this quotient, and so on till no whole number remains to multiply.*

IV. *Add the several results together; the sum will be the quotient (the whole number being on the left of the point, and the remainder on the right), subject to the following changes, when required:*

V. *If any number has been carried from the remainder in adding, multiply this number also by the complement of the divisor, point off as above, and add the result to the quotient.*

VI. *If the remainder equal or exceed the divisor, reduce it to a whole or mixed number, and add the same to the quotient.*

EXAMPLES.

1. Divide 36714 by 99.

367.14, quotient by 100.
3.67, 367 × 1 ÷ 100.
3, 3 × 1 ÷ 100.
$370\frac{84}{99}$, *Ans.*

EXPLANATION.—36714 ÷ 100 = 367 and 14 remainder. Now, as 100 = 99 + 1, 99 is contained in the same number 367 times, with a remainder of $\overline{367 \times 1} + 14$; or the quotient may be written, $367 + \frac{367 + 14}{99} = 367\frac{14}{99} + \frac{367}{99}$. Therefore, $367\frac{14}{99}$ is taken as a partial quotient, and 367 remains to be divided. Proceeding as before, 367 ÷ 100 = 3.67; hence, 367 ÷ 99 = 3, with 3 + 67 remainder. Therefore, $3\frac{67}{99}$ is another part of the true quotient, while 3, divided in the same manner, gives $\frac{3}{99}$ for the final part of the remainder. Adding these partial quotients, we have $370\frac{84}{99}$, the true quotient.

2. Divide 1673458 by 996.

1673.458, quotient by 1000.
6.692, 16.73 × 4 ÷ 1000.
24, 6 × 4 ÷ 1000.
4, 1 (to be carried) × 4 ÷ 1000.
$1680\frac{178}{996}$

EXPLANATION.—Reasoning as in the last case, we continually multiply the whole number of the quotient obtained by dividing by 1000, by 4, the complement of the divisor, and place the products under the first. After the third division, no whole number remains; but, we observe that, in adding, 1 must be carried from the sum of the partial remainders; therefore, we multiply this 1 by 4, divide it by 1000, and place the quotient under the other remainders. The whole quotient is then found to be $1680\frac{178}{996}$.

3. Divide 64578 by 98.
4. Divide 36725 by 97.
5. Divide 19872 by 96.
6. Divide 87432567 by 997.
7. Divide 23476931 by 996.
8. Divide 9645395 by 995.

DIVISORS, FACTORS, AND MULTIPLES.

50. An **Exact Divisor,** or **Measure** of a number, is any whole number that will divide it without a remainder. Thus, 2, 3, 4, and 6 are exact divisors of 12.

51. The **Factors** of a number are the whole numbers which multiplied together will produce it. Thus, 3 and 4 are factors of 12.

52. Every factor of a number must be a divisor of it; hence, *divisors* and *factors* are convertible terms; a divisor is a factor, and *vice versa.*

53. A **Multiple** of a number is any number which is exactly divisible by it. Thus, 6, 9, and 12 are multiples of 3.

54. A **Prime Number** is a number which has no exact divisor, except itself, and 1; that is, which cannot be resolved into two or more integral factors; as 2, 5, 7, 9, 11, 13, etc.

55. A **Composite Number** is a number which has other exact divisors besides itself and 1; as 4, 8, 10, 12.

56. The **Prime Factors** of a number are the prime numbers which multiplied together produce that number. Thus, 2, 3, and 5 are the prime factors of 30.

57. A **Composite Factor** of a number is a factor which is a composite number. Thus, 15 is a composite factor of 30.

METHOD OF FACTORING NUMBERS.

58. A number is said to be resolved into its prime factors when it is separated into factors which are all prime numbers.

59. A factor is a divisor; therefore, it is obvious that the prime factors of a number are all of its exact prime divisors.

60. To find the prime factors of a composite number.

EXAMPLE.

What are the prime factors of 15015?

OPERATION.

5)15015

3)3003

7)1001

11)143

13

ANALYSIS.—By dividing by any exact prime divisor, we separate the given number into two factors, one of which at least (the divisor) is prime. If the quotient be also prime, it is the only other prime factor, and the work is ended; if not, it contains some exact divisor which is another of the prime factors of the original number. So, by continuing the division till a prime quotient is obtained, all the divisors and the quotient will constitute all the prime factors of the given number; 5 × 3 × 7 × 11 × 13 = 15015. Hence, the

RULE.—*Divide the given number successively by its exact prime divisors, till a prime quotient is obtained; the divisors and the final quotient will be its prime factors.*

61. There are no certain tests of the divisibility of numbers, except in a few cases, beyond which the divisors can be found only by trial.

A number is exactly divisible

By 2, if it is an even number.

By 3, if the sum of its digits is exactly divisible by 3.

By 4, if its two right-hand figures are ciphers, or express a number which is exactly divisible by 4.

By 5, if it terminates with 0 or 5.

By 6, if it is an even number of which 3 is an exact divisor.

By 8, if its three right-hand figures are ciphers, or express a number which is exactly divisible by 8.

By 9, if the sum of its digits is divisible by 9.

EXAMPLES FOR PRACTICE.

1. Find the prime factors of 31570.

2. Find the prime factors of 28665.

3. What are the prime factors of 2754?

4. Resolve 45045 into its prime factors.

5. Resolve 6120 into its prime factors.

6. Find the prime factors of 323. Of 429. Of 1761. Of 147. Of 133.

GREATEST COMMON DIVISOR.

62. A **Common Divisor,** or **Measure,** of two or more numbers, is any number that will divide each of them without a remainder.

63. The **Greatest Common Divisor,** or **Measure,** of two or more numbers, is the greatest number that will divide each of them without a remainder.

Numbers which have no common divisor are said to be *prime to each other.*

From these definitions, and the properties of factors and divisors, it follows that

1. A common divisor of two or more numbers, is any prime or composite factor common to all the numbers; and the greatest common divisor is a number which contains all of those factors, and no others. Hence,

2. *The greatest common divisor of two or more numbers is the product of all their common factors.*

To find the greatest common divisor of two or more numbers.

EXAMPLE.

What is the greatest common divisor of 1050, 3150, and 5250?

OPERATION.

50)1050 . .	3150 . .	5250
7)21 . .	63 . .	105
3)3 . .	9 . .	15
1	3	5

ANALYSIS.—We perceive that 50 is a common divisor; and, dividing by it, we obtain the quotients 21, 63, and 105. Now, as these quotients are not prime to each other, 50 is not the greatest common divisor, but only a factor of it. Therefore we must proceed further, to find another factor. 7 is a common divisor of the quotients, and the division gives a second set of quotients, which contain still another factor; hence, 50 × 7 is not the greatest common divisor. Finally, dividing by 3, we find the resulting quotients to be prime to each other; and, since there are no other common factors, $50 \times 7 \times 3 = 1050$, must be the greatest common divisor of the given numbers. Hence, the

RULE.—I. *Write the given numbers in a line, and divide them all by any common factor.*

II. *Divide the quotients thus obtained in like manner, and so continue to divide, till the resulting quotients are prime to each other; and the product of all the divisors will be the greatest common divisor sought.*

EXAMPLES FOR PRACTICE.

1. What is the greatest common divisor of 15, 20, 35, and 70?

2. What is the greatest common divisor of 105, 75, 60, and 30?

3. Find the greatest common divisor of 26, 104, 65, and 39.

4. Find the greatest common divisor of 57, 38, 114, and 171.

5. Required the greatest common divisor of 36, 96, 72, and 54.

6. Required the greatest common divisor of 46, 69, 102, and 230.

64. When the given numbers are large, and not readily factored, another method may be adopted, which depends upon the following principles:

1. *Every number which is a factor of another number, is also a factor of any multiple of that number.*

For, the product of two numbers must contain all the factors of both. For example, 3 is a factor of 12; it is also a factor of any number of times 12, as 24, 36, etc.

2. *A common divisor of two numbers, is also a common divisor of their* SUM *and* DIFFERENCE.

For, the sum or difference of two multiples of a number must be a multiple of that number. For example, 5 is a common divisor of 25 and 35; it is also a common divisor of 60, their sum, and of 10, their difference.

3. *The greatest common divisor of two numbers, is the greatest common divisor of the smaller and the remainder found by dividing the larger by the smaller.*

For, by division, a multiple of the divisor is subtracted from the dividend, and this multiple must contain every factor of the divisor; hence, the greatest factor of the divisor and the remainder, the other part of the dividend, must be the largest number which exactly measures both numbers. Thus, if 475 be divided by 15, the remainder will be 10; and 5, the greatest common divisor of 10 and 15, is the greatest common divisor of 15 and 475.

EXAMPLE 1. What is the greatest common divisor of 325 and 6275?

OPERATION.

```
325)6275(19
    325
    ----
    3025
    2925
    ----
     100)325(3
         300
         ---
          25)100(4
             100
```

ANALYSIS.—Since 325 must be a divisor of itself and of the largest multiple of itself found in 6275 (Prin. I), we divide by it. Now, if there were no remainder, 325 would be the greatest common divisor required. But, as there is a remainder, the greatest common divisor of the given numbers, is the greatest common divisor of the remainder and the divisor (Prin. III). Hence, we proceed as before, and divide 325 by 100, or the divisor by the remainder, and find a second remainder of 25; hence, 100 is not a divisor of 325; and, therefore, is not the greatest common divisor sought. Again, dividing the last divisor by the last remainder, we find that 25 is contained in 100 without a remainder; and since 25 is the largest number that will exactly divide itself, it is the greatest common divisor of itself and 100.

Now, since 25 is the greatest common divisor of 25 and 100, it is the greatest common divisor of 25 and 300 (a multiple of 100), and hence of 100 and 300 + 25, or 325. Therefore (Prin. III), it is the greatest common divisor of the given numbers. Hence, the following

RULE.—I. To find the greatest common divisor of two numbers, *divide the greater by the less, and if there be a remainder, divide the divisor by it, and continue dividing the last divisor by the last remainder, till nothing remains. The last divisor will be the greatest common divisor sought.*

II. To find the greatest common divisor of more than two numbers, *first find the greatest common divisor of two of them, then of this divisor and one of the other numbers, and so on, to the last. The last divisor will be the greatest common divisor required.*

EXAMPLE 2. Required the greatest common divisor of 836, 1650, and 2783.

OPERATION.

836)1650	22)2783
836	22
814)836	58
814	44
22)814	143
66	132
154	11)22
154	22 *Ans.* 11.

NOTE.—The quotients are omitted as unnecessary. The greatest common divisor of 836 and 1650 is 22; and of 23 and 2780, 11.

EXAMPLES FOR PRACTICE.

1. Find the greatest common divisor of 150, 246, and 114.

2. What is the greatest common divisor of 304, 703, 133, and 1653?

3. Find the greatest common divisor of 945, 1560, and 22683.

4. Find the greatest common divisor of 1426, 322, and 598.

5. What is the greatest common divisor of 11729, 7892, and 4958?

6. Find the greatest common divisor of 1177, 1391, and 1819.

CANCELLATION.

65. **Cancellation** is the process of rejecting equal factors from a dividend and its divisor. It is most frequently applied to shorten the work, when the continued product of several numbers is to be divided by the continued product of several other numbers, as in the multiplication of fractions.

Cancellation depends upon the following principles:

1. If a number be both multiplied and divided by the same number, the quotient will be the number multiplied.

2. If both divisor and dividend be divided by the same number, the quotient will not be changed.

If a factor of any number be canceled, the number will be divided by that factor. Thus, 21 is equal to 7 × 3; and canceling 7, or 3, is the same as dividing by 7 or 3; for 21 is the product of these two factors; and $21 \div 7 = 3$, or $21 \div 3 = 7$.

$$\frac{21}{7} = \frac{7 \times 3}{7} = \frac{\cancel{7} \times 3}{\cancel{7}} = 3.$$

NOTE.—A canceled factor is simply divided by itself, and not absolutely *stricken out:* therefore, the quotient 1, and not 0, takes its place, though it need not be written, except in the final result, when all the factors of the

divisor or dividend, or of both, have been canceled. When all the factors of both divisor and dividend are canceled the result is $\frac{1}{1} = 1$.

EXAMPLE.

Divide the product of 7, 8, 11, and 36, by the product of 2, 4, 12, and 14.

OPERATION.

$$\frac{\cancel{7} \times \cancel{8} \times 11 \times \overset{3}{\cancel{36}}}{\cancel{2} \times \cancel{4} \times \cancel{12} \times \underset{2}{\cancel{14}}} = 16\tfrac{1}{2} \textit{ Ans.}$$

Or,

$$\begin{array}{r|l} \cancel{2} & \cancel{7} \\ \cancel{4} & \cancel{8} \\ \cancel{12} & 11 \\ 2\ \cancel{14} & \cancel{36}\ 3 \end{array}$$

$$2)\underline{33}$$

$$16\tfrac{1}{2} \textit{ Ans.}$$

ANALYSIS.—Having first indicated the operations to be performed, we observe that 8 in the dividend is equal to 2 × 4 in the divisor; therefore, we cancel these numbers. Again, 12 is a factor of 36; and 36 ÷ 12 = 3, the other factor of 36; therefore, we cancel 12 and 36, and write the quotient above. Lastly, 7 is a factor of 14, and we cancel 7, and write 2, the other factor of 14, below. We then have the result $\frac{11 \times 3}{2} = 16\frac{1}{2}$, *Ans.* Hence the

RULE.—I. *Write the factors of the dividend and those of the divisor in form to express the required multiplications and division.*

II. *Cancel all the factors common to the dividend and divisor.*

III. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

1. What is the quotient of 9 × 18 × 36 × 72 divided by 3 × 6 × 12 × 24?

2. Divide 3 × 4 × 21 × 6 by 8 × 2 × 3 × 7.

3. How many times are 9 times 5 contained in 45 times 15?

4. Divide the product of 13, 26, 48, and 22, by the product of 39, 52, 44, and 11.

5. Divide $3 \times 6 \times 17 \times 8$ by $2 \times 12 \times 51 \times 2$.

6. Multiply 18 by 3, and divide the product by 3 times 9, multiply this quotient by 36 times 2 divided by 4 times 3.

7. How many bushels of corn, at $1.20 per bushel, must be given for 20 yards of cloth at $1.80 per yard?

8. Find the quotient of $\frac{24 \times 15 \times 7 \times 32 \times 27}{9 \times 16 \times 18 \times 42.}$.

9. Find the quotient of $\frac{9 \times 16 \times 24 \times 80}{160 \times 27 \times 48 \times 3}$.

10. Find the quotient of $\frac{6 \times 8 \times 4 \times 3 \times 5}{7 \times 18 \times 12 \times 9 \times 30}$.

11. Divide $16 \times 8 \times 3$ by $19 \times 4 \times 2 \times 2$.

12. Divide $11 \times 11 \times 13$ by $22 \times 33 \times 2$.

67. The process of cancellation may be applied to the terms of a proportion.

Take, for example, any proportion, as,

$$3 : 6 :: 12 : 24.$$

Any factor common to either ratio, or couplet, may be canceled. Thus, canceling 3 from the first and second terms, we have

$$1 : 2 :: 12 : 24.$$

And, canceling 12 from the third and fourth terms, the result is

$$1 : 2 :: 1 : 2.$$

Again, equal factors may be canceled from the *homologous* terms; that is, from the antecedents and consequents. Thus, canceling 3 from the first and third terms, we have

$$1 : 6 :: 4 : 24.$$

Or, canceling 6 from the second and fourth terms, the result is

$$3 : 1 :: 12 : 4.$$

NOTE.—For an explanation of the principles involved, and their application, see Proportion.

LEAST COMMON MULTIPLE.

68. A **Common Multiple** of two or more numbers, is any number which is exactly divisible by each of them. Thus, 32 is a common multiple of 2, 4, 8, and 16.

69. The **Least Common Multiple** of two or more numbers, is the least number which is exactly divisible by each of them. Thus, 12 is the least common multiple of 2, 3, and 4.

70. Since a product is a multiple of all its factors, it is plain that *a common multiple of two or more numbers may be found by multiplying them together;* and if the numbers are prime to each other, this product will be their least common multiple.

GENERAL PRINCIPLES.

1. A multiple of a number contains all the prime factors of that number.

2. A common multiple of two or more numbers contains all the prime factors of each of them. Hence,

3. The least common multiple of two or more numbers, is the least number which contains all of the prime factors of each of them.

71. To find the least common multiple of two or more numbers.

FIRST METHOD.—*By Factoring.*

EXAMPLE. Find the least common multiple of 21, 18, 12, and 24.

SOLUTION.

$21 = 3 \times 7$

$18 = 3 \times 3 \times 2$

$12 = 3 \times 2 \times 2$

$24 = 3 \times 2 \times 2 \times 2$

$3 \times 7 \times 3 \times 2 \times 2 \times 2 = 504$, *Ans.*

ANALYSIS.—The required multiple must contain all the prime factors of each of the numbers taken separately, and no others. Therefore, it must contain all their *different* prime factors, each repeated as many times as it is repeated in any of the given numbers. The different prime factors are, 3, 7, and 2. The greatest number of times that these occur, respectively, in any of the numbers, is 3 twice, 7 once, and 2 three times. Hence, the least common multiple will be composed of two 3's, one 7, and three 2's.

$$3 \times 3 \times 7 \times 2 \times 2 \times 2 = 504, \textit{ Ans.}$$

72. Another method of arriving at the same result, is to multiply any one of the numbers (the largest is most convenient) by such prime factors of the other numbers as are not found in that number. Thus,

$$24 \times 3 \times 7 = 504, \textit{ Ans.}$$

73. SECOND METHOD.—*By Division.*

EXAMPLE. What is the least common multiple of 8, 16, 18, and 24?

SOLUTION.

$$\begin{array}{r} 2)\underline{\cancel{8} \;..\; 16 \;..\; 18 \;..\; 24} \\ 2)\underline{8 \;..\; 9 \;..\; 12} \\ 2)\underline{4 \;..\; 9 \;..\; 6} \\ 3)\underline{2 \;..\; 9 \;..\; 3} \\ 2 \;..\; 3 \;..\; 1 \end{array}$$

$$2 \times 2 \times 2 \times 3 \times 2 \times 3 = 144, \textit{ Ans.}$$

ANALYSIS.—Having written the numbers in a horizontal line, we cancel 8, because it is contained in one of the other numbers. Now, since the remaining numbers are not prime to each other, two or more of them must contain a common prime factor, which must, of course, be a factor of the required multiple. 2 is such a factor; and dividing by it, we write the quotients underneath. For the same reason, we divide the resulting quotients by 2, and bring down the undivided number 9. In like manner we continue the division, till the quotients and undivided numbers are prime to each other. These, with the divisors, constitute the prime factors of the least common multiple of the given numbers.

THIRD METHOD.—*By Cancellation.*

74. From the foregoing illustrations and explanations, we deduce the following

PROPOSITION.—*The least common multiple of two or more numbers, is the product of any one of the numbers and such prime factors of the other numbers as are not contained in that number.*

Hence, the process of cancellation may be applied.

Example. Find the least common multiple of 12, 18, 24, 27, and 36.

SOLUTION.

$$\cancel{12}\;..\;\cancel{18}\;..\;\cancel{24}\;..\;\cancel{27}\;..\;36$$

$$2 \times 3 \times 36 = 216, \textit{Ans.}$$

Analysis.—First, cancel 12 and 18, because they are factors of 36. Then, cancel 12, the greatest common factor of 24 and 36, from 24, and write the remaining factor 2, underneath. Lastly, cancel 9, the greatest common factor of 27 and 36, from 27, and write the remaining factor, 3, underneath. The product of these remaining factors and the undivided number, 36, is the least common multiple required.

75. If any of the given numbers can be divided into two or more factors, one of which is contained in a second number, another in a third, and so on, the whole number so divisible must be canceled.

Example. What is the least common multiple of 21, 13, 18, 14, and 63?

SOLUTION.

$$\cancel{21}\;..\;13\;..\;\cancel{18}\;..\;\cancel{14}\;..\;63$$

$$13 \times 2 \times 63 = 1638, \textit{Ans.}$$

Analysis.—Cancel 21, because it is a factor of 63. Bring down 13, because it is a prime number, and is not a factor of any of the other numbers. Now, 18 is composed of two factors, 9 and 2, one of which is cantained in 63, and the other in 14; therefore, cancel 18. Finally, cancel 7, the greatest common factor of 14 and 63, from either number. The remaining factors, $13 \times 2 \times 63 = 1638$, *Ans.*

Note.—Care must be taken, in this process, to avoid canceling a factor too many times. For example, take the numbers 9, 24, and 48. 9 may be canceled, for the reason that one of its factors, 3 and 3, is found in 24, and the other in 48. But, as 3 occurs only once as a factor in either of these numbers, it must not be canceled from either; and the largest other factor which can be canceled is 8, not 12, or 6.

76. From the foregoing examples and illustrations, we derive the following

Rule.—I. *Resolve the given numbers into their prime factors, or ascertain their separate prime factors by any of the foregoing methods.*

II. *Multiply together all their separate prime factors, and the product will be the least common multiple required.*

EXAMPLES FOR PRACTICE.

1. Find the least common multiple of 11, 22, 15, 10, 30, and 35.

2. Find the least common multiple of the denominators of the following fractions: $\frac{1}{14}$, $\frac{1}{7}$, $\frac{1}{19}$, $\frac{1}{57}$, $\frac{1}{42}$, and $\frac{1}{18}$.

3. Required the least common multiple of 27, 13, 39, 7, 6, and 4.

4. Required the least common multiple of 8, 16, 45, 50, and 5.

5. Find the least common multiple of 19, 57, 3, and 5.

6. Find the least common multiple of 16, 18, 34, 17, and 54
Ans. 7344.

7. What is the least common multiple of 3, 7, 18, 9, and 27?

8. What is the least common multiple of 25, 14, 17, 51, and 99?

9. What is the least common multiple of 9, 17, 6, and 27?

10. Find the least common multiple of 13, 52, and 39.

11. Find the least common multiple of 13, 17, and 31.

12. Three cog-wheels run together; the first, having 25 cogs, drives the second, which has 120 cogs, and the second drives the third, which has 360 cogs: how many revolutions from any given position must each wheel make before they all will be in the same position as when they started?
Ans. The first, 72 revolutions; the second, 15; and the third, 5.

13. A can walk around a certain island in 2 days, B in 3 days, and C in 4 days. Suppose they all start at the same time, from one point, and travel at these rates till they all arrive together at the starting-point: how long will they travel?
Ans. 12 days.

COMMON FRACTIONS.

77. A **Fraction** is a part of a thing, number, or quantity.

Fractions arise from division. If a thing be divided into two equal parts, each of these parts is called one *half;* two of them are called two *halves.* If it be divided into three equal parts, the parts are called *thirds;* if into four equal parts, they are called *fourths,* and so on.

78. A **Fractional Unit**, or the **Unit of a Fraction**, is one of the equal parts into which a thing, number, or quantity is divided.

ILLUSTRATION.—If an apple be cut into any number of equal parts, one of those parts is a fraction of the entire apple, but in itself a *fractional unit*, or a *whole thing*,—a piece or part. One third of a yard is a fractional unit of a yard, but a *whole foot*. A square rod, viewed as a certain quantity of surface, is an integral unit, but regarded as a part of an acre, it is a fractional unit.

79. A **Written Fraction** is an expression denoting one or more fractional units.

80. Fractions are of the *same unit* when they express fractional units of the same kind or value, whether abstract or concrete.

81. Fractions are of two kinds, *Common* and *Decimal.* They are distinguished by their forms: thus, $\frac{1}{2}$ is a common fraction; and .5 $(=\frac{5}{10}=\frac{1}{2})$ is a decimal fraction.

82. A Simple Common Fraction is expressed by two numbers, written one above the other, with a line between them. Thus, one half is written $\frac{1}{2}$; three fourths, $\frac{3}{4}$.

83. The number above the line is called the *Numerator,* because it numbers the fractional units. It shows how many are taken or considered.

84. The number below the line is called the *Denominator,* because it denominates, or names the fractional units. It shows their size, or how many make an integral unit.

85. The **Terms** of a fraction are the numerator and denominator, taken together.

86. A **Proper Fraction** is a fraction whose numerator is less than its denominator. Its value is less than a unit; as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$.

87. An **Improper Fraction** is a fraction whose numerator equals or exceeds its denominator. Its value is equal to, or greater than, a unit; as, $\frac{7}{4}$, $\frac{9}{5}$, $\frac{2}{2}$, $\frac{13}{8}$.

88. A **Mixed Number** consists of a whole number and a fraction; as, $15\frac{3}{4}$, $4\frac{1}{2}$.

89. A **Simple Fraction** is a fraction whose numerator and denominator are both whole or simple numbers; as, $\frac{5}{8}$, $\frac{13}{25}$.

90. A **Compound Fraction** is a fraction of a fraction, or two or more fractions connected by the word *of*, or with the sign of multiplication between them; as, $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{5}{6}$; $\frac{17}{25} \times \frac{1}{5} \times \frac{4}{11}$.

91. A **Complex Fraction** is a fraction which has a fraction in one or both of its terms; as, $\frac{4\frac{1}{5}}{6\frac{1}{3}}$, $\frac{\frac{1}{7}}{\frac{8}{9}}$, $\frac{7\frac{1}{2}}{14}$, $\frac{12}{3\frac{1}{2}}$.

92. The **Reciprocal of a Fraction** is the fraction inverted; thus, $\frac{5}{3}$ is the reciprocal of $\frac{3}{5}$.

93. The Reciprocal of a whole number is 1 divided by the number. Thus, the reciprocal of 25 is $\frac{1}{25}$.

General Principles.

94. Fractions are most commonly regarded as indicating division *which cannot be performed.* It will relieve the whole subject of very many perplexities to view them as denoting division *actually performed,* according to the fact. For example, the quotient of 5 divided by 9 is $\frac{5}{9}$, which truly expresses $\frac{1}{9}$ of 5, for $5 = 45$ ninths, and 45 ninths $\div 9 = 5$ ninths.

A fraction simply expresses a number of fractional units which are the result of the division of an integral number. The numerator shows their number, and the denominator their denominational value.

Therefore, a fraction is multiplied, or divided, like a simple number, by multiplying or dividing the numerator, because this multiplies or divides the units, without changing their magnitude. 2 times 1 third are 2 thirds; 3 times 4 sevenths are 12 sevenths One half of 8 ninths is 4 ninths.

But, a fraction may also be multiplied, or divided, by performing the reverse operation on the denominator. Dividing the denominator multiplies the fraction by increasing the denominational value of its units, and multiplying the denominator divides the fraction by diminishing the denominational value of its units, without changing their number. 1 half divided by 2 is 1 fourth, because $\frac{1}{4}$ is one half the magnitude of $\frac{1}{2}$. 5 sixths multiplied by 2 is 5 thirds, for thirds are twice the value of sixths.

These principles may be enumerated as follows :—

I. *Multiplying the numerator multiplies the fraction, and dividing the numerator divides the fraction.*

II. *Multiplying the denominator divides the fraction, and dividing the denominator multiplies the fraction.*

III. *Multiplying or dividing both terms by the same number, does not change the value of the fraction.*

Exercises in Writing Fractions.

95. Express the following fractions in figures:

1. One half.
2. Two thirds.
3. Four sevenths.
4. Nineteen twenty-firsts.
5. Eighteen thirty-ninths.
6. Two hundred and twenty five four-hundredths.
7. One thousand two hundred and seventeen ten-thousand-four-hundred-and thirteenths.
8. Nine hundred and seventy eight millionths.
9. Write 16 in fractional form.

96. Write in figures the following mixed numbers :—

1. Seventy five, and one half.

2. Eighteen, and seven elevenths.

3. Three hundred, and twenty-four twenty-fifths.

4. One thousand four hundred and twenty, and forty forty-firsts.

5. Twenty-seven hundred, and twenty-seven hundredths.

6. One million, and one millionth.

7. Three hundred and fourteen, and eighteen ten-thousandths.

8. Seventeen, and three hundred-thousandths.

97. Write as improper fractions :—

1. One-fourth of seven. *Ans.* $\frac{7}{4}$.

2. Two-thirds of eighteen. *Ans.* $\frac{36}{3} = \frac{12}{1}$.

3. One-eighth of seventeen.

4. Three-fourths of twenty-four.

5. $\frac{1}{2}$ of 8. $\frac{2}{3}$ of 12. $\frac{3}{5}$ of 20. $\frac{1}{6}$ of 37.

REDUCTION OF FRACTIONS.

98. Reduction of Fractions is the process of changing them to equivalent fractions of different denominations. Thus, $\frac{4}{8}$ may be changed to $\frac{2}{4}$, or $\frac{1}{2}$, without altering its value; or, the denomination may be raised, as $\frac{1}{3}$ to $\frac{2}{6}$, $\frac{4}{12}$, etc.

GENERAL PRINCIPLE.—*Multiplying or dividing both terms of a fraction by the same number does not alter its value.*

CASE I.

99. To reduce a fraction to its lowest terms.

A fraction is in its lowest terms when its numerator and denominator are prime to each other; that is, when they are the least whole numbers which will express its value.

EXAMPLE.

Reduce $\frac{18}{54}$ to its lowest terms.

OPERATION.

$6)\frac{18}{54} = \frac{3}{9}$

$3)\ \frac{3}{9} = \frac{1}{3}$, *Ans.*

Or,

$18)\frac{18}{54} = \frac{1}{3}$, *Ans.*

ANALYSIS.—The division of both terms of a fraction by the same number does not change its value. Hence, by the first method, we divide both terms of the given fraction by 6, a common factor, and obtain $\frac{3}{9}$, and then both terms of this result by 3, and get $\frac{1}{3}$, which is in its lowest terms, for the numerator and denominator are prime to each other.

By the second method we obtain the same result by dividing both terms by 18, their greatest common divisor. Hence the following

RULE.—*Divide the numerator and denominator successively by any of their common factors:*

Or, *divide both terms by their greatest common divisor.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{20}{35}$ to its lowest terms. *Ans.* $\frac{4}{7}$.
2. Reduce $\frac{85}{145}$ to its lowest terms.
3. Reduce $\frac{12}{144}$ to its lowest terms.
4. Reduce $\frac{756}{2940}$ to its lowest terms. *Ans.* $\frac{9}{35}$.
5. Reduce $\frac{24}{472}$ to its lowest terms. *Ans.* $\frac{3}{59}$.

Reduce the following fractions to their lowest terms:—

1.	$\frac{9}{12}$	4.	$\frac{57}{171}$	7.	$\frac{258}{282}$	10.	$\frac{831}{1143}$
2.	$\frac{14}{24}$	5.	$\frac{123}{369}$	8.	$\frac{57}{111}$	11.	$\frac{216}{432}$
3.	$\frac{17}{68}$	6.	$\frac{184}{261}$	9.	$\frac{92}{96}$	12.	$\frac{102}{486}$

CASE II.

100. **To reduce a fraction to an equivalent fraction of any given denomination.**

It has been shown that fractions are reduced to lower terms by *division;* and, conversely, they may be raised to higher terms by *multiplication.*

EXAMPLE.

Change $\frac{1}{4}$ to its equivalent in 12ths.

OPERATION.

$$12 \div 4 = 3$$

$$\frac{1 \times 3}{4 \times 3} = \frac{3}{12}, \textit{ Ans.}$$

ANALYSIS.—Since both terms must be multiplied by that number which will make the denominator 12, we divide 12 by the denominator of the given fraction, to find the required multiplier. $12 \div 4 = 3$, and multiplying both terms of $\frac{1}{4}$ by 3, we obtain $\frac{3}{12}$, *Ans.* Hence the

RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

1. Change $\frac{7}{8}$ to a fraction whose denominator is 40.

2. Reduce $\frac{3}{4}$ to 36ths.

3. Reduce $\frac{80}{100}$ to 75ths. *Ans.* $\frac{60}{75}$.

NOTE.—Under the rule, the multiplier in this case is $\frac{75}{100} = \frac{3}{4}$. See Example 3, page 41. We may first reduce the fraction to its lowest terms, $\frac{4}{5}$, or to $\frac{20}{25}$, and then raise the denomination by multiplying both terms by 15, or by 3.

4. Reduce $\frac{8}{16}$ to its equivalent in 68ths. *Ans*, $\frac{34}{68}$.

5. Reduce $\frac{89\frac{1}{3}}{100}$ to 75ths. *Ans.* $\frac{67}{75}$.

101. By the same rule, with the word *numerator* substituted for *denominator*, a fraction may be changed to another of equal value, having a given numerator.

EXAMPLES.

1. Reduce $\frac{3}{7}$ to an equivalent fraction having 15 for a numerator. *Ans.* $\frac{15}{35}$.

2. Reduce $\frac{8}{9}$ to a fraction whose numerator is 6. *Ans.* $\frac{6}{6\frac{3}{4}}$.

3. Change $3\frac{3}{5}$ to a fraction whose numerator is 9. *Ans.* $\frac{9}{2\frac{1}{2}}$.

CASE III.

102. **To reduce a mixed number to an improper fraction.**

EXAMPLE. Reduce $13\frac{5}{9}$ to an improper fraction.

OPERATION.

$13\frac{5}{9}$

9

$\frac{122}{9}$, *Ans.*

ANALYSIS.—Since there are 9 ninths in 1, there are 13 times 9 ninths in 13; $13 \times 9 = 117$, and 117 ninths + 5 ninths = 122 ninths = $\frac{122}{9}$, *Ans.* Hence the

RULE.—*Multiply the whole number by the denominator of the fraction, to the product add the numerator, and place the sum over the denominator.*

NOTE.—To reduce a whole number to a fraction of a given denomination, multiply the whole number by the required denominator, and write the product over the denominator. Thus, 7 reduced to thirds is $\frac{21}{3}$.

EXAMPLES FOR PRACTICE.

1. Reduce $23\frac{4}{7}$ to an improper fraction.
2. Reduce $162\frac{3}{4}$ to fourths.
3. In $723\frac{1}{14}$, how many fourteenths?
4. Reduce $126\frac{3}{25}$ to an improper fraction.
5. Reduce $450\frac{4}{7}$ to an improper fraction.
6. Reduce 27 to a fraction having 4 for its denominator.
7. How many fifths in 14? In 47? In 132?

Reduce the following mixed numbers to improper fractions:—

1. $17\frac{5}{8}$
2. $29\frac{5}{6}$
3. $11\frac{2}{11}$
4. $131\frac{4}{9}$
5. $721\frac{2}{15}$
6. $347\frac{4}{17}$
7. $99\frac{1}{9}$
8. $222\frac{1}{3}$
9. $147\frac{3}{5}$
10. $398\frac{5}{12}$
11. $719\frac{2}{5}$
12. $26\frac{4}{31}$

CASE IV.

103. **To reduce an improper fraction to a whole or mixed number.**

EXAMPLE. Reduce $\frac{57}{16}$ to a mixed number.

OPERATION.

$\frac{57}{16} = 57 \div 16 = 3\frac{9}{16}$, *Ans.*

ANALYSIS.—The numerator denotes the number of fractional units, and the denominator shows that 16 of these units make an integral unit. Therefore the value of the fraction in integers is found by dividing 57 by 16. $57 \div 16 = 3\frac{9}{16}$.

RULE.—*Divide the numerator by the denominator.*

NOTE.—When there is a remainder, write it over the denominator, at the right of the integral part of the quotient, as a fraction. In general, first reduce this fraction to its lowest terms.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{148}{12}$ to a whole or mixed number. *Ans.* $12\frac{1}{3}$.

2. Reduce $\frac{1067}{11}$ to a whole or mixed number. *Ans.* 97.

3. Find the integral value of $\frac{1150}{9}$.

4. How many units are there in $\frac{3600}{75}$?

5. Reduce $\frac{65}{65}$ to a whole number.

6. If 465 dollars be divided equally among 17 persons, how much will each receive?

Reduce the following fractions to whole or mixed numbers:—

1. $\frac{42}{14}$
2. $\frac{372}{21}$
3. $\frac{288}{99}$
4. $\frac{1000}{1000}$
5. $\frac{25}{5}$
6. $\frac{114}{19}$
7. $\frac{4167}{13}$
8. $\frac{3762}{1000}$
9. $\frac{1870}{25}$
10. $\frac{1927}{14}$
11. $\frac{376}{211}$
12. $\frac{7342}{195}$

CASE V.

104. **To reduce fractions to a common denomination.**

Several fractions have a common denominator, or are of the same denomination, when their denominators are alike.

A common denominator of two or more fractions may be any multiple of all their denominators. The continued product of the denominators must be a multiple of them (Art. 70).

EXAMPLE.

Reduce $\frac{5}{9}$, $\frac{4}{7}$, and $\frac{2}{3}$ to the same denomination.

OPERATION.—*First Method.*

$9 \times 7 \times 3 = 189$, common denominator.
$\frac{5}{9}$ of $189 = 105$. $\frac{5}{9} = \frac{105}{189}$
$\frac{4}{7}$ of $189 = 108$. $\frac{4}{7} = \frac{108}{189}$
$\frac{2}{3}$ of $189 = 126$. $\frac{2}{3} = \frac{126}{189}$

ANALYSIS. — Having found the continued product of the denominators to be 189, we take this as the common denominator. Each fraction is then reduced to 189ths by taking for its numerator such part of 189 as the fraction is of 1, or by dividing by the denominator and multiplying by the numerator. Thus, to obtain a new numerator for $\frac{5}{9}$, we divide 189 by 9 and multiply the quotient 5. The result, 105, placed over the common denominator, gives a fraction which is equal to $\frac{5}{9}$; for, $\frac{5}{9} = \frac{5}{9}$ of $1 = \frac{5}{9}$ of $\frac{189}{189} = \frac{105}{189}$.

Second Method.

$9 \times 7 \times 3 = 189$, common denominator.
$5 \times 7 \times 3 = 105$, new numerator of $\frac{5}{9}$.
$4 \times 9 \times 3 = 108$, new numerator of $\frac{4}{7}$.
$2 \times 7 \times 9 = 126$, new numerator of $\frac{2}{3}$. *Ans.* $\frac{105}{189}$, $\frac{108}{189}$, $\frac{126}{189}$.

ANALYSIS.—By this method we multiply each numerator by all the denominators, except its own, and write the results, respectively, over the common denominator. And, since the product found by multiplying each denominator by all the others is the common denominator, it follows that the terms of each fraction are both multiplied by the same number; hence its value is not changed. The process may be illustrated as follows: $\frac{5 \times 7 \times 3}{9 \times 7 \times 3} = \frac{105}{189}$; $\frac{4 \times 9 \times 3}{7 \times 9 \times 3} = \frac{108}{189}$; $\frac{2 \times 7 \times 9}{3 \times 7 \times 9} = \frac{126}{189}$. Hence we derive the following

RULE.—*Multiply all the denominators together for a common denominator; and the numerator of each fraction by all the denominators except its own for the new numerators.*

Or, *take such a part of the common denominator for the numerator of each fraction as the fraction is of a unit.*

NOTES.—1. Fractions not in their lowest terms, may first be reduced, to simplify the work.

2. Compound and complex fractions should first be reduced to simple fractions.

EXAMPLES FOR PRACTICE

1. Reduce $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{8}$ to the same denomination. *Ans.* $\frac{24}{144}$, $\frac{48}{144}$, $\frac{18}{144}$.

2. Reduce $\frac{4}{11}$, $\frac{7}{16}$, and $\frac{3}{7}$ to the same denomination.

3. Reduce $\frac{2}{7}$, $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{2}{10}$ to the same denomination.

4. Reduce $\frac{3}{11}$, $\frac{4}{7}$, and $\frac{9}{14}$ to the same denomination.

5. Reduce $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{9}{12}$ to the same denomination.

CASE VI.

105. To reduce fractions to their least common denomination.

The Least Common Denominator of two or more fractions, is the least common multiple of their denominators.

EXAMPLE. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to their least common denomination.

OPERATION.

$$\begin{array}{cccr} \cancel{3} & \cancel{4} & \cancel{6} & 8 \\ & & & 3 \\ \hline & & & 24 \end{array}$$

$$\frac{2}{3} = \frac{16}{24}$$
$$\frac{3}{4} = \frac{18}{24}$$
$$\frac{5}{6} = \frac{20}{24}$$
$$\frac{7}{8} = \frac{21}{24}$$

ANALYSIS.—The least common multiple of the denominators, which we find to be 24, is the least common denominator. $\frac{2}{3}$ of $24 = 16$, which written over 24 gives $\frac{16}{24} = \frac{2}{3}$. In the same manner we find the value of each of the other fractions in 24ths, each of the new numerators being such a part of 24 as the fraction is of a unit.

By this process the values of the fractions are not changed, for the terms of each are multiplied by the same number. Hence the

RULE.—I. *Find the least common multiple of the denominators for the least common denominator.*

II. *Take such part of this common denominator for the numerator of each fraction as the fraction is of a unit.*

NOTE.—All the fractions should first be reduced to their lowest terms.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{5}{8}$, $\frac{4}{5}$, $\frac{2}{14}$, and $\frac{3}{10}$ to their least common denomination. *Ans.* $\frac{175}{280}$, $\frac{224}{280}$, $\frac{40}{280}$, $\frac{84}{280}$.

2. Reduce $2\frac{1}{2}$, $4\frac{1}{3}$, and $2\frac{3}{4}$ to their least common denomination.

Ans. $2\frac{6}{12}$, $4\frac{4}{12}$, $2\frac{9}{12}$.

3. Reduce $\frac{2}{15}$, $\frac{3}{5}$, $\frac{4}{30}$, and $\frac{19}{45}$ to their least common denomination.

4. Reduce $\frac{3}{32}$, $\frac{2}{16}$, $\frac{3}{8}$, and $\frac{1}{4}$ to their least common denomination.

5. Reduce $\frac{7}{8}$, $\frac{4}{5}$, $\frac{3}{20}$, and $\frac{3}{4}$ to their least common denomination.

6. Reduce $\frac{3}{7}$, $\frac{9}{14}$, $\frac{1}{3}$, and $\frac{11}{21}$ to their least common denomination.

7. Reduce $\frac{21}{36}$, $\frac{4}{9}$, $\frac{3}{4}$, and $\frac{19}{12}$ to their least common denomination.

Reduce the fractions in the following examples to equivalent fractions having the least common denominator:—

1.	$\frac{7}{8}$, $\frac{5}{9}$, $\frac{3}{18}$, $\frac{4}{5}$	5.	$\frac{2}{5}$, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{7}{10}$
2.	$3\frac{5}{6}$, $14\frac{2}{3}$, $19\frac{1}{2}$	6.	$\frac{5}{11}$, $\frac{3}{13}$, $\frac{4}{7}$, $\frac{8}{15}$
3.	$18\frac{7}{10}$, $200\frac{1}{4}$, $\frac{3}{20}$	7.	$\frac{50}{60}$, $\frac{70}{40}$, $\frac{30}{80}$, $\frac{10}{20}$
4.	$\frac{3}{7}$, $\frac{4}{9}$, $\frac{5}{6}$, $\frac{1}{13}$	8.	$\frac{3}{15}$, $\frac{1}{5}$, $\frac{6}{30}$, $\frac{75}{375}$

CASE VII.

106. To reduce fractions to a common denomination by inspection.

From the preceding cases and analyses we derive the following

RULE.—*Multiply or divide both terms of the given fractions, respectively, by any number that will make their denominators alike.*

EXAMPLE. Reduce $\frac{7}{12}$ and $\frac{4}{15}$ to the same denomination.

OPERATION.

$$\frac{7 \times 5}{12 \times 5} = \frac{35}{60}$$

$$\frac{4 \times 4}{15 \times 4} = \frac{16}{60}$$

ANALYSIS.—By inspecting the denominators we see that $5 \times 12 = 60$, and $15 \times 4 = 60$. Therefore, by multiplying both terms of $\frac{7}{12}$ by 5, and both terms of $\frac{4}{15}$ by 4, the fractions are changed to the same denomination.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{14}{50}$ and $\frac{16}{75}$ to a common denomination.
2. Reduce $\frac{7}{8}$ and $\frac{1}{6}$ to the same denomination.
3. Reduce $\frac{9}{12}$, $\frac{3}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ to their least common denomination.
4. Reduce $\frac{6}{16}$, $\frac{6}{24}$, and $\frac{4}{32}$ to their equivalents in 8ths.
5. Reduce $\frac{4}{12}$, $\frac{1}{33}$, and $\frac{10}{66}$ to the denomination of 33ds.
Ans. $\frac{11}{33}$, $\frac{1}{33}$ $\frac{5}{33}$.

ADDITION OF FRACTIONS.

107. The **Addition of Fractions** is the process of finding the sum of two or more fractions.

Fractions can be added only when they are of the same denomination and the same unit.

When several *abstract* fractions have a common denominator, their numerators express fractional units of the same value, and the operation of adding them is like the addition of simple numbers. Thus, if it be required to find the sum of $\frac{3}{4}$ and $\frac{2}{4}$, the process may be represented as follows:—

$$\begin{array}{r} 3 \text{ fourths,} \\ +\ 2 \text{ fourths,} \\ \hline =\ 5 \text{ fourths.} \end{array}$$

It is just as evident that 3 fourths and 2 fourths are 5 fourths, as that 3 horses and 2 horses are 5 horses. We add the *fractional units* precisely like simple numbers.

When the denominators are unlike, the fractions must first be reduced to the same denomination. For example, $\frac{3}{4}$ and $\frac{7}{8}$ cannot be added in their present form, for fourths and eighths are not like fractional units. But, $\frac{3}{4} = \frac{6}{8}$ and $\frac{6}{8} + \frac{7}{8} = \frac{13}{8} = 1\frac{5}{8}$.

Concrete fractions must not only be reduced to the same denomination, but to the same unit. For example, $\frac{3}{4}$ of a yard added to $\frac{2}{4}$ of a foot, is neither $\frac{5}{4}$ of a yard nor $\frac{5}{4}$ of a foot. But $\frac{3}{4}$ of a yard is equal to $\frac{9}{4}$ of a foot; and $\frac{9}{4} + \frac{2}{4} = \frac{11}{4} = 2\frac{3}{4}$ feet.

CASE I.

108. To add fractions.

EXAMPLE 1. What is the sum of $\frac{1}{8}$, $\frac{3}{8}$, and $\frac{5}{8}$?

OPERATION.

$$\frac{1}{8}+\frac{3}{8}+\frac{5}{8}=\frac{1+3+5}{8}=\frac{9}{8}=1\tfrac{1}{8}, \textit{ Ans.}$$

ANALYSIS.—Since the denominators are alike, the numerators express fractional units of the same value. Therefore, we add the numerators like simple numbers, and write their sum over the common denominator.

EXAMPLE 2. Add $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{3}$, and $\frac{7}{8}$.

OPERATION.

$$\frac{3}{4}+\frac{5}{6}+\frac{2}{3}+\frac{7}{8}=\frac{18}{24}+\frac{20}{24}+\frac{16}{24}+\frac{21}{24}=\frac{75}{24}=3\tfrac{3}{24}=3\tfrac{1}{8}, \textit{ Ans.}$$

ANALYSIS.—In this example the denominators are unlike, and hence the fractions cannot be added in their original form. We, therefore, first reduce them to the same denomination, and then add them as in the preceding example.

EXAMPLE 3. Find the sum of $\frac{19}{36}$, $\frac{2}{9}$, $\frac{3}{12}$, and $\frac{1}{18}$.

OPERATION.

$$\begin{array}{r} 19 \\ 8 \\ 9 \\ 2 \\ \hline \end{array}$$

$$\frac{38}{36}=1\tfrac{1}{18}, \textit{ Ans.}$$

ANALYSIS.—This illustrates a rapid method of adding fractions. The numerators, reduced to 36ths by inspection, are written and added like simple numbers.

From these illustrations we derive the following

RULE.—*Reduce the fractions, if necessary, to a common denomination; then add the numerators, and write the sum over the common denominator.*

NOTE.—In adding mixed numbers, find the sum of the fractions and whole numbers separately, and then add their sums. Practically, add the fractions first, and *carry* the resulting integers, if any, to the units' column.

EXAMPLES FOR PRACTICE.

1. Add $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{5}$, and $\frac{4}{5}$.

2. Add $\frac{9}{11}$, $\frac{2}{33}$, $\frac{4}{7}$, and $\frac{7}{14}$. *Ans.* $1\frac{439}{462}$.

3. What is the sum of $141\frac{3}{5}$ and $19\frac{3}{7}$?

4. What is the sum of $27\frac{4}{5}$, $\frac{3}{7}$, $4\frac{1}{8}$, and $42\frac{1}{3}$?

5. Add $\frac{2}{3}$, $\frac{3}{13}$, $\frac{3}{4}$, and $\frac{4}{18}$.

6. Add $\frac{7\frac{1}{2}}{30}$ to $\frac{1}{4}$ of 30.

7. What is a third and half a third of 10?

SOLUTION.—$\frac{1\frac{1}{2}}{3} = \frac{1}{2}$; $\frac{1}{2}$ of $10 = 5$. *Ans.* 5.

Find the sum of the following:—

1. $\frac{3}{4} + \frac{5}{6} + \frac{2}{3} + \frac{7}{8}$
2. $4\frac{1}{3} + 3\frac{1}{2} + 9\frac{1}{4} + \frac{1}{2}$
3. $17\frac{1}{9} + 19 + 200\frac{2}{18}$
4. $\frac{1}{3} + \frac{6}{7} + 4\frac{1}{2} + \frac{2}{13}$
5. $\frac{11}{21} + 3\frac{19}{21} + 75\frac{17}{21} + 18$
6. $198\frac{17}{36} + 400\frac{9}{17} + 217\frac{17}{34}$
7. $37\frac{3}{4} + 18\frac{7}{8} + 35\frac{1}{2} + \frac{7}{16}$
8. $\frac{3}{4} + \frac{5}{7} + \frac{11}{16} + \frac{3}{14} + \frac{5}{16}$

9. What is the sum of $\frac{3}{23}$, $\frac{4}{46}$, $\frac{5}{11}$, and $\frac{7}{92}$? 10. Of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$, and $\frac{11}{32}$? 11. Of $1\frac{7}{8}$, $13\frac{4}{5}$, $15\frac{1}{8}$, and $\frac{5}{8}$?

12. Add $\frac{4}{9}$, $\frac{11}{18}$, $\frac{2}{3}$, and $\frac{1}{2}$
13. Add $3\frac{1}{2}$, $14\frac{7}{8}$, and $19\frac{3}{4}$
14. Add $11\frac{1}{19}$, $13\frac{7}{18}$, and $\frac{13}{18}$
15. Add $\frac{3}{5}$, $\frac{4}{10}$, and $\frac{5}{20}$
16. Add $425\frac{3}{4}$ and $1781\frac{9}{12}$
17. Add $1000\frac{1}{2}$ and $275\frac{1}{60}$
18. Add $3046\frac{37}{60}$ and $2758\frac{17}{80}$
19. Add $275\frac{1}{4}$, $37\frac{1}{2}$, and $116\frac{1}{8}$
20. Add $300\frac{16}{17}$ and $19\frac{17}{34}$
21. Add $26758\frac{9}{20}$ and $48\frac{4}{5}$
22. Add 75, $62\frac{1}{2}$ and $87\frac{3}{16}$
23. Add $\frac{3}{11}$, $\frac{4}{33}$, 17, and $6\frac{1}{6}$
24. Add $9\frac{1}{10}$ and $17\frac{2}{20}$
25. Add 206 and $354\frac{1}{18}$
26. Add $21\frac{4}{7}$, $32\frac{3}{8}$, and $47\frac{5}{14}$
27. Add $\frac{11}{18}$, $3\frac{1}{3}$, and $18\frac{3}{4}$

CASE II.

109. To add two fractions having 1 for their numerator.

Example. Add $\frac{1}{7}$ and $\frac{1}{9}$.

OPERATION.

$$\tfrac{1}{7} + \tfrac{1}{9} = \frac{9 + 7}{9 \times 7} = \tfrac{16}{63}, \textit{ Ans.}$$

Analysis.—The process of reducing the fractions to the same denomination gives 1 × 9 for the new numerator of $\frac{1}{7}$, and 1 × 7 for the new numerator of $\frac{1}{9}$; that is, 9 and 7, the denominators of the given fractions, form the new numerators, while their product is the common denominator. Hence the

Rule.—*Write the sum of the denominators over their product.*

Examples for Practice.

Add the following fractions:—

1. $\frac{1}{5} + \frac{1}{8}$
2. $\frac{1}{6} + \frac{1}{7}$
3. $\frac{1}{20} + \frac{1}{30}$
4. $\frac{1}{11} + \frac{1}{12}$
5. $\frac{1}{13} + \frac{1}{4}$
6. $\frac{1}{15} + \frac{1}{3}$
7. $\frac{1}{8} + \frac{1}{9}$
8. $\frac{1}{12} + \frac{1}{5}$

Case III.

110. To add two fractions whose numerators are alike.

Example. Find the sum of $\frac{3}{5}$ and $\frac{3}{8}$.

OPERATION.

$$\tfrac{3}{5} + \tfrac{3}{8} = \frac{(5 + 8) \times 3}{5 \times 8} = \tfrac{39}{40}, \textit{ Ans.}$$

Analysis.—By the process of reducing the fractions to the same denomination, we should obtain for the numerator of the first, 3 × 8, and of the second, 3 × 5, and for the common denominator, 5 × 8. But 3 × 8 + 3 × 5 = (8 + 5) × 3. Hence the

Rule.—*Multiply the sum of the denominators by the common numerator, and write the product over the product of the denominators.*

Examples for Practice.

Find the sum of the following fractions:—

1. $\frac{3}{5} + \frac{3}{7}$
2. $\frac{4}{5} + \frac{4}{15}$
3. $\frac{5}{8} + \frac{5}{11}$
4. $\frac{6}{9} + \frac{6}{12}$
5. $\frac{7}{8} + \frac{7}{9}$
6. $\frac{8}{11} + \frac{8}{16}$
7. $\frac{2}{12} + \frac{2}{13}$
8. $\frac{10}{13} + \frac{10}{30}$

SUBTRACTION OF FRACTIONS.

111. Subtraction of Fractions is the process of finding the difference between two fractions, or numbers, when either or both are fractional.

One fraction can be subtracted from another only when both are of the same denomination and the same unit.

When the minuend and subtrahend have a common denominator, the numerators of the fractions express fractional units of the same value, and the process of subtraction is like that of simple numbers. For example, let the difference between $\frac{7}{12}$ and $\frac{5}{12}$ be required. The analogy of the process to the subtraction of simple numbers may be illustrated as follows:—

7 twelfths
— 5 twelfths
= 2 twelfths.

7 twelfths less 5 twelfths equals 2 twelfths. There is no practical difference between the operation of taking $\frac{5}{12}$ from $\frac{7}{12}$, and the subtraction of 5 units from 7 units, or $\frac{5}{24}$ from $\frac{7}{24}$, or 5 yards from 7 yards. In each case the remainder is 2 of the given denomination.

When the fractions are of different denominations, they must first be reduced to the same denomination. For example, to subtract $\frac{5}{16}$ from $\frac{3}{8}$, we must reduce $\frac{3}{8}$ to 16ths, or both fractions to some other denomination. $\frac{3}{8} = \frac{6}{16}$, and $\frac{6}{16} - \frac{5}{16} = \frac{1}{16}$.

CASE I.

112. To subtract a fraction from a fraction.

EXAMPLE 1. From $\frac{9}{15}$ take $\frac{7}{15}$.

OPERATION.

$\frac{9}{15} - \frac{7}{15} = \frac{2}{15}$, *Ans.*

ANALYSIS.—The fractions being of the same denomination, we have only to subtract the numerator 7 from the numerator 9, and write the remainder over the common denominator, 15; 7 fifteenths from 9 fifteenths leaves 2 fifteenths.

Example 2. From $\frac{7}{8}$ take $\frac{4}{7}$.

OPERATION.

$$\tfrac{7}{8} - \tfrac{4}{7} = \tfrac{49}{56} - \tfrac{32}{56} = \tfrac{17}{56},\ \textit{Ans.}$$

Analysis.—The given fractions being of different denominations, must first be reduced to the same. $\frac{7}{8} = \frac{49}{56}$; $\frac{4}{7} = \frac{32}{56}$; and 49 fifty-sixths minus 32 fifty-sixths = 17 fifty-sixths. Hence the following

Rule.—*Reduce the fractions, if necessary, to a common, or their least common denomination. Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference over the common denominator.*

EXAMPLES FOR PRACTICE.

1. From $\frac{11}{12}$ take $\frac{5}{18}$
2. From $\frac{17}{18}$ take $\frac{1}{6}$
3. From $\frac{3}{8}$ take $\frac{1}{16}$
4. From $\frac{99}{111}$ take $\frac{75}{100}$
5. From $\frac{5}{13}$ take $\frac{4}{14}$
6. From $\frac{4}{10}$ take $\frac{1}{5}$
7. From $\frac{3}{11}$ take $\frac{2}{9}$
8. From $\frac{8}{17}$ take $\dfrac{7\frac{1}{2}}{17}$

Find the value of the following fractions:—

1. $\frac{1}{2} + \frac{3}{4} - (\frac{3}{7} + \frac{1}{8})$
2. $\frac{3}{8} + \frac{2}{5} - \frac{1}{2} + \frac{2}{30}$
3. $\frac{7}{16} - \frac{3}{48} + \frac{1}{5} + \frac{2}{7}$
4. $\frac{11}{12} - \frac{3}{4} + 7 - \frac{3}{8}$
5. $\frac{1005}{1000} - \frac{7}{100}$
6. $\frac{7}{16} + \frac{9}{16} - \frac{11}{39} + \frac{21}{32}$
7. $\frac{17}{33} + \frac{18}{33} - \frac{1}{66} + \frac{5}{132}$
8. $\frac{9}{8} - \frac{1}{8} - \frac{1}{32} - \frac{21}{64}$

CASE II.

113. **To subtract one fraction from another when each has 1 for a numerator.**

Example. Subtract $\frac{1}{9}$ from $\frac{1}{7}$.

OPERATION.

$$\tfrac{1}{7} - \tfrac{1}{9} = \frac{9-7}{7 \times 9} = \tfrac{2}{63},\ \textit{Ans.}$$

Analysis.—In reducing the fractions to a common denomination by the ordinary process, we should obtain for the new numerators 9 × 1 and 7 × 1, or 9 and 7, the denominators of the given fractions; and, for a common denominator, 9 × 7 = 63. Hence the

Rule.—*Write the difference of the denominators over their product.*

EXAMPLES FOR PRACTICE.

1. From $\frac{1}{4}$ take $\frac{1}{8}$
2. From $\frac{1}{16}$ take $\frac{1}{17}$
3. From $\frac{1}{12}$ take $\frac{1}{13}$
4. From $\frac{1}{4}$ take $\frac{1}{5}$
5. From $\frac{1}{10}$ take $\frac{1}{11}$
6. From $\frac{1}{100}$ take $\frac{1}{101}$

CASE III.

114. **To subtract one fraction from another when their numerators are alike.**

EXAMPLE. From $\frac{3}{5}$ take $\frac{3}{16}$.

OPERATION.

$$\tfrac{3}{5} - \tfrac{3}{16} = \frac{(16-5)\times 3}{5\times 16} = \tfrac{33}{80},\ \textit{Ans.}$$

ANALYSIS.—The process of reducing the fractions to the same denomination would give for the numerator of he first 16 × 3, and of the second, 5 × 3; but, $16 \times 3 - 5 \times 3 = (16 - 5) \times 3 = 33$; and the common denominator is $5 \times 16 = 80$. Hence the

RULE.—*Multiply the difference of the denominators by the common numerator, and write the product over the product of the denominators.*

EXAMPLES FOR PRACTICE.

1. From $\frac{7}{12}$ take $\frac{7}{24}$
2. From $\frac{8}{15}$ take $\frac{8}{30}$
3. From $\frac{4}{5}$ take $\frac{4}{7}$
4. From $\frac{6}{13}$ take $\frac{6}{15}$

CASE IV.

115. **To subtract a fraction or a mixed number from a whole number.**

EXAMPLE 1. From 18 take $\frac{5}{6}$.

OPERATION.

$$\begin{array}{r} 18 = 17\tfrac{6}{6} \\ \tfrac{5}{6} \\ \hline 17\tfrac{1}{6} \end{array}$$

ANALYSIS.—Since there is no fraction in the minuend from which to subtract the $\frac{5}{6}$, we borrow 1 from 8, which is equal to $\frac{6}{6}$, and from this take $\frac{5}{6}$; and, as we have taken 1 from 8, the whole number remaining is 17, and $17 + \frac{1}{6} = 17\frac{1}{6}$, *Ans.*

EXAMPLE 2. From 37 take $3\frac{3}{4}$.

OPERATION.

$$\begin{array}{r} 37 \\ 3\frac{3}{4} \\ \hline 33\frac{1}{4} \end{array}$$

ANALYSIS.—Borrow 1 from 37, reduce it to 4ths, and subtract $\frac{3}{4}$ from it; $1 = \frac{4}{4}$, and $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$. Then carry 1 to 3, and subtract 4 from 37. *Ans.* $33\frac{1}{4}$.

Hence the following

RULE.—*Borrow* 1 *from the minuend, and reduce it to a fraction of the same denomination as that of the given fraction. From this subtract the fraction; then carry* 1, *and subtract as in simple numbers.*

EXAMPLES FOR PRACTICE.

1. From 256 take $\frac{3}{19}$
2. From 423 take $181\frac{9}{13}$
3. From 675 take $112\frac{15}{27}$
4. From 100 take $\frac{1}{99}$
5. From 75 take $\frac{18}{37}$
6. From 125 take $39\frac{1}{5}$

7. A man having 342 acres of land, sold $210\frac{3}{4}$ acres. How much had he left?

8. From 1 dollar take $\frac{1}{8}$ of a dollar. What fraction of a dollar will remain?

CASE V.

116. **To subtract a mixed number from a mixed number.**

EXAMPLE 1. From $37\frac{7}{8}$ take $18\frac{2}{3}$.

OPERATION.

$$\begin{array}{r} 37\frac{7}{8} = 37\frac{21}{24} \\ 18\frac{2}{3} = 18\frac{16}{24} \\ \hline 19\frac{5}{24}, \textit{ Ans.} \end{array}$$

ANALYSIS.—In this example the fraction of the minuend is greater than that of the subtrahend. Therefore, having reduced the fractions to the same denomination, we take $\frac{16}{24}$ from $\frac{21}{24}$, and then 18 from 37. The whole remainder is $19\frac{5}{24}$, *Ans.*

EXAMPLE 2. From $83\frac{1}{3}$ take $46\frac{4}{9}$.

OPERATION.

$$\begin{array}{r} 83\frac{1}{3} = 83\frac{3}{9} = 82\frac{12}{9} \\ 46\frac{4}{9} \\ \hline 36\frac{8}{9}, \textit{ Ans.} \end{array}$$

ANALYSIS.—In this case, since we cannot take $\frac{4}{9}$ from $\frac{3}{9}$, we borrow 1 from 83, and, reducing it to 9ths, add it to $\frac{3}{9}$, and then from the sum subtract $\frac{4}{9}$. $\frac{9}{9} + \frac{3}{9} = \frac{12}{9}$, and $\frac{12}{9} - \frac{4}{9} = \frac{8}{9}$. Then carrying 1 to 6, we subtract 47 from 83. Hence the following

RULE.—*Reduce the fractions, if necessary, to the same denomination, and then subtract as above explained.*

NOTE.—We may reduce the mixed numbers to improper fractions, and then proceed according to the rule for subtracting a fraction from a fraction.

EXAMPLES FOR PRACTICE.

1. From $812\frac{3}{4}$ take $641\frac{11}{12}$
2. From $100\frac{7}{8}$ take $2\frac{1}{2}$
3. From $259\frac{9}{16}$ take $125\frac{1}{4}$
4. From $999\frac{8}{9}$ take $110\frac{10}{11}$
5. From $7\frac{1}{2}$ take $3\frac{1}{13}$
6. From $1\frac{20}{21}$ take $1\frac{1}{17}$
7. From $18\frac{4}{5}$ take $3\frac{3}{5}$
8. From $375\frac{7}{23}$ take $216\frac{45}{46}$

9. What number increased by $19\frac{3}{4}$ will be equal to $39\frac{1}{2}$?

10. What number must be added to $2\frac{1}{8}$ to make the sum $4\frac{1}{18}$?

11. The sum of two numbers is $28\frac{3}{4}$, and the less is $9\frac{7}{8}$; what is the greater?

12. The difference between two fractions is $\frac{17}{28}$, and the greater is $\frac{3}{4}$; what is the less?

13. From $81\frac{7}{9}$ take $\frac{1}{4}$
14. From 816 take $1\frac{11}{12}$
15. From $37\frac{3}{4}$ take $18\frac{3}{4}$
16. From $217\frac{5}{8}$ take $31\frac{3}{5}$
17. From $19\frac{1}{2}$ take $18\frac{7}{8}$
18. From 419 take $\frac{7}{88}$
19. From $275\frac{1}{117}$ take $3\frac{5}{118}$
20. From $16\frac{17}{18}$ take $9\frac{1}{4}$
21. From $8\frac{1}{8}$ take $7\frac{7}{8}$
22. From $100\frac{1}{9}$ take $11\frac{1}{19}$

MULTIPLICATION OF FRACTIONS.

117. Multiplication of Fractions is the process of finding the product of two or more fractions, or numbers, when one or more of them are fractional.

GENERAL PRINCIPLE.—*Multiplying the numerator, or dividing the denominator, multiplies the fraction.*

CASE I.

multiply a fraction by a whole number.

EXAMPLE. Multiply $\frac{7}{12}$ by 4.

FIRST METHOD.

$$\frac{7}{12}\times 4=\frac{28}{12}=2\tfrac{1}{3}.$$

SECOND METHOD.

$$\frac{7}{12\div 4}=\frac{7}{3}=2\tfrac{1}{3}.$$

THIRD METHOD.

$$\frac{7}{\underset{3}{\cancel{12}}}\times\frac{\cancel{4}}{1}=\frac{7}{3}=2\tfrac{1}{3}.$$

ANALYSIS.—*First Method.* Multiplying the numerator multiplies the fraction, by taking the number of fractional units as many times as there are units in the multiplier. 4 times 7 twelfths are 28 twelfths.

Second Method. Dividing the denominator also multiplies the fraction, by increasing the denominational value of the fractional units, or making each four times as large as it was before, without changing their number. Thirds are four times the value of twelfths.

Third Method. This method is identical in principle with the first. The operation is expressed, and the process of cancellation applied. Hence we have the following

RULE.—*Multiply the numerator, or divide the denominator.*

NOTE.—It is better to divide the denominator when it contains the multiplier without a remainder, in order to get the result in lower terms.

EXAMPLES FOR PRACTICE.

1. Multiply $\frac{8}{17}$ by 5
2. Multiply $\frac{23}{45}$ by 18
3. Multiply $\frac{9}{11}$ by 11
4. Multiply $\frac{4}{5}$ by 2
5. Multiply $\frac{7}{16}$ by 16
6. Multiply $\frac{475}{868}$ by 31
7. Multiply $\frac{61}{81}$ by 9
8. Multiply $\frac{37}{75}$ by 25
9. At $\frac{5}{8}$ of a dollar a yard, what will 15 yards of cloth cost?
10. What are 156 bushels of oats worth, at $\frac{3}{8}$ of a dollar per bushel?

CASE II.

119. **To multiply a whole number by a fraction.**

To multiply by a fraction is to take such a part of the multiplicand as the multiplier is of unity. Thus, multiplying by $\frac{1}{4}$ is taking $\frac{1}{4}$ of the multiplicand, or dividing by 4; $12\times\frac{1}{4}$, or $\frac{1}{4}$ of 12, is $12\div 4=3$.

Example. Multiply 32 by $\frac{3}{8}$.

FIRST METHOD.

$$\begin{array}{r} 8)\underline{32} \\ 4 \\ \underline{3} \\ 12, \textit{ Ans.} \end{array}$$

Analysis.—By the first method $\frac{1}{8}$ of 32 is found by dividing by 8. Then the quotient 4 multiplied by 3 equals 12, or $\frac{3}{8}$ of 32.

SECOND METHOD.

$$\begin{array}{r} 32 \\ \underline{3} \\ 8)\underline{96} \\ 12, \textit{ Ans.} \end{array}$$

By the second method we multiply by the numerator, and then divide by the denominator. The result is the same, for $\frac{3}{8}$ of 32 = $\frac{1}{8}$ of 32 × 3.

THIRD METHOD.

$$\frac{\overset{4}{\cancel{32}}}{1} \times \frac{3}{\cancel{8}} = \frac{12}{1} = 12, \textit{ Ans.}$$

The third method is the same in principle as the other two. The operation is expressed, and the result obtained by cancellation. Hence we derive the following

Rule.—*Divide by the denominator and multiply by the numerator.*

Note.—When it is not evident that the denominator is an exact divisor of the multiplicand, it is generally better to multiply by the numerator first.

Examples for Practice.

1. Multiply 42 by $\frac{6}{7}$
2. Multiply 371 by $\frac{2}{3}$
3. Multiply 285 by $\frac{14}{21}$
4. Multiply 27 by $\frac{1}{3}$
5. Multiply 236 by $\frac{7}{8}$
6. Multiply 375 by $\frac{3}{4}$
7. Multiply 4130 by $\frac{7}{10}$
8. Multiply 217 by $\frac{5}{6}$

9. What is $\frac{3}{7}$ of 21? $\frac{8}{9}$ of 91? $\frac{3}{8}$ of 17? $\frac{4}{11}$ of 19?

10. What is the difference between 24 times $\frac{7}{8}$ and $\frac{7}{8}$ of 24?

11. A person having 135 acres of land, sold $\frac{3}{5}$ of it to one man, and $\frac{1}{4}$ of it to another. How many acres had he left?

CASE III.

120. To multiply a fraction by a fraction.

EXAMPLE 1. Multiply $\frac{7}{8}$ by $\frac{3}{4}$.

OPERATION.

$\frac{7}{8} \times \frac{3}{4} = \frac{21}{32}$, *Ans.*

ANALYSIS.—To multiply a number, whether whole or fractional, by a fraction, is to take such a part of it as the multiplier is of a unit. Now, one fourth part of $\frac{7}{8}$ is a fraction having the same numerator, and a denominator four times as large; that is, to divide $\frac{7}{8}$ by 4, we multiply the denominator by 4 (Prin. II). Therefore, $\frac{1}{4}$ of $\frac{7}{8} = \frac{7}{8 \times 4} = \frac{7}{32}$; and $\frac{3}{4}$ of $\frac{7}{8} = \frac{7}{32} \times 3 = \frac{21}{32}$, *Ans.* This operation is equivalent to multiplying the numerators together for the numerator of the product, and the denominators together for the denominator.

The work may often be shortened by canceling all factors common to numerators and denominators.

EXAMPLE 2. Multiply $\frac{8}{21}$ by $\frac{3}{4}$.

OPERATION.

$$\overset{2}{\frac{\cancel{8}}{\underset{7}{\cancel{21}}}} \times \frac{\cancel{3}}{\cancel{4}} = \frac{2}{7}, \textit{ Ans.}$$

ANALYSIS.—The canceling of equal factors from the numerator and denominator of the expressed result, is identical in principle with the process of reducing a fraction to its lowest terms; for a common factor of any numerator and any denominator, is a common factor of the terms of the product.

121. The operation of finding the continued product of several fractions is the same. This is generally called the *Reduction of Compound Fractions to Simple ones.*

EXAMPLE 3. Multiply together $\frac{9}{12}$, $\frac{6}{8}$, and $\frac{2}{3}$.

OPERATION.

$$\overset{3}{\frac{\cancel{9}}{\cancel{12}}} \times \frac{\cancel{6}}{8} \times \frac{\cancel{2}}{\cancel{3}} = \frac{3}{8}, \textit{ Ans.}$$

NOTE.—Multiplying together $\frac{9}{12}$, $\frac{6}{8}$, and $\frac{2}{3}$, is equivalent to taking that fractional part of one of them which is indicated by the other two. Thus, we may regard $\frac{9}{12}$ as multiplied by $\frac{6}{8}$ of $\frac{2}{3} = \frac{1}{2}$, or $\frac{2}{3}$ as multiplied by $\frac{9}{12}$ of $\frac{6}{8} = \frac{9}{16}$. Therefore, the process is the same as that of multiplying a fraction by a fraction. Hence we have the following

RULE.—*Multiply together the numerators for the numerator of the product, and the denominators for the denominator.*

EXAMPLES FOR PRACTICE.

1. Multiply $\frac{7}{9}$ by $\frac{4}{5}$
2. Multiply $\frac{2}{11}$ by $\frac{11}{18}$
3. Multiply $\frac{3}{7}$ by $\frac{19}{21}$
4. Multiply $\frac{13}{25}$ by $\frac{14}{17}$
5. Multiply $\frac{21}{23}$ by $\frac{9}{11}$
6. Multiply $\frac{43}{67}$ by $\frac{3}{5}$
7. Multiply $\frac{18}{35}$ by $\frac{70}{93}$
8. Multiply $\frac{37}{40}$ by $\frac{49}{50}$
9. Find the value of $\frac{1}{8} \times \frac{6}{10} \times \frac{8}{12} \times \frac{20}{24}$.
10. Find the value of $\frac{3}{16}$ of $\frac{4}{11}$ of $\frac{3}{7}$ of $\frac{32}{36}$.

NOTE.—The word *of* between fractions signifies multiplication.

11. Find the value of $3\frac{1}{4} \times \frac{1}{2} \times \frac{7}{12} \times \frac{1}{13} \times 2\frac{1}{2}$.

NOTE.—First reduce the mixed numbers to improper fractions.

12. Find the value of $\overline{7\frac{1}{2} \times \frac{1}{3}} + \overline{\frac{3}{4} \text{ of } \frac{4}{5}} + \overline{9\frac{3}{4} \times \frac{1}{6} \times 5} - \frac{1}{8}$ of 2.
Ans. $10\frac{39}{40}$.

CASE IV.

122. **To multiply a mixed number by a whole number or a whole number by a mixed number.**

EXAMPLE 1. Multiply $29\frac{3}{4}$ by 5.

OPERATION.

$$\begin{array}{r} 29\frac{3}{4} \\ 5 \\ \hline 145 \\ 3\frac{3}{4} \\ \hline 148\frac{3}{4}, \textit{ Ans.} \end{array}$$

ANALYSIS.

$$\begin{array}{lr} 5 \text{ times } 29 = & 145 \\ 5 \text{ times } \frac{3}{4} = & 3\frac{3}{4} \\ \hline 145 + 3\frac{3}{4} = & 148\frac{3}{4}, \textit{ Ans.} \end{array}$$

EXAMPLE 2. Multiply 186 by $6\frac{3}{4}$.

OPERATION.

$$\begin{array}{r} 186 \\ 6\frac{3}{4} \\ \hline 1116 \\ 139\frac{1}{2} \\ \hline 1255\frac{1}{2}, \textit{ Ans.} \end{array}$$

ANALYSIS.

$$\begin{array}{lr} 6 \text{ times } 186 = & 1116 \\ \frac{3}{4} \text{ of } 186 = 139\frac{2}{4} = & 139\frac{1}{2} \\ \hline 1116 + 139\frac{1}{2} = & 1255\frac{1}{2}, \textit{ Ans.} \end{array}$$

Hence the following

Rule.—*Multiply (or multiply by) the whole and fractional part separately, and add the products.*

Examples for Practice.

1. Multiply 210 by $3\frac{3}{4}$
2. Multiply $347\frac{1}{8}$ by 17
3. Multiply $876\frac{3}{13}$ by 16
4. Multiply 1216 by $12\frac{1}{2}$
5. Multiply $17\frac{7}{8}$ by 12
6. Multiply 16 by $13\frac{3}{4}$
7. Multiply $621\frac{1}{2}$ by 4
8. Multiply 246 by $7\frac{2}{3}$

9. What is the cost of 346 yards of white muslin at $21\frac{1}{2}$ cents per yard?

10. Find the value of 472 tons of coal at \$$6\frac{3}{4}$ per ton.

11. What is the cost of 16 tons of hay at \$$22\frac{5}{8}$ per ton?

12. What is the cost of 210 barrels of flour at \$$9\frac{7}{8}$ per barrel?

Case V.

123. **To multiply a mixed number by a mixed number.**

Example. Multiply $18\frac{1}{5}$ by $3\frac{1}{3}$.

FIRST METHOD.

$18\frac{1}{5} = \frac{91}{5}$; $3\frac{1}{3} = \frac{10}{3}$.

$$\frac{91}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{3} = \frac{182}{3} = 60\tfrac{2}{3}, \textit{ Ans.}$$

SECOND METHOD.

$$\begin{array}{ll}
18\frac{1}{5} & \\
\;\;3\frac{1}{3} & \\
\hline
54\frac{3}{5} & = 18\frac{1}{5} \times 3 \\
\;\;6\frac{1}{15} & = 18\frac{1}{5} \times \frac{1}{3} \\
\hline
60\frac{2}{3}, \textit{ Ans.} &
\end{array}$$

Analysis.—*First Method.* By this method we first reduce the mixed numbers to improper fractions, and then proceed as in Case III.

Second Method. In this solution we first multiply both the whole number and the fraction by 3, and then by $\frac{1}{3}$. 3 times 18 are 54, and 3 times $\frac{1}{5}$ are $\frac{3}{5}$; $\frac{1}{3}$ of 18 is 6, and $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$. The sum of these results is the total product. This is a practical method, quite as simple, and shorter than the first. Of course, we may multiply either by the fraction or whole number first. Hence the following

RULE.—*Reduce the mixed numbers to improper fractions, and then multiply numerator by numerator and denominator by denominator.* Or,

Multiply both the whole number and the fraction of the multiplicand by the whole number and the fraction of the multiplier separately, and add the products.

EXAMPLES FOR PRACTICE.

1. Multiply $3\frac{5}{8}$ by $4\frac{1}{3}$
2. Multiply $17\frac{1}{6}$ by $13\frac{1}{4}$
3. Multiply $201\frac{1}{8}$ by $7\frac{1}{5}$
4. Multiply $18\frac{3}{4}$ by $7\frac{1}{2}$
5. Multiply $26\frac{2}{3}$ by $9\frac{3}{4}$
6. Multiply $121\frac{3}{7}$ by $6\frac{7}{8}$
7. Multiply $19\frac{1}{8}$ by $17\frac{1}{5}$
8. Multiply $42\frac{1}{13}$ by $6\frac{5}{6}$
9. What will $33\frac{1}{4}$ yards of print cost at $16\frac{1}{8}$ cents per yard?
10. At \$$23\frac{3}{8}$ per ton, what is $9\frac{1}{2}$ tons of hay worth?

NOTE.—For short practical methods of multiplication, see pages 16 to 36, inclusive.

DIVISION OF FRACTIONS.

124. **Division of Fractions** is the process of finding how many times one number is contained in another, when one or both are fractional.

GENERAL PRINCIPLE.—*Dividing the numerator, or multiplying the denominator, divides the fraction.*

CASE I.

125. To divide a fraction by a whole number.

EXAMPLE. Divide $\frac{6}{8}$ by 3.

FIRST METHOD.

$$\tfrac{6}{8} \div 3 = \frac{6 \div 3}{8} = \tfrac{2}{8} = \tfrac{1}{4}, \textit{ Ans.}$$

SECOND METHOD.

$$\tfrac{6}{8} \div 3 = \frac{6}{8 \times 3} = \tfrac{6}{24} = \tfrac{1}{4}, \textit{ Ans.}$$

ANALYSIS.—*First Method.* Dividing the numerator divides the fraction, for $\frac{1}{3}$ of 6 is 2; and upon the same principle, $\frac{1}{3}$ of 6 eighths is 2 eighths.

Second Method. Multiplying the denominator also divides the fraction, by diminishing the denominational value of

THIRD METHOD.

$$\tfrac{6}{8} \div 3 = \frac{\cancel{6}}{\underset{4}{\cancel{8}}} \times \frac{1}{\cancel{3}} = \tfrac{1}{4},\ \textit{Ans.}$$

the fractional units, or making each one-third its former size. Twenty-fourths are one-third the value of eighths.

Third Method. This method is substantially the same as the second, the dividend being multiplied by the reciprocal of the divisor. Hence the

RULE.—*Divide the numerator or multiply the denominator.*

NOTE.—It is better to divide the numerator when it exactly contains the divisor, as this will give the quotient in lower terms.

EXAMPLES FOR PRACTICE.

1. Divide $\frac{8}{45}$ by 8
2. Divide $\frac{7}{23}$ by 17
3. Divide $\frac{12}{37}$ by 6
4. Divide $\frac{970}{1562}$ by 10
5. Divide $\frac{118}{119}$ by 9
6. Divide $\frac{242}{363}$ by 11
7. Divide $\frac{1}{2}$ by 1
8. Divide $\frac{26}{67}$ by 13

CASE II.

126. To divide a whole number by a fraction.

EXAMPLE. Divide 18 by $\frac{3}{8}$.

FIRST METHOD.

$$18 \div \tfrac{3}{8} = \frac{18 \times 8}{3} = 48,\ \textit{Ans.}$$

SECOND METHOD.

$$18 \div \tfrac{3}{8} = 18 \div 3 \times 8 = 48,\ \textit{Ans.}$$

THIRD METHOD.

$$18 \div \tfrac{3}{8} = \frac{\overset{6}{\cancel{18}}}{1} \times \frac{8}{\cancel{3}} = 48,\ \textit{Ans.}$$

ANALYSIS.—*First Method* Since 18 contains $\frac{1}{8}$ as many times as there are eighths in 18, it will contain $\frac{3}{8}$ one third of that number of times. 18 = 144 eighths, and 144 ÷ 3 = 48.

Second Method. 18 contains 3 six times; but 6 is only $\frac{1}{8}$ of the true quotient, for 3 is eight times $\frac{3}{8}$. Therefore, the quotient is 6 × 8 = 48.

Third Method. Multiplying by the reciprocal of the divisor. is the same operation in principle, and gives the same result.

In each of these solutions we multiply by the denominator and divide by the numerator. Hence we have the following

RULE.—*Multiply by the denominator and divide by the numerator.*

EXAMPLES FOR PRACTICE.

1. Divide 9 by $\frac{11}{12}$
2. Divide 13 by $\frac{32}{71}$
3. Divide 8 by $\frac{112}{647}$
4. Divide 6 by $\frac{18}{36}$
5. Divide 15 by $\frac{14}{25}$
6. Divide 11 by $\frac{22}{27}$
7. Divide 4 by $\frac{1}{4}$
8. Divide 3 by $\frac{3}{7}$
9. Divide 324 by $\frac{13}{98}$
10. Divide 1728 by $\frac{14}{27}$
11. Divide 1964 by $\frac{13}{17}$
12. Divide 4659 by $\frac{3}{11}$

CASE III.

127. To divide a fraction by a fraction.

EXAMPLE. Divide $\frac{7}{13}$ by $\frac{6}{11}$.

OPERATION.

$$\tfrac{7}{13} \div \tfrac{6}{11} = \tfrac{7}{13} \times \tfrac{11}{6} = \tfrac{77}{78}.$$

ANALYSIS.—We have seen that a whole number is divided by a fraction by dividing by the numerator and multiplying by the denominator. A fraction is divided by a fraction upon the same principle. $\frac{7}{13} \div 6 = \frac{7}{78}$; and $\frac{7}{78} \times 11 = \frac{77}{78}$. Or, the process may be explained as follows:—6 is contained in $\frac{7}{13}$ one-sixth of $\frac{7}{13}$ times, equals $\frac{7}{78}$ of a time; but $\frac{6}{11}$ being only $\frac{1}{11}$ of 6, is contained in it 11 times as many times; and $\frac{7}{78} \times 11 = \frac{77}{78}$, *Ans.* This operation is equivalent to multiplying the dividend by the divisor inverted. Hence the

RULE.—*Multiply the dividend by the divisor inverted.*

NOTE.—It follows from the principles of cancellation, that when the fractions have a common denominator, it can be rejected, and the division performed, as in whole numbers, by dividing the numerator of the dividend by the numerator of the divisor. Thus, $\frac{21}{35} \div \frac{7}{35} = 21 \div 7 = 3$. Evidently, 3 thirty-fifths are contained in 21 thirty-fifths as many times as 7 is contained in 21, or 3 times.

EXAMPLES FOR PRACTICE.

1. Divide $\frac{1}{4}$ by $\frac{7}{8}$
2. Divide $\frac{11}{16}$ by $\frac{3}{5}$
3. Divide $\frac{14}{15}$ by $\frac{27}{30}$
4. Divide $\frac{77}{99}$ by $\frac{11}{99}$
5. Divide $\frac{17}{39}$ by $\frac{12}{39}$
6. Divide $\frac{24}{8}$ by $\frac{3}{8}$
7. Divide $\frac{19}{16}$ by $\frac{15}{23}$
8. Divide $\frac{1}{2}$ by $\frac{1}{2}$

9. Divide $\frac{3}{4}$ of $\frac{5}{12}$ by $\frac{8}{189}$ of $\frac{13}{24}$.

10. Divide $\frac{7}{8}$ of $\frac{4}{5}$ by $\frac{3}{7}$ of $\frac{6}{11}$ of $\frac{14}{24}$.

11. What fraction multiplied by $\frac{27}{68}$ will produce $\frac{7}{8}$?

CASE IV.

128. To divide a mixed number by a whole number.

EXAMPLE. Divide $48\frac{5}{8}$ by 7.

OPERATION.

$$\frac{7)48\frac{5}{8}}{6}, \quad \text{Rem. } 6\frac{5}{8} = \frac{53}{8};$$

$$\frac{53}{8} \div 7 = \frac{53}{56};\ 6 + \frac{53}{56} = 6\frac{53}{56}, \textit{ Ans.}$$

ANALYSIS.—We first divide the whole number, 48, by 7, as in simple numbers, and obtain the integral part of the quotient, 6, with a remainder of $6\frac{5}{8}$. This remainder we reduce to an improper fraction, and divide as in Case I.

Or, the entire dividend may be reduced to an improper fraction, and divided as in Case I. Hence the

RULE.—*Divide the integral and fractional parts separately, and unite the quotients. When there is a remainder on dividing the whole number, prefix it to the fraction, reduce to an improper fraction, and divide as in Case I.* Or,

Reduce the mixed number to an improper fraction, and divide as in Case I.

EXAMPLES FOR PRACTICE.

1. Divide $14\frac{2}{7}$ by 2
2. Divide $21\frac{3}{11}$ by 6
3. Divide $39\frac{4}{7}$ by 12
4. Divide $88\frac{1}{2}$ by 22
5. Divide $3487\frac{1}{3}$ by 14
6. Divide $435\frac{7}{8}$ by 27
7. Divide $312\frac{1}{5}$ by 5
8. Divide $1\frac{1}{2}$ by 3
9. Divide $342\frac{1}{3}$ by 17
10. Divide $243\frac{9}{18}$ by 162
11. Divide $237\frac{3}{8}$ by 2
12. Divide $18\frac{1}{2}$ by 6

CASE V.

129. **To divide a whole number by a mixed number.**

EXAMPLE. Divide 3876 by $187\frac{1}{2}$.

OPERATION.

$187\frac{1}{2})3876$
$2 2$

$375\ )7752(20\frac{252}{375}$, *Ans.*
750

252

ANALYSIS.—We first multiply the divisor and dividend by 2, to reduce both to halves, because the divisor contains the fraction $\frac{1}{2}$, and then divide as in whole numbers. The true quotient is thus obtained, for the multiplication of both divisor and dividend by the same number does not change their relation to each other. And, according to the principles of cancellation, we may reject the common denominator, and divide numerator by numerator. See note, Case III. Hence the

RULE.—*Reduce both divisor and dividend to the denomination of the fraction of the divisor, and then divide as in whole numbers.*

EXAMPLES FOR PRACTICE.

1. Divide 4000 by $13\frac{7}{8}$
2. Divide 127 by $25\frac{1}{3}$
3. Divide 18 by $1\frac{1}{6}$
4. Divide 475 by $11\frac{3}{4}$
5. Divide 1870 by $9\frac{8}{11}$
6. Divide 1362 by $100\frac{1}{4}$
7. Divide 206 by $2\frac{1}{2}$
8. Divide 118 by $17\frac{3}{5}$
9. Divide 14 by $2\frac{1}{3}$
10. Divide 396 by $11\frac{3}{7}$
11. Divide 2162 by $19\frac{3}{4}$
12. Divide 162 by $18\frac{4}{5}$

CASE VI.

130. **To divide a mixed number by a mixed number.**

EXAMPLE. Divide $18\frac{1}{6}$ by $3\frac{1}{3}$.

FIRST METHOD.

$18\frac{1}{6} = \frac{109}{6}$
$3\frac{1}{3} = \frac{10}{3}$

$$\frac{109}{6} \div \frac{10}{3} = \frac{109}{\underset{2}{\cancel{6}}} \times \frac{\cancel{3}}{10} = \frac{109}{20} = 5\frac{9}{20}.$$

ANALYSIS.—*First Method.* Having first reduced the mixed numbers to improper fractions, we proceed as in Case III.

SECOND METHOD.

$$\begin{array}{rl} 3\frac{1}{3}) & 18\frac{1}{6} \\ 6 & 6 \\ \hline 20) & 109(5\frac{9}{20} \\ & 100 \\ \hline & 9 \end{array}$$

Second Method. By this method we reduce both the divisor and the dividend to 6ths, by multiplying both by 6, the least common multiple of the denominators of the fractions. The division is then performed as in whole numbers. Hence we have the following

Rule.—*Reduce the mixed numbers to improper fractions, and then proceed as in the division of a fraction by a fraction.* Or,

Multiply both divisor and dividend by the least common multiple of the denominators of the fractions, and then divide as in whole numbers.

Examples for Practice.

1. Divide $24\frac{7}{8}$ by $4\frac{1}{2}$
2. Divide $132\frac{1}{9}$ by $14\frac{1}{3}$
3. Divide $926\frac{3}{18}$ by $12\frac{1}{5}$
4. Divide $100\frac{1}{4}$ by $25\frac{1}{8}$
5. Divide $21\frac{1}{2}$ by $1\frac{1}{4}$
6. Divide $14\frac{1}{5}$ by $6\frac{1}{7}$
7. Divide $99\frac{1}{4}$ by $9\frac{1}{4}$
8. Divide $113\frac{3}{4}$ by $14\frac{1}{8}$

9. If a pound of tea cost $1\frac{1}{4}$ dollars, how many pounds can be bought for $17\frac{1}{2}$ dollars?

10. If $4\frac{7}{8}$ tons of coal cost \$$36\frac{9}{16}$, what is the price per ton?

11. How many turkeys at \$$1\frac{7}{8}$ each can be bought for \$$16\frac{7}{8}$?

REDUCTION OF COMPLEX FRACTIONS.

131. A Complex Fraction merely expresses division, either the numerator or denominator, or both being fractional, or indicating certain operations to be performed to reduce them to simpler forms.

Hence, to reduce complex to simple fractions, or whole or mixed numbers, we have only to *perform the operations indicated by rules already given.*

EXAMPLES.

1. Reduce $\frac{\frac{3}{4}}{\frac{5}{6}}$ to a simple fraction.

SOLUTION: $\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$, *Ans.*

2. Reduce $\frac{\frac{9}{10}}{3\frac{1}{2}}$ to a simple fraction.

SOLUTION: $\frac{9}{10} \div 3\frac{1}{2} = \frac{9}{10} \times \frac{2}{7} = \frac{18}{70} = \frac{9}{35}$, *Ans.*

3. Reduce $\frac{\frac{3}{8} - \frac{1}{4}}{\frac{3}{16} + \frac{1}{2}}$ to its simplest form.

4. Reduce $\frac{\frac{1}{2} \times \frac{4}{5}}{\frac{1}{3} \div 3\frac{1}{3}}$ to its simplest form.

5. Reduce $\frac{18 \div \frac{1}{6}}{9 \times \frac{1}{4}}$ to its simplest form.

A complex fraction whose numerator and denominator are in simple form, may be reduced to a simple fraction by multiplying both terms by a multiple of the denominators of their fractional parts, or by the denominator of the fractional part, if only one term is fractional.

EXAMPLES.

1. Reduce $\frac{5\frac{2}{3}}{3\frac{3}{4}}$ to its simplest form.

SOLUTION: $\frac{5\frac{2}{3} \times 12}{3\frac{3}{4} \times 12} = \frac{68}{45} = 1\frac{23}{45}$, *Ans.*

2. Reduce $\frac{14\frac{1}{5}}{4}$ to its simplest form.

3. Reduce $\frac{125\frac{3}{4}}{18\frac{7}{8}}$ to its simplest form.

In certain cases both terms can be divided by some number that will clear both of fractions.

4. Reduce $\frac{4\frac{1}{2}}{9}$ to a simple fraction.

SOLUTION: $4\frac{1}{2} \,\Big|\, \frac{4\frac{1}{2}}{9} = \frac{1}{2}$, *Ans.*

5. Reduce $\frac{3\frac{1}{2}}{13\frac{1}{2}}$ to a simple fraction

GREATEST COMMON DIVISOR OF FRACTIONS.

132. The **Greatest Common Divisor** of two or more fractions, is the greatest fraction that will divide each of them, and give a whole number for the quotient.

Since in dividing by a fraction, we multiply the dividend by the reciprocal of the divisor, it follows that one fraction will be an exact divisor of another when the numerator of the divisor is an exact divisor of the numerator of the dividend, and the denominator of the divisor exactly contains, or is a multiple of the denominator of the dividend. For example: $\frac{15}{48}$, $\frac{15}{64}$, $\frac{5}{32}$, and $\frac{3}{80}$, are exact divisors of $\frac{15}{16}$, since each of the denominators is a multiple of 16, and each of the numerators an exact divisor of 15. In other words, after inverting the divisor, the denominators may be canceled, and the quotient will be a whole number. Thus,

$$\frac{15}{16} \div \frac{15}{48} = \frac{\cancel{15}}{\cancel{16}} \times \frac{\overset{3}{\cancel{48}}}{\cancel{15}} = 3.$$

Again, we have seen that when one fraction is to be divided by another having the same denominator, the division can be performed by dividing the numerator of the dividend by the numerator of the divisor, as in simple numbers. Hence, it is evident that to obtain an exact divisor of two or more fractions, we have only to find a fraction whose denominator is a common multiple of all the denominators of the given fractions, and whose numerator is an exact divisor of all the numerators. That fraction having the *largest numerator* and the *least denominator* which will answer these conditions, in any given case, is the greatest common divisor required. Hence,

133. To find the greatest common divisor of two or more fractions.

RULE.—*Find the greatest common divisor of their numerators for a numerator, and the least common multiple of their denominators for a denominator. This fraction will be the greatest common divisor sought.*

EXAMPLES.

1. What is the greatest common divisor of $\frac{3}{8}$, $\frac{9}{16}$, and $\frac{3}{4}$?

SOLUTION.—Greatest common divisor of the numerators $= 3$
Least common multiple of the denominators $= \overline{16}$

Therefore, $\frac{3}{16}$ is the greatest common divisor required.

2. Required the greatest common divisor of $3\frac{1}{2}$, $\frac{3}{32}$, and $2\frac{3}{4}$.
Ans. $\frac{1}{32}$.

NOTES.—1. First reduce the mixed numbers to improper fractions.

2. In this case, the numerators are prime to each other, and therefore, their greatest common divisor is 1. Hence, the greatest common divisor of two or more fractions whose numerators are prime to each other, is a fraction whose numerator is one, and whose denominator is the least common multiple of the denominators.

3. Find the greatest common divisor of $\frac{9}{12}$, $\frac{18}{25}$, and $\frac{27}{50}$.

4. What is the greatest common divisor of $2\frac{7}{8}$, $3\frac{4}{5}$, and $\frac{1}{2}$?

5. A gentleman wishes to inclose his yard, which is 56 feet 3 inches long and 40 feet 6 inches wide, with an iron fence, the parts or sections of which shall be of equal lengths: what is the length of the longest sections that can be used?
Ans. 2 feet 3 inches.

LEAST COMMON MULTIPLE OF FRACTIONS.

134. The **Least Common Multiple** of two or more fractions, is the least number that can be divided by each of them, and give a whole number for the quotient.

A fraction will contain another fraction a whole number of times only when, with the divisor inverted, the denominators may be canceled; or in other words, the numerator of the dividend must exactly contain the numerator of the divisor, and the denominator of the divisor must exactly contain the denominator of the dividend. Thus, $\frac{3}{4}$ is a multiple of $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{16}$, etc.; and $\frac{6}{9}$ is a multiple of $\frac{1}{9}$, $\frac{6}{18}$, $\frac{3}{9}$, $\frac{3}{18}$, etc.

Hence, a multiple of several fractions having a common denominator, is a fraction having the same denominator, or a multiple of it, and a numerator which exactly contains, or is a multiple of all the numerators.

That fraction having the *least numerator*, and the *largest denominator* that will answer these conditions, in any given case, is the least common multiple required. Hence,

135. To find the least common multiple of two or more fractions, we have the following

RULE.—*Find the least common multiple of their numerators for a numerator, and the greatest common divisor of their denominators for a denominator. This fraction will be the least common multiple sought.*

EXAMPLES.

1. What is the least common multiple of $\frac{1}{12}$, $\frac{3}{4}$, and $\frac{7}{8}$?

SOLUTION: Least common multiple of the numerators $= 21$
Greatest common divisor of the denominators $= \overline{4}$

Therefore, $\frac{21}{4} = 5\frac{1}{4}$, is the least common multiple sought.

NOTES.—1. When the denominators are prime to each other, the least common multiple is a whole number.

2. The least whole number that will contain two or more fractions in their lowest terms, an exact number of times, is the least common multiple of their numerators.

2. What is the least common multiple of $3\frac{1}{8}$, $\frac{2}{3}$, and $6\frac{1}{7}$?

3. Find the least common multiple of $\frac{3}{7}$, $\frac{4}{9}$, and $\frac{6}{10}$.

4. A, B, and C start together from the same point to travel around a mile race-course, in the same direction, till they all meet at one point. A walks $2\frac{3}{4}$ miles per hour; B, $3\frac{1}{2}$ miles; and C rides $10\frac{1}{2}$ miles. How many hours must they travel, and how many times must each go around the course?

Ans. $115\frac{1}{2}$ hours.
A, $317\frac{5}{8}$ times.
B, $404\frac{1}{4}$ times.
C, $1212\frac{3}{4}$ times.

136. Miscellaneous Examples in Fractions.

1. Add $\frac{3}{14}$, $\frac{7}{9}$, $\frac{3}{28}$, and $\frac{1}{18}$.

2. From $275\frac{3}{8}$ take $27\frac{13}{16}$.

3. Reduce $\frac{7}{28}$, $\frac{31}{186}$, and $\frac{350}{750}$ to their lowest terms.

4. Reduce $\frac{8768}{435}$ to a mixed number.

5. What is the sum of $\frac{1}{3}$, $\frac{3}{4}$, $\frac{7}{12}$, $\frac{9}{24}$, and $\frac{11}{36}$?

6. From $\frac{1}{9}$ of $\frac{7}{8}$ take $\frac{2}{3}$ of $\frac{3}{216}$.

7. Find the value of $\frac{3}{4}$ of $\frac{4}{11}$ of $\frac{11}{23}$ of $\frac{16}{33}$.

8. Find the value of $(3\frac{1}{2} \times \frac{3}{5}) + 2\frac{1}{2} \times (3\frac{1}{5} - 2\frac{1}{2})$.

9. Multiply $4\frac{1}{2}$ by $9\frac{1}{3}$.

10. Multiply $445\frac{3}{4}$ by 17.

11. Multiply $1256\frac{1}{2}$ by $14\frac{1}{7}$.

12. Multiply $32\frac{3}{4}$ by $\frac{1}{8}$.

13. Divide $16\frac{1}{5}$ by $17\frac{1}{8}$.

14. Divide $\frac{3}{8}$ of $3\frac{1}{2}$ by $\frac{4}{5}$ of $16\frac{1}{4}$.

15. A man had $\frac{5}{6}$ of $\frac{2}{3}$ of $960, and paid $\frac{7}{8}$ of $\frac{2}{3}$ of it for a horse: how much had he left?

16. When apples are worth $$4\frac{5}{8}$ per barrel, what is $\frac{3}{4}$ of a barrel worth?

17. A person having a certain sum of money, paid $\frac{7}{16}$ of $\frac{1}{2}$ of it to B, and $\frac{1}{4}$ of $\frac{1}{8}$ of it to C, and then had $500 remaining: how much had he at first?

18. What number diminished by $\frac{1}{7}$ and $\frac{3}{14}$ of itself, leaves a remainder of 1358?

19. 376 is $\frac{3}{11}$ of what number?

20. What number is that $\frac{1}{3}$ of $\frac{3}{4}$ of which is 240?

21. What must be added to $631\frac{4}{5}$ that the sum will be $20\frac{1}{4}$ times $42\frac{1}{2}$?

22. Find the value of $\dfrac{13\frac{1}{3} + \frac{1}{5}}{\frac{1}{2} + \frac{2}{7}} \times \dfrac{\frac{7}{8} \text{ of } \frac{3}{11}}{\frac{1}{4} \text{ of } \frac{6}{11}}$.

23. A man being asked the value of his farm, replied that its value increased by $\frac{3}{25}$ of itself and $\$367\frac{5}{8}$ more, would be $\$3072\frac{7}{10}$: what was the value of his farm?

24. A and B own together 836 sheep: how many has each, if A has $2\frac{1}{6}$ times as many as B?

25. A merchant purchased 150 yards of cloth for $675, and sold $\frac{2}{3}$ of it at a profit of $\$\frac{5}{8}$ per yard, and the remainder at a loss of $\$\frac{1}{16}$ per yard: how much did he gain?

26. If $\frac{7}{8}$ of a stick of timber 14 feet long be sawed into five sections of equal length, what fractional part of the whole length will one of the sections be? What will be the length of each section?

27. If $6\frac{1}{2}$ tons of hay cost $\$124\frac{1}{2}$, how many tons can be bought for $\$373\frac{1}{3}$, at the same rate?

28. A can do a certain piece of work in 12 days, and B can do the same in 9 days: in how many days can both do it?

29. James has 36 cents, which is $\frac{1}{2}$ of $\frac{3}{4}$ of William's money: how much money has William?

30. If 18 horses consume $14\frac{1}{2}$ bushels of oats in $32\frac{1}{2}$ days. how many bushels will ten horses consume in $7\frac{1}{2}$ days?

31. If $\frac{7}{8}$ of a dollar will purchase $2\frac{1}{2}$ lbs. of coffee, how many pounds can be purchased for $5\frac{1}{2}$ dollars?

32. The sum of two fractions is $\frac{27}{32}$, and their difference $\frac{1}{8}$. what are the fractions?

33. $3\frac{1}{2}$ times a certain number added to 1 equals $\frac{7}{8}$ of the quotient obtained by dividing $4\frac{3}{7}$ by $1\frac{4}{5}$: what is the number?

Ans. $\frac{148}{567}$.

34. A gentleman invested $\frac{1}{2}$ of his money in real estate, $\frac{1}{9}$ of it in railroad stock, and the rest he deposited in a savings bank. The amount which he deposited in bank was $4500 more than he invested in stocks. How much money had he?

Ans. $16200.

35. The product of the sum and difference of two fractions is $\frac{161}{400}$, and their sum is $1\frac{3}{20}$: what are the fractions?

Ans. $\frac{3}{4}$ and $\frac{2}{5}$.

DECIMAL FRACTIONS.

137. A Decimal Fraction is a fraction whose denominator is some power of 10; as, 10, 100, 1000, etc.

Decimal Fractions are briefly called Decimals. The word *decimal* is derived from the Latin *decimalis*, from *decimus*, tenth, from *decem*, ten.

138. Decimals are distinguished from common fractions by the mode of their expression. The numerator of a decimal only is written, with a dot (.) called the *decimal point*, or *separatrix* (and ciphers when necessary) before it, to indicate the denominator. Thus:

$\frac{3}{10}$	is written	.3	$\frac{8}{100}$	is written	.08
$\frac{25}{100}$	" "	.25	$\frac{5}{1000}$	" "	.005

139. Decimals arise from successive divisions by 10, or by dividing a unit into 10, 100, 1000, etc. equal parts. A unit divided by 10, is *one tenth;* one tenth divided by 10, is *one hundredth;* one hundredth divided by 10, is *one thousandth,* and so on. Ten thousandths make one hundredth; ten hundredths, one tenth; ten tenths, a unit or one.

NOTATION AND NUMERATION.

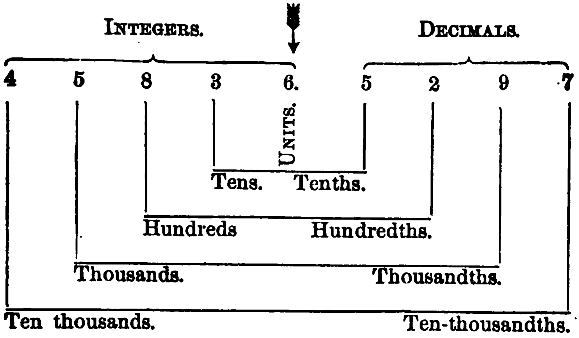
140. The system of **Decimal Notation** corresponds with that of integers. It is, in fact, but an extension of that system downward in the scale; hence its great practical convenience and utility. Decimals, alone or together with integers, may be operated upon in precisely the same manner as whole numbers.

In decimals, as well as integers, a unit of any order is ten times the value of a unit of the next lower order, and one-tenth the value of a unit of the next higher order. A decimal is, therefore, expressed by writing the numerator in a certain position with reference to the decimal point. Mixed numbers are written with the decimal point between the integers and the decimals. These principles are illustrated by the following

NOTATION AND NUMERATION TABLE.

Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.	Ten-Millionths.	Hundred-Millionths.	Billionths.
6	2	5	9	7	3	4	8	7	5	.	4	3	0	5	8	2	9	1	7
10th order.	9th order.	8th order.	7th order.	6th order.	5th order.	4th order.	3d order.	2d order.	1st order.		1st order.	2d order.	3d order.	4th order.	5th order.	6th order.	7th order.	8th order.	9th order.
INTEGERS.											DECIMALS.								

The first figure on the right of the decimal point expresses *tenths;* the second, *hundredths;* the third, *thousandths,* etc. And, beginning with units and numerating both ways, we find, left and right, integers and decimals of corresponding names equidistant from the units' place, as shown by the following diagram:



141. It follows from this law of local value that, in writing decimals, vacant orders must be filled with ciphers. For example, to express five hundredths, 5 must be written in the second decimal place, and a cipher in the first; thus, .05.

142. We also observe that every decimal place indicates a cipher in the denominator, when the decimal is written as a common fraction; that is, *the denominator is always* 1 *with as many ciphers annexed as there are places in the decimal.*

143. Since the value of a figure depends upon its position with reference to the decimal point, the removal of a figure one place to the left, by changing the location of the point, *multiplies* the number it represents by 10; and the removal of it one place to the right *divides* the number by 10. Hence,

1. *The prefixing of a cipher to a decimal* (*on the right of the point*) *divides it by* 10, *for it removes all the figures one place to the right.*

2. *The annexing of a cipher to a decimal changes its denomination, but does not alter its value, for it does not change the place of any figure.*

ILLUSTRATIONS.—1. If a cipher be prefixed to .5 it becomes .05, which is the quotient of .5 divided by 10, for $\frac{5}{10} \div 10 = \frac{5}{10} \times \frac{1}{10} = \frac{5}{100} = .05$.

2. If a cipher be annexed to .5 it becomes .50, which is of the same value as .5, for a cipher is thus added to the denominator; that is, both terms of the fraction are multiplied by the same number; $.50 = \frac{50}{100} = \frac{5}{10} = .5$.

144. From these principles we deduce the

RULE FOR DECIMAL NOTATION.

Write the numerator as a whole number, and prefix the decimal point, with ciphers if necessary, so as to make as many decimal places as the denominator contains ciphers.

And also the

RULE FOR DECIMAL NUMERATION.

Read the significant figures as a whole number expressing so many units of the lowest denomination contained in the decimal.

NOTES.—1. A decimal may be read otherwise. Thus, .234 may be read. two tenths, three hundredths, and four thousandths.

2. A common fraction is frequently connected with a decimal; as, $.45\frac{1}{3}$. In such a case, the common fraction is a fraction of a unit of the lowest decimal order. In this instance, it is one-third of one hundredth. The whole is read, forty five and one third hundredths.

EXERCISES IN NOTATION.

145. Write the following numbers according to the decimal system of notation:—

1. Nine tenths.
2. Seven hundredths.
3. Fourteen thousandths.
4. Two hundred and twelve thousandths.
5. Fifty-four, and three tenths.
6. One hundred and sixty, and six hundredths.
7. Eighteen, and nineteen thousandths.
8. One thousand, and one thousandth.
9. Write the following fractions decimally: $\frac{7}{10}$, $\frac{15}{100}$, $\frac{201}{1000}$, $\frac{3}{10000}$, $\frac{865}{1000000}$, $\frac{1300}{1000}$, $\frac{186}{10}$, $\frac{195}{1000}$.
10. Express decimally, $3\frac{5}{10}$, $18\frac{75}{1000}$, $223\frac{475}{10000}$, $4000\frac{1}{1000}$, $555\frac{55}{1000}$, $16\frac{25}{100}$, $1\frac{1}{1000000}$.
11. How many millionths are equal to five thousandths?

Ans. 5000.

EXERCISES IN NUMERATION.

Read the following:—

1. .4
2. .37
3. .381
4. .6567
5. 18.75
6. 23.001
7. 91.225
8. 62.12$\frac{1}{2}$
9. 10000.001
10. 10001.10001
11. 145658.888
12. 3762.000202

REDUCTION OF DECIMALS.

CASE I.

146. To reduce a decimal to a common fraction.

This operation consists simply in changing the form of the expression. Hence the

RULE.—*Erase or omit the decimal point and ciphers prefixed, if there be any, and write the significant figures of the decimal over its denominator, which is a unit with as many ciphers annexed as there are places in the decimal.*

EXAMPLES.

1. Reduce .9 to a common fraction. *Ans.* $\frac{9}{10}$.
2. Reduce .125 to a common fraction.
3. Reduce .003 to a common fraction.
4. Reduce .00375 to a common fraction.
5. Reduce .12$\frac{1}{2}$ to a common fraction.
6. Reduce .46$\frac{1}{3}$ to a common fraction.
7. Reduce .33$\frac{1}{3}$ to a common fraction.
8. Reduce 3.66$\frac{2}{3}$ to a common fraction.

CASE II.

147. To reduce a common fraction to a decimal.

EXAMPLE. Reduce $\frac{3}{4}$ to a decimal fraction.

OPERATION.

$$\begin{array}{r} 4)\underline{3.00} \\ .75, \textit{ Ans.} \end{array}$$

ANALYSIS.—Since the fraction $\frac{3}{4}$ is less than a unit, and is equal to $\frac{1}{4}$ of 3, we first reduce the numerator to tenths by annexing a cipher, and then divide. 4 is contained in 30 7 times, or, $\frac{1}{4}$ of 30 tenths is 7 tenths, with a remainder of 2. To continue the division we annex another cipher, which reduces this remainder to hundredths, and find that 4 is contained in 20 5 times. Now, since we have annexed two ciphers, or multiplied the numerator by 100, we have reduced it to hundredths, and $\frac{1}{4}$ of 300 hundredths equals 75 hundredths = .75, *Ans.*

To prove in another way the correctness of this process, multiply both terms of the fraction by 100, and divide both terms by 4. Thus, $\frac{3}{4} = \frac{300}{400} = \frac{75}{100} = .75$. Hence the following

RULE.—*Annex one or more ciphers to the numerator, and divide by the denominator. Point off in the quotient as many decimal places as are equal to the number of ciphers annexed.*

NOTES.—1. When an exact decimal cannot be obtained, the remainder may be written as a common fraction, or the sign + affixed, to show that the division does not terminate.

2. A common fraction cannot be reduced to an exact equivalent decimal, if, when in its lowest terms, the denominator contains any other prime factor than 2 or 5, because no other number is a measure of 10 or any of its multiples.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{5}{8}$ to a decimal. *Ans.* 625.

2. Reduce $\frac{3}{16}$ to a decimal.

3. Reduce $\frac{1}{125}$ to a decimal.

4. Reduce $.25\frac{1}{8}$ to an equivalent simple decimal.

5. Reduce $\$54\frac{3}{4}$ to an equivalent mixed decimal.

6. Reduce $\frac{99}{100}$ to a decimal.

7. Reduce $\frac{33}{888}$ to a decimal correct to the fifth place.

8. Can $\frac{5}{27}$ be accurately expressed in decimals? If not, reduce it to thousandths, and express the remainder as a common fraction.

9. Reduce the following fractions to decimals: $\frac{175}{320}$, $\frac{21}{442}$, $\frac{368}{640}$, $\frac{8}{25}$, $\frac{1}{3}$, $\frac{13}{14}$, $\frac{756}{12800}$, $\frac{46}{85}$, $\frac{900}{1200}$, $\frac{37\frac{1}{2}}{80}$, $\frac{1625}{5000}$, $\frac{400}{1502}$, $\frac{2\frac{1}{2}}{4}$, $\frac{1\frac{1}{3}}{10}$.

ADDITION OF DECIMALS.

148. Since decimals are expressed according to the same system of notation as integers, with respect to the law of local value, they may be added, subtracted, multiplied, and divided, in the same manner as integers.

149. In ADDITION, it is necessary to observe that, as in whole numbers, only units of like orders can be added. Hence, units must be written under units, tenths under tenths, etc. We then add as in whole numbers, beginning with the lowest decimal order.

Example. Find the sum of 3.75, 77.956, 40.04, and 56.1258.

OPERATION.

$$\begin{array}{r} 3.75 \\ 78.956 \\ 40.04 \\ 56.1258 \\ \hline 178.8718 \end{array}$$

Analysis.—Writing the numbers so that the decimal points fall in the same vertical line, the figures of the different orders stand, respectively, in the same columns; that is, units under units, tenths under tenths, etc. Then, commencing at the right, we add as in simple numbers, and place the decimal point in the result directly under the points in the numbers added. Hence the

Rule.—I. *Write the numbers so that the decimal points shall stand in the same vertical line.*

II. *Add as in whole numbers, and place the decimal point in the sum directly under the points in the numbers added.*

Examples for Practice.

1. Add .63$\frac{1}{3}$, 1.37$\frac{3}{4}$, 2.1875, 38.005625, and 5.53$\frac{1}{3}$.

OPERATION.

$$\begin{array}{rcl} .63\frac{1}{3} & = & .633333\frac{1}{3} \\ 1.37\frac{3}{4} & = & 1.3775 \\ 2.1875 & = & 2.1875 \\ 38.005625 & = & 38.005625 \\ 5.53\frac{1}{3} & = & 5.533333\frac{1}{3} \\ \hline & & 47.737281\frac{2}{3} \end{array}$$

Note.—In a case like this, expand the decimals which terminate with common fractions, to as many places as that one has which contains the largest number.

2. Add 3.65, 42.475, 37.55, 18.09, and .75.

3. Add 165.05, 326.75, 116.125, 300, and .5.

4. Add .12$\frac{1}{2}$, 4.6$\frac{3}{4}$, 12.67$\frac{1}{8}$, and 9.75$\frac{3}{16}$.

5. What is the sum of fifteen millions and ten, three thousand and six, one millionth, twenty, and five tenths?

Ans. 15003036.500001.

6. Find the sum of $75.50, 126.37\frac{1}{2}$, 150.62\frac{1}{2}$, $1850.05, and $195.46?

7. What is the sum of one, one tenth, one hundredth, one

thousandth, one ten thousandth, and one one hundred thousandth?

8. Find the sum of $\frac{18}{19}$, $\frac{162}{376}$, and $\frac{13}{17}$, in decimals, correct to the fourth place.

SUBTRACTION OF DECIMALS.

150. In **Subtraction of Decimals,** we have only to arrange the given numbers so that the figures of the subtrahend shall stand under those of the minuend of like orders, and then proceed as in the subtraction of simple numbers.

Example. From 2395.45 take 750.875.

OPERATION.

```
2395.45
 750.875
────────
1644.595, Ans.
```

Analysis.—We write the numbers so that the decimal point of the subtrahend stands under that of the minuend. This brings units under units, tenths under tenths, etc. We then subtract as in whole numbers, and place the decimal point in the remainder directly under the points in the given numbers. A cipher is supposed to occupy the place of thousandths in the minuend, which, if supplied, would reduce the fractions to the same denomination. Hence the

Rule.—I. *Write the numbers so that the decimal points shall stand one under the other.*

II. *Subtract as in whole numbers, and place the decimal point in the remainder directly under the points of the minuend and subtrahend.*

Examples for Practice.

1. From 319.05 take 75.5. *Ans.* 243.55.

2. From 2.95 take 1.8385.

3. From 5675 take one-tenth.

4. From one thousandth subtract one millionth.

5. What is the difference between 7 and .00007?

6. From fifty-three, and ninety thousandths, take ten, and three one-hundred-thousandths.

7. From 27.16 acres subtract 6.75 acres.

8. From $150 take $1.50.

9. From 1.8125 take $.33\frac{1}{3}$.

10. From $$1.12\frac{1}{2}$ take $$0.37\frac{3}{4}$.

11. Reduce $\frac{198}{3127}$ and $\frac{26}{999}$ to decimals, correct to the fourth place, and find the difference between them.

12. From one million take one-millionth.

13. From .2645+ take .03752+.

14. What is the difference between 13.475 and 10.125?

15. From 1000 take .00001.

MULTIPLICATION OF DECIMALS.

151. Decimals are multiplied like integers; but the location of the decimal point depends on principles which are illustrated by the following examples.

EXAMPLE 1. Multiply .25 by .5.

OPERATION.

$$\begin{array}{r} .25 \\ .5 \\ \hline .125,\ \textit{Ans.} \end{array}$$

ANALYSIS.—First, multiplying as though both factors were whole numbers, we obtain the product 125. But the multiplicand is *twenty-five hundredths*, and the multiplier *five tenths*, and the product of hundredths multiplied by tenths is thousandths; for, $\frac{1}{100} \times \frac{1}{10} = \frac{1}{1000}$; and $\frac{25}{100} \times \frac{5}{10} = \frac{125}{1000} = .125$. Therefore, we must point off from the right three decimal places, or as many as there are in both factors.

EXAMPLE 2. Multiply .038 by .15.

OPERATION.

$$\begin{array}{r} .038 \\ .15 \\ \hline .00570,\ \textit{Ans} \end{array}$$

ANALYSIS.—In this case there are not as many figures in the product as there are decimals in the factors; therefore, two ciphers must be prefixed to supply the deficiency. This gives the correct result; for, $\frac{38}{1000} \times \frac{15}{100} = \frac{570}{100000}$. Hence the following

RULE.—*Multiply as in whole numbers, and point off from the right of the product as many figures for decimals as there are decimal places in both factors. If there be not so many in the product, supply the deficiency by prefixing ciphers.*

NOTE.—To multiply a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the right as there are ciphers in the multiplier. When the number of figures in the multiplicand is less than the number of ciphers in the multiplier, annex ciphers to supply the deficiency. Thus, $.25 \times 10 = 2.5$; $.5 \times 100 = 50$.

EXAMPLES FOR PRACTICE.

1. Multiply .475 by 1.35.

2. Multiply 218.62 by 7.48.

3. Multiply .08 by .001.

4. Multiply 637.125 by 3.816.

5. Multiply .31 by .200001.

6. Multiply twenty-five ten-thousandths by two hundred and seventy-five, and fourteen one-hundredths.

7. What is the product of $325.5 \times 4.3 \times .12\frac{1}{2}$?

8. What is the value of .25 of $\frac{9}{12}$?

9. What is .01 of $\frac{3}{4}$?

10. At \$0.87½ per pound, what will be the cost of .5 of a pound of tea?

11. Multiply 98.75 by 100.

12. What will 142 yards of broadcloth cost at \$5.62½ per yard?

13. A man purchased a farm containing 342.5 acres, at \$54.12½ per acre: how much did the farm cost him?

14. What is one tenth of four times three hundredths?

Find the value of the following:

1. 287.7 × 1000
2. 372 × .0001
3. .9 × 10
4. .9 × 1000
5. 636.85 × 141.01
6. 10001 × .100001
7. 25 × .13½
3. 70000 × 70001.001

DIVISION OF DECIMALS.

152. We have seen that, if one decimal fraction be multiplied by another, the product will contain as many decimal places as there are in both factors. Now, since division is the reverse of multiplication, the dividend corresponding to the product, and the divisor and quotient to the factors, it is evident that the dividend must contain as many decimal places as the divisor and quotient together. Therefore, there must be as many decimal places in the quotient as the number in the dividend exceeds the number in the divisor.

153. Three different cases occur, of which the following are examples:

CASE I.

When the number of decimal places in the dividend *exceeds* the number in the divisor.

EXAMPLE. Divide 65.856 by 6.72.

OPERATION.

6.72)65.856(9.8, *Ans.*
6048
5376
5376

ANALYSIS.—Dividing as in whole numbers, we obtain the quotient 98. But, the dividend is 65856 thousandths, and the divisor 672 hundredths; and, thousandths divided by hundredths gives a quotient in tenths; for $\frac{1}{1000} \div \frac{1}{100} = \frac{1}{1000} \times \frac{100}{1} = \frac{1}{10}$; and $\frac{65856}{1000} \div \frac{672}{100} = \frac{65856}{1000} \times \frac{100}{672} = \frac{98}{10} = 9.8$. Therefore, we must point off one figure for a decimal, or as many as the number of decimal places in the dividend exceeds the number in the divisor.

CASE II.

When the number of decimal places in the dividend *equals* the number in the divisor.

Example. Divide 61.04 by 4.36.

OPERATION.

```
4.36)61.04(14, Ans.
     436
     ----
     1744
     1744
```

Analysis.—In this case the quotient is a whole number, for one fraction is contained in another of the same denomination a whole number of times, if the dividend be greater than the divisor. Proof: $\frac{6104}{100} \div \frac{436}{100} = \frac{6104}{100} \times \frac{100}{436} = \frac{6104}{436} = 14$.

Case III.

When the number of decimal places in the dividend is *less* than the number in the divisor.

Example. Divide 60.25 by 3.125.

OPERATION.

```
3.125)56.250(18, Ans.
      3125
      -----
      25000
      25000
```

Analysis.—We first equalize the decimal places of the divisor and dividend, by annexing ciphers to the dividend, which does not alter its value. Then, performing the division, the quotient is a whole number, upon the principle explained in the last example. Hence we derive the following

Rule.—*Divide as in simple numbers, and point off, from the right of the quotient, as many figures for decimals as the decimal places in the dividend exceed those in the divisor.*

Notes.—1. Should the number of figures in the quotient be insufficient for the required number of decimal places, supply the deficiency by prefixing ciphers.

2. When the divisor has the greater number of decimal places, annex as many ciphers to the dividend as will make its decimal places equal to those of the divisor. If then the dividend be greater than the divisor, the quotient will be a whole number, till after all the figures of the dividend shall have been used in the division.

3. When the dividend is less than the divisor, ciphers must be annexed to the former, and the quotient will be entirely decimal.

4. To continue the division when there is a remainder, annex ciphers, and consider them decimal places of the dividend.

5. When the decimal is not exact, the sign + may be affixed to show that the division may be continued.

6. A decimal may be divided by 10, 100, 1000, etc., by removing the decimal point as many places to the left as there are ciphers in the divisor.

EXAMPLES FOR PRACTICE.

1. Divide .16 by .2.

2. Divide 32.864 by .08.

3. Divide .00075 by .015. *Ans.* .05.

4. Divide .005 by .000005.

5. Divide 9 by 1350,

6. Divide 892.7 by 1000. *Ans.* .8927.

7. Divide $400.50 by .5.

8. Divide 1000 by .01.

9. Divide 156 by 15.6.

10. Divide 400 thousandths by 800 ten-thousandths.

11. Find the quotient of 4 divided by 87, correct to the fourth decimal place.

12. Express $\frac{7}{8}$ of $1 decimally.

13. How many times will .12 of 12 be contained in .24 of 72?

14. At $1.25 per bushel, how many bushels of corn can be bought for $1875?

15. Divide 1000001 by 1.0000001.

16. Divide 56 by .008.

17. Reduce $\frac{3}{25}$ to a decimal.

18. Divide $\frac{3}{8}$ of .24 by $\frac{3}{14}$ of .42.

19. Divide .3$\frac{1}{3}$ by .2 correctly to the fifth quotient figure.

20. Divide .0001 by 1000.

21. Divide 13.5 by 100
22. Divide .1 by 10
23. Divide 4.06 by 1000
24. Divide 218.6 by .1
25. Divide 437.562 by 22.01
26. Divide 89.6 by 10000
27. Divide .0001 of 10 by .1 of 1000
28. Divide 10.0100 by 101.001
29. Divide 41675 by 18.5

CIRCULATING DECIMALS.

154. A **Circulating**, or **Repeating Decimal**, is a decimal in which a figure, or period of figures, is repeated continually to infinity; as, 333, etc.; 123123, etc.

155. Circulating decimals arise from the reduction of common fractions, when they cannot be accurately expressed in decimals. With the first, or after a certain number of divisions, a figure, or number of figures in regular rotation, will recur, or repeat; hence the name.

156. The figure, or period repeated, is called a *Repetend,* and is marked by a point placed over a single figure, or by points over the first and last figures of a period; thus, .333, etc., and .123123, etc., are written $.\dot{3}$ and $.\dot{1}2\dot{3}$.

157. The reduction of any common fraction whose denominator, when in its lowest terms, contains any other prime factors than 2 or 5, will produce a repeating decimal; for, annexing a cipher to the numerator multiplies it by 10, and neither 10, nor any of its multiples, is divisible by any other prime number, without a remainder.

158. If decimal figures precede the repetend, they are called the *finite* part of the decimal.

159. When a decimal begins with a repetend, it is called a *Pure Circulating Decimal;* as, $\dot{4}$, $\dot{7}\dot{2}$. When it consists of a finite part, followed by a repetend, it is called a *Mixed Circulating Decimal;* as, $.2\dot{6}$, $.08\dot{3}$.

REDUCTION OF CIRCULATING DECIMALS TO COMMON FRACTIONS.

160. It has been found that every repetend is equal to a common fraction whose numerator consists of the figure or figures of the repetend, and whose denominator is as many 9's as there are places in the repetend.

The numerical value of a fraction thus derived depends on the local value of the repeating figures or period. Thus, $.\dot{4} = \frac{4}{9}$; but $.1\dot{6} = \frac{1}{10} + \frac{6}{90} = \frac{9}{90} + \frac{6}{90} = \frac{15}{90} = \frac{1}{6}$. The repetend commences in the place of hundredths, and its value is $\frac{1}{10}$ of $\frac{6}{9} = \frac{6}{90}$. Hence, for every figure, or place of a finite decimal, a cipher must be annexed to the 9's in the denominator.

EXAMPLE 1. Reduce $.\dot{3}\dot{6}$ to a common fraction.

OPERATION.

$.\dot{3}\dot{6} = \frac{36}{99} = \frac{12}{33}$. *Ans.* $\frac{12}{33}$.

ALGEBRAICAL SOLUTION.

Let $d = \dot{3}\dot{6}$	Put d for *decimal* and form an equation.
$100\,d = 36.\dot{3}\dot{6}$	Multiply both members of the equation by 100.
$d = .\dot{3}\dot{6}$	Subtract the first equation.
$99\,d = 36$	Divide both members of the equation by 99.
$d = \frac{36}{99}$	Hence the decimal $.\dot{3}\dot{6} = \frac{36}{99} = \frac{12}{33}$.

EXAMPLE 2. Reduce $.57\dot{8}$ to a common fraction.

OPERATION.

$.57\dot{8} = \frac{57}{100} + \frac{8}{900} = \frac{513}{900} + \frac{8}{900} = \frac{521}{900}$, *Ans.*

Or, $.57\dot{8} = .57\frac{8}{9} = \dfrac{57\frac{8}{9}}{100} = \frac{521}{900}$.

Or, 578, the given decimal taken as a whole number.
57, finite part subtracted.

521, the required numerator.
900, the proper denominator written underneath.

ANALYSIS.—We first write the finite part and the repetend separately, as common fractions, and then add them. As explained before, the repetend is not $\frac{8}{9}$ but $\frac{8}{9}$ of $\frac{1}{100} = \frac{8}{900}$. This is also shown in the second solution.

The third, or abbreviated method, is derived from the mode of reducing the fractions to a common denomination. We multiply both terms of 57 by 9; but multiplying by 10, and subtracting the multiplicand is equivalent to multiplying by 9. Therefore, $57 \times 9 = 570 - 57$; and $\overline{570 - 57} + 8 = 578 - 57 = 521$, the required numerator.

ALGEBRAICAL SOLUTION.

Let $d = .57\dot{8}$

$1000\,d = 578.8$ Multiply both members by 1000.

$100\,d = 57.8$ Multiply both members by 100, and subtract.

$900\,d = 521$ Divide both members by 900.

$d = \frac{521}{900}$ Hence, $.57\dot{8} = \frac{521}{900}$.

Hence, to reduce a circulating decimal to a common fraction, we have the following

RULE.—I. *If the decimal be a pure circulating decimal, write the figures of the repetend, with the points and decimal sign omitted, for the numerator, and as many 9's as there are places in the repetend, for the denominator.*

II. *If the decimal be a mixed circulating decimal, subtract the finite part from the whole decimal, treating both as whole numbers. The remainder will be the required numerator. For the denominator, write as many 9's as there are figures in the repetend, with as many ciphers annexed as there are places in the finite part.*

EXAMPLES FOR PRACTICE.

1. Reduce $.0\dot{4}\dot{5}$ to a common fraction.

2. Reduce $.08\dot{3}$ to a common fraction.

3. Reduce $.\dot{6}3\dot{4}$ to a common fraction.

4. Reduce $2.5\dot{6}\dot{4}$ to a mixed number in common fractions.

5. Reduce $.7\dot{8}5\dot{2}$ to a common fraction.

6. What is the value of $.02\dot{6} \times \frac{3}{4}$?

7. Find the value of $241.\dot{7}2\dot{3}$.

NOTE.—Circulating decimals admit of changes in form and character, without altering their value. They may be expanded: thus, $.\dot{4}\dot{5} = .\dot{4}5454\dot{5}$; and $.\dot{3}2\dot{4} = .\dot{3}2432\dot{4}$. So the points may be removed to the right: thus, $.\dot{3}2\dot{4} = .32\dot{4}3243\dot{2}$; or, $.3\dot{2}4\dot{3}$. Hence, dissimilar decimals may be made similar and conterminous, so that they may be added and subtracted.

without reduction to common fractions. Yet as this branch of the subject is of very little importance, the process being seldom used, a discussion of it here is omitted. Therefore, to add, subtract, multiply, or divide circulating decimals, *reduce them to common fractions, and then perform the required operation.*

MONEY AND CURRENCY.

161. **Money** is the *medium of exchange* used by individuals and nations in transferring property from one to another, and passes as an equivalent or artificial measure of its value.

162. Money is known, commercially, as *Cash, Specie, Currency, Bullion,* etc.

163. **Cash** is a general term applied to money as a circulating medium.

164. **Specie** is coin.

165. **Currency** is a term applied to all kinds of money, both coin and paper, which constitute the circulating medium of a country. In a limited commercial sense, it means paper money only.

166. **Paper Money** consists of bills or notes issued by banks and the Government, as substitutes or representatives of coin.

167. **Bullion** is gold or silver in bars, or ingots, or in the mass.

168. The expression *Money of Account* denotes the denominations of money in which *accounts* are kept in different countries. These denominations correspond with certain denominations of the current money—usually that which is taken as the unit and its subdivisions; as, in the United States, dollars and cents; and, in England, pounds, shillings, and pence.

COINS.

169. **Coins** are pieces of metal (generally gold, silver, or copper, in combination with some alloy) impressed or stamped with devices and

figures, by authority of the government, to give them legal and fixed values as current money.

In comparison with our own standards, foreign coin may be said to have four values, viz.:—

1st. The **Intrinsic Value,** which is the value of the pure metal it contains.

2d. The **Legal Value,** which is the value fixed by law, to be allowed for it in the payment of duties, and all other business transactions with the government.

3d. The **Commercial** or **Market Value,** which is the price it commands when offered for sale or exchange, or in payment for goods.

4th. The **Exchange Value,** which is the value at which it is reckoned in the purchase and sale of bills of exchange.

170. For foreign silver coins there is no fixed legal valuation, the law providing for the changing of the price, according to the condition of demand and supply. The present price of standard silver is 122½ cents per ounce.

171. The Coins of the United States are a legal tender in any payment, according to their nominal value, as follows:

Gold Coins, for any amount.

Silver Coins, except three-cent pieces, for any amount not exceeding five dollars.

Three-cent pieces, Silver, to the amount of thirty cents.

Three-cent pieces, Nickel, to the amount of sixty cents.

Five-cent pieces, Nickel, to the amount of one dollar.

One and two-cent pieces, to the amount of four cents.

TABLES OF FOREIGN AND UNITED STATES COINS.

172. The following Tables were prepared by the Director of the United States Mint at Philadelphia, and accompany the United States Treasurer's Report on Finance for 1866, and may, therefore, be regarded as reliable.

EXPLANATIONS.—Only the principal denominations of Foreign Coins are given, except when the sizes are not proportional; and in that case the deviation is stated.

The weight of each piece is expressed in the decimal of a Troy ounce; the fineness in thousands, or the number of parts of pure gold in one thousand parts of the coin.

In the fifth column is shown the value as compared with the amount of fine gold in our coins or the value for any other purpose than recoinage.

The sixth column gives the value in exchange for our coins at the Mint, after the uniform deduction of one-half of one per cent.

FOREIGN GOLD COINS.

COUNTRY.	DENOMINATION.	WEIGHT.	FINENESS.	VALUE.	VALUE AFTER DEDUC'N.
		Oz. Dec.	*Thous*		*Price paid at Mint.*
Australia	Pound of 1852	0.281	916.5	$5.32 37	$5.29.71
do	Sovereign of 1855-60	0.256.5	916	4.85.58	4.83.16
Austria	Ducat	0.112	986	2.28.28	2.27.04
do	Souverain	0.363	900	6.75.35	6.71.98
do	New Union crown (assumed).	0.357	900	6.64.19	6.60.87
Belgium	Twenty-five francs	0.254	899	4.72.03	4.69.67
Bolivia	Doubloon	0.867	870	15.59.25	15 51.46
Brazil	Twenty milreis	0.575	917.5	10.90.57	10.85.12
Central America	Two escudos	0.209	853.5	3.68.75	3.66.91
do do	Four reals.	0.027	875	0.48.8	0.48.6
Chili	Old doubloon	0.867	870	15.59.26	15.51.47
do	Ten pesos	0.492	900	9 15.35	9.10.78
Denmark	Ten thaler	0.427	895	7.90.01	7.86.06
Ecuador	Four escudos	0.433	844	7.55.46	7.51.69
England	Pound or sovereign, new	0.256.7	916.5	4.86.34	4.83.91
do	Pound or sovereign, average.	0.256.2	916	4.84.92	4.82.50
France	Twenty francs, new	0.207.5	899.5	3.85.83	3.83.91
do	Twenty francs, average	0.207	899	3.84.69	3.82.77
Germany, North.	Ten thaler	0.427	895	7.90.01	7.86.06
do do	Ten thaler, Prussian	0.427	903	9.97.07	7.93.09
do do	Krone (crown)	0.357	900	6.64.20	6.60.88
Germany, South.	Ducat	0.112	986	2.28.28	2.27.14
Greece	Twenty drachms	0.185	900	3.44.19	3.42.47
Hindostan	Mohur	0.374	916	7.08.18	7.04.64
Italy	Twenty lire	0.207	898	3.84.26	3.82.34
Japan	Old Cobang	0.362	568	4.44.0	4.41.8
do	New Cobang	0.289	572	3.57.6	3.55.8
Mexico	Doubloon, average	0.867.5	866	15.52.98	15.45.22
do	Doubloon, new	0.867.5	870.5	15.61.05	15.53.25
Naples	Six ducati, new	0.245	996	5.04.43	5.01.91
Netherlands	Ten guilders	0.215	899	3.99.56	3.97.57
New Grenada	Old doubloon, Bogota	0.868	870	15.61.06	15.53.26
do do	Old doubloon, Popayan	0.867	858	15.37.75	15.30.07
do do	Ten pesos, new	0.525	891.5	9.67.51	9.62.68
Peru	Old doubloon	0.867	868	15.55.67	15.47.90
do	Twenty soles	1.035	898	19.21.8	19.12.2
Portugal	Gold crown	0.308	912	5.80.66	5.77.76
Prussia	New Union crown (assumed).	0.357	900	6.64.19	6.60.87
Rome	Two-and-a-half scudi, new	0.140	900	2.60.47	2 59.17
Russia	Five roubles	0.210	916	3.97.64	3.95.66
Spain	One hundred reals.	0.268	896	4.96.39	4.93.91
do	Eighty reals	0.215	869.5	3.86.44	3.84.51
Sweden	Ducat	0.111	975	2.23.72	2.22.61
Tunis	Twenty-five piastres	0.161	900	2.99.54	2.98.05
Turkey	One hundred piastres	0.231	915	4.36.93	4.34.75
Tuscany	Sequin	0.112	999	2.31.29	2.30.14

WEIGHT AND VALUE OF UNITED STATES GOLD COINS.

	DENOMINATION.	WEIGHT.	FINENESS.	VALUE.	WEIGHT IN GRAINS.
		Oz. Dec.	*Thous.*		
United States	Dollar (legal)	0 053.75	900	$1.00	25.8
	Quarter eagle	0.134.37	900	2.50	64.5
	Three dollar	0.161.25	900	3.00	77.4
	Half eagle	0.268.75	900	5.00	129
	Eagle	0.537.5	900	10.00	258
	Double eagle	1.075	900	20.00	516

FOREIGN SILVER COINS.

COUNTRY.	DENOMINATION.	WEIGHT.	FINENESS.	VALUE.
		Oz Dec	*Thous.*	
Austria	Old rix dollar	0.902	833	$1.02.27
do	Old scudo	0.836	902	1.02.64
do	Florin, before 1858	0.451	833	.51.14
do	New florin	0.397	900	.48.63
do	New Union dollar	0.596	900	.73.01
do	Maria Theresa dollar, 1780	0.895	838	1.02.12
Belgium	Five francs	0.803	897	.98.04
Bolivia	New dollar	0.643	903.5	.79.07
do	Half dollar	0.432	667	.39.22
Brazil	Double milreis	0.820	918.5	1.02.55
Canada	Twenty cents	0.150	925	.18.87
Central America	Dollar	0.866	850	1.00.19
Chili	Old dollar	0.864	908	1.06.79
do	New dollar	0.801	900.5	.98.17
Denmark	Two rigsdaler	0.927	877	1.10.65
England	Shilling, new	0.182.5	924.5	.22.96
do	Shilling, average	0.178	925	.22.41
France	Five franc, average	0.800	900	.98.00
Germany, North	Thaler, before 1857	0.712	750	.72.67
do do	New thaler	0.595	900	.72.89
Germany, South	Florin, before 1857	0.340	900	.41.65
do do	Five drachms	0.719	900	.88.08
Hindostan	Rupee	0.374	916	.46.62
Japan	Itzebu	0.279	991	.37.63
do	New itzebu	0.279	890	.33.80
Mexico	Dollar, new	0.867.5	903	1.06.62
do	Dollar, average	0.866	901	1.06.20
Naples	Scudo	0.844	830	.95 84
Netherlands	Two-and-a-half guild	0.804	944	1.03.31
Norway	Specie daler	0.927	877	1.10.65
New Granada	Dollar of 1857	0.803	896	.97.92
Peru	Old dollar	0.866	901	1.06.20
do	Dollar of 1858	0.766	909	.94.77
do	Half dollar, 1835–38	0.433	650	.38.31
Prussia	Thaler, before 1857	0.712	750	.72.68
do	New thaler	0.595	900	.72.89
Rome	Scudo	0.864	900	1.05.84
Russia	Rouble	0.667	875	.79.44
Sardinia	Five lire	0.800	900	.98.00
Spain	New pistareen	0.166	809	.20.31
Sweden	Rix dollar	1.092	750	1.11.48
Switzerland	Two francs	0.323	899	.39.52
Tunis	Five piastres	0.511	898.5	.62.49
Turkey	Twenty piastres	0.770	830	.86.98
Tuscany	Florin	0.220	925	.27.60

WEIGHT AND VALUE OF UNITED STATES SILVER COINS.

	DENOMINATION.	WEIGHT.	FINENESS.	WEIGHT IN GRAINS.
		Oz. Dec.	*Thous.*	
United States	Dollar (legal)	0.859.375	900	412.5
	Half dollar	0.400	900	192
	Quarter dollar	0.200	900	96
	Dime	0.080	900	38.4
	Half dime	0.040	900	19.2
	Three cent	0.024	900	11.52

UNITED STATES MONEY.

173. The **Legal Standard Currency** of the United States, otherwise called Federal Money, consists of a decimal coinage, the dollar being the unit.

Government notes, known as legal tender notes and "greenbacks," may also be considered legal currency, but must be distinguished from the standard money.

174. The coins are as follows:

GOLD			SILVER		
Double Eagle	=	20 dollars	Dollar	=	100 cents
Eagle	=	10 dollars	Half Dollar	=	50 cents
Half Eagle	=	5 dollars	Quarter Dollar	=	25 cents
Quarter Eagle	=	2½ dollars	Dime	=	10 cents
Three Dollar Piece	=	3 dollars	Half Dime	=	5 cents
One Dollar Piece	=	1 dollar	Three Cent Piece	=	3 cents

NICKEL			COPPER		
Half Dime	=	5 cents	Two Cent Piece	=	2 cents
Three Cent Piece	=	3 cents	One Cent Piece	=	1 cent

NOTES.—1. Gold 50 dollar pieces (octagonal) were issued by the United States Assay Office in California, previous to the establishment of the Branch Mint at San Francisco; and, also, ½ and ¼ dollar gold pieces, by Moffat & Co., all of which are of the standard value, but they are not legal coins.

2. In addition to these there are in circulation 5, 2½, and 1 dollar gold pieces, issued by the Private Mints of Carolina and Georgia, which are worth about 5 per cent. less than the value expressed on their face.

3. The Mormons, in Utah, have also coined 20, 10, 5, and 2½ dollar gold pieces; but they are very deficient both in weight and fineness, and are not, of course, legal.

175. TABLE OF UNITED STATES MONEY.

10 Mills	make	1 Cent
10 Cents	"	1 Dime
10 Dimes	"	1 Dollar
10 Dollars	"	1 Eagle
20 Dollars	"	1 Double Eagle

The only denominations used in practice are *dollars*, *cents*, and *mills*. Hence,

176. The system of NOTATION AND NUMERATION corresponds with that of Decimal Fractions. Dollars, or units, are written as integers at the left of the decimal point, and cents and mills, or *hundredths* and *thousandths* of a dollar, as decimals, at the right.

177. The character $, called the dollar mark, denotes dollars, or United States Money, and is placed before the number.

178. To express cents or mills only, a cipher is generally written between the symbol, $, and the decimal point, and the proper decimal after it. To express any number of cents less than 10, a cipher must be placed between the figure and the decimal point, for cents are hundredths of a dollar. Thus,

1 mill is written $0.001.
1 cent is written $0.01.
1 dime, or ten cents, is written $0.10.
1 dollar is written $1.
1 dollar and twenty-five cents is written $1.25.

NOTE.—In business, cents are frequently expressed by common fractions, particularly in the bodies of notes and drafts.

179. EXAMPLES FOR PRACTICE.

1. Write twelve dollars, fifteen cents.
2. Write five dollars, five cents.
3. Write thirty-seven dollars, ten cents.
4. Write four dollars, nine cents.
5. Write five mills.
6. Write three hundred and fifty-five cents.
7. Write nine dollars, eight cents, and five mills.
8. Write two thousand mills.
9. Read $100.25, $75.375, $0.015, $18.003.

REDUCTION.

180. \$1 = 100 cents = 1000 mills; therefore,

1. To reduce dollars to cents, *annex two ciphers;* to mills, *three ciphers;* cents to mills, *one cipher.*

2. To reduce cents to dollars, *point off two figures from the right;* mills to dollars, *three figures;* and mills to cents, *one figure.*

NOTE.—The first rule must, of course, be understood with reference to an entire change of the expression; for, \$1 with two ciphers annexed would be \$100, *one hundred dollars;* and with a decimal point and two ciphers following, it would still express *one dollar,* and not one hundred cents.

OPERATIONS IN UNITED STATES MONEY.

181. Since Federal Money is a decimal currency, it may be added, subtracted, multiplied, and divided in the same manner as decimals.

EXAMPLES FOR PRACTICE.

1. What is the sum of \$132.37, \$275.86, \$1000, \$0.75, and \$9.09?

2. From \$436.75 take \$203.35.

3. Multiply \$37.50 by 16½.

4. Add together fifty-seven dollars, eighteen dollars and sixty cents, and forty-five dollars and twenty-five cents.

5. Find the difference between one hundred dollars and eleven dollars and eleven cents.

6. A farmer sold 10 doz. eggs, at 26 cents per doz., 36 lbs. of poultry, at 18 cents per lb., and 45 lbs. of butter, at 35½ cents per lb.: what did the whole amount to?

7. If ⅝ of a cord of wood cost \$5.50, what is the price per cord?

8. A man sold a horse for \$175, which was \$35 more than he paid for him: what did the horse cost him?

9. What will 37½ hundredths of a ton of coal cost, at $7.50 per ton?

10. A man sold 50 bushels of corn, at $1.37½ per bushel, and received in payment 4 yards of broad-cloth, at $4½ per yard, 10 lbs. of coffee, at 31 cents per lb., and the balance in cash: how much cash did he receive?

11. If ½ of .5 of a ton of hay is worth $5½, what is ¾ of .125 of a ton worth at the same rate?

NOTE.—In business calculations, mills in a final result are disregarded when less than 5, but are usually called a cent when 5 or more than 5.

12. How many yards of cloth, at $3.50 per yard, can be purchased for $31.50?

13. A speculator bought two pieces of land, one containing 275½ acres, at $52.50 per acre, and the other containing 140¾ acres, at $37.50 per acre, and sold the whole at a profit of $1258.12½: at what price per acre did he sell it?

ALIQUOT PARTS.

182. An **Aliquot Part** of a number is any fractional part of it which can be expressed by a simple fraction whose numerator is 1; as, ½, ⅓, ¼, ⅕, ⅒, etc. Thus, 2, 2½, 3⅓, and 5 are aliquot parts of 10. So 12½, 16⅔, 25, etc., are aliquot parts of 100.

The following are some of the aliquot parts of a dollar:

50 cents = ½ of $1
33⅓ cents = ⅓ of $1
25 cents = ¼ of $1
20 cents = ⅕ of $1

16⅔ cents = ⅙ of $1
12½ cents = ⅛ of $1
6¼ cents = $\frac{1}{16}$ of $1
5 cents = $\frac{1}{20}$ of $1

When the price of an article is an aliquot part of a dollar, and the cost of a quantity is required, the operation may be shortened by using the fraction which denotes the aliquot part as a multiplier.

EXAMPLE. What is the cost of 48 yards of print, at 12½ cents per yard?

SOLUTION.
8)48
—
6, *Ans.* $6.

ANALYSIS.—Since the price of 1 yard is ⅛ of $1, the cost will be ⅛ as many dollars as there are yards. Hence the

RULE.—*Take such part of the number of articles, or quantity, as the price is of* $1, *and the result will be the cost in dollars.*

183. When any aliquot part of a dollar occurs as a divisor, reverse this process:

RULE.—*To divide by an aliquot part of a dollar, divide by the fraction which expresses the part.*

EXAMPLE. How many pounds of sugar, at 16⅔ cents per pound, can be bought for $13?

OPERATION.
13
6
—
98 lbs., *Ans.*

ANALYSIS.—Since the price is ⅙ of a dollar per lb., $1 will buy 6 lbs., and $13 dollars will buy 6 times 13 lbs., or $13 \div \frac{1}{6} = 13 \times 6 = 98$.

EXAMPLES FOR PRACTICE.

1. Find the cost of 75 lbs. of dried apples, at 16⅔ cents per lb.

2. How many doz. oranges, at 25 cents per doz., can be bought for $5.

3. What is the cost of 27 yards of cambric, at 33⅓ cents per yard?

4. How many pounds of coffee, at 33⅓ cents per pound, can be bought for $14.50?

184. To find the cost of articles sold by the hundred or thousand.

Example. Required the cost of 762 feet of pine boards, at $35 per M.

OPERATION.

762
35
———
$26.670, *Ans.*

Analysis.—At $35 per foot the cost would be 762 × 35 = 26670 dollars; but this result is 1000 times too great, because the price is $35 per *thousand* feet; therefore we divide 26670 by 1000 by pointing off three figures at the right.

Note.—When the price is given by the hundred. point off two figures for decimals. Hence the

Rule.—*Multiply the quantity by the price, and divide by* 100 *or* 1000, *as the case may require.*

Examples.

1. What is the cost of 25 thousand envelopes, at $2.50 per M.?
2. Find the value of 1786 feet of square timber, at $35 per M.
3. What is the cost of 2684 bricks, at $8.50 per M?

185. **To find the cost of articles sold by the ton of 2000 pounds.**

Example. What is the cost of 1450 pounds of coal, at $8 per ton?

SOLUTION.

1450 or 725
4 8
——— ———
$5.800, *Ans.* $5.800

Analysis.—Since $8 is the price of 2000 lbs., we may take one-half of $8 as the price of 1000 lbs., and then proceed according to the previous rule; or when it is more convenient we may take one-half the quantity, multiply it by the whole price, and point off three figures for decimals. Hence the

Rule.—*Multiply the quantity in pounds by half the price, or half the quantity by the price, and divide by* 1000.

Examples.

1. Required the cost of 2860 lbs. of hay, at $20 per ton.
2. What is the freight of 6275 pounds of iron, at $1.50 per ton?

COMPOUND NUMBERS.

186. An **Abstract Number** is a number used without reference to a particular object; as, 5, 7.

187. A **Concrete,** or **Denominate Number,** is a number associated with, or applied to, a particular object or quantity; as, 5 dollars, 7 days, 3 yards.

188. A **Compound Number** is a number composed of concrete numbers of different denominations, according to a varying scale of units; as, 2 tons, 3 cwt., 1 qr.

189. A **Unit of Measure** is the fundamental unit adopted in any system of weights and measures, money, etc., by which its several denominations are regulated, and which is itself defined by comparison with some known, or assumed standard; as, the dollar, the yard, the gallon, the metre.

WEIGHTS AND MEASURES.

190. **Measure** is a standard of dimension, or that by which extent, capacity, or amount is ascertained.

The different kinds of quantity to which measures are applied are as follows:—

1. Lines, or Lengths;
2. Surfaces, or Areas;
3. Solidities, or Volumes;
4. Weights, or Force of Gravity;
5. Angles;
6. Time, or Duration;
7. Values, of which Money is the artificial measure.

There are also intensities, or forces; but, as these are measured by weights or lengths, they are included in the above classification.

ORIGIN OF STANDARDS.

191. **In order to define and express the measure of a quantity of any kind, we must first find in nature, or assume arbitrarily, some standard with which it may be compared.**

Absolutely invariable standards have, however, never yet been, and in the nature of things cannot be, attained; while to secure measures of the utmost exactness, the least degree of variability, and capable of being reproduced, if lost or destroyed, is a problem upon which a vast amount of scientific research, ingenuity, and labor have been expended.

Among the earlier measures of length of various nations were such as the finger's length, the digit (or finger's breadth), the palm, the span, the cubit (length of the forearm), the nail, the *orgyia* (stretch of the arms), the foot, pace, etc., whose origin, as indicated by their names, is supposed to have been the application of parts of the human body to define extension. The "grain" occurring in Troy Weight, the "barleycorn" in Long Measure, show what were the original, or "natural" units resorted to in forming these measures.

In the progress of men's minds toward exactness of conception and reasoning, these extremely variable and indefinite measures became, to a certain degree, fixed and established by conventional or arbitrary standards. This stage was doubtless reached by the Greeks and Romans, and, at an early period, by the English.

In England, whose system of weights and measures was early adopted by the United States, the first important step toward establishing definite and national standards was taken in the years 1736 and 1742, when experiments were made, under the auspices of the Royal Society, with the pendulum, with a view of deriving from the operation of an *unchangeable* law of nature, the *force of gravity*, an *invariable standard of length.*

These experiments were conducted by George Graham, who determined the length of a pendulum vibrating seconds of mean time, in the latitude of London, at the temperature of 62° F., in a vacuum, at the level of the sea, to be in the then known units of measure, 39.13 English inches. From this measurement he constructed a standard yard of 36 inches, of which, under the direction of the House of Commons, Mr. Bird prepared two accurate copies, respectively marked Standard Yard of "1758," and "1760." He also carefully determined and prepared the Troy pound, the original of that now used. Though these standards were afterward destroyed by the burning of the Parliament Houses, they were restored from copies, and are, without any intentional change, still in use in England and the United States.

Subsequent to this period, the French philosophers of the time of the revolution sought also for an *invariable natural standard,* on which to found a system of weights and measures. They selected a quadrant of the earth's meridian, or the distance from the equator to the pole, which was computed from a very minute survey of an arc of the meridian between Dunkirk and Barcelona. A ten-millionth part of this was taken as the fundamental unit of length, and was called the *metre.* The metre is equal to 39.37079 English inches, and is, therefore, very nearly the same as the length of the pendulum found in the manner above described. The system founded on this basis is called the metric system, and was first promulgated in 1799, and finally adopted, or made compulsory, in 1837.

The efforts of the French called attention in England to the desirableness of having standards commensurable with a natural unit; and, from investigations already made, the seconds pendulum was believed to afford the best and most tangible basis. This standard was, therefore, selected; but, before its final adoption, further experiments were made which resulted in the discovery of some slight errors in the work of Mr. Graham, or at least in defining the length of the pendulum more accurately than he had done, for it was declared to be 39.1393 inches instead of 39.13, as determined by him. Reports to the House of Commons, based on these experiments and comparisons made in 1816, 1818, and 1820, led to the adoption of the *Imperial Measures and Standards*, under the Act of Parliament, 5 George IV., which was passed in 1824, and took effect January 1st, 1826. This Act adopted the Standard Yard of 1760, and also the Troy pound of 1758, but introduced changes in the measures of capacity,—the gallon and bushel. With reference to linear measure, it prescribed the following rule:—"Take a pendulum which will vibrate seconds in London, on a level of the sea, in a vacuum, at a temperature of 62° F.; divide all that part thereof which lies between the axis of suspension and the centre of oscillation, into 391393 equal parts; then will 10000 of these parts be an imperial inch, 12 whereof make a foot, and 36 whereof make a yard."

Previous to the passage of this Act, the "Bird Standards" and the former English Standards of Dry and Liquid Measures had become established in the United States, and have since been legalized by Act of Congress. The only difference, therefore, between our own and the "Imperial System," is to be found in the standards for Liquid and Dry Measures, as hereafter explained.

UNITED STATES STANDARDS.

192. The United States Standards of Weights and Measures, originally taken from the English System, were finally established by Act of Congress in the year 1834.

193. The fundamental units are:—

1. The *Yard* = 3 feet = 36 inches.
2. The *Troy Pound* = 12 oz. = 240 pwt. = 5760 grs.
3. The *Gallon* = 231 cubic inches.
4. The *Bushel* = 2150.42 cubic inches.

194. The National Standard of Length is the *Yard*, as marked upon a brass scale 82 inches long, prepared by Troughton of London, and which is deposited in the Treasury Department at Washington. It is the same as the Imperial yard, or $\frac{360000}{391393}$ of the pendulum rod beating seconds at London.

195. The Standard of Weight is the *Troy Pound*, procured by the American Minister at London in 1827, and is a copy of the English standard pound of 1758. It was determined, according to the Act of George IV, as follows:—A cubic inch of distilled water, weighed in air by brass weights, at 62° F., the barometer being at 30 inches, is equal to 252.458 grains, of which the standard Troy pound contains 5760. This standard is kept at the United States Mint at Philadelphia.

196. The *Avoirdupois Pound* is 7000 grains Troy.

197. The Units of Capacity are the gallon for Liquid, and the bushel for Dry Measure.

198. The *Gallon* is the capacity of a vessel containing 58372.2 grains Troy of the standard pound, of distilled water, at its maximum density (40° F.), weighed in air of the temperature of 62°, under a barometric pressure of 30 inches, and is thus a small fraction over 231 cubic inches. The weight expressed in pounds, is 8.3388 + pounds Avoirdupois, or 10.134 + pounds Troy.

This is the old English, or *Winchester Wine Gallon* (so called because its standard was long preserved at Winchester), and is .83388 + of the Imperial; or 6 of the former very nearly equal 5 of the latter.

199. The *Bushel* is the capacity of a measure containing 543391.89 standard grains Troy (= 94.339 − lbs. Troy, or 77.6274 + lbs. Avoirdupois) of distilled water, under the same conditions as those just named, which is equal to 2152.405 + cubic inches. But the nearest practical representative of this measure is a cylinder 18½ inches in diameter and 8 inches deep, containing 2150.42 cubic inches, which is consequently taken as the true standard.

This is the *Winchester Bushel,* and is .9696262 + of the Imperial; so that 33 United States Bushels are about equal to 32 Imperial Bushels.

ENGLISH STANDARDS.

200. The English Linear, Square, and Cubic Measures, and Troy and Avoirdupois Weights, are the same as those of the United States.

201. The Standard for Measures of Capacity, whether Dry or Liquid, is the *Gallon,* containing 10 pounds Avoirdupois (70000 grains Troy) of distilled water, weighed in air, at 62° F., barometer showing 30 inches, which is equal to 277.274 cubic inches.

202. The *Imperial Bushel* contains 8 gallons.

203. From a comparison of the English and United States Standards, we derive the following :

Rule.—I. To reduce Imperial Gallons to United States Gallons, *divide by* .83388; and to reduce Imperial Bushels to United States Bushels, *divide by* .969626.

II. To reduce United States Gallons and Bushels to Imperial, *multiply by these numbers.*

204. Denominations, etc.—The denominations and values of the measures of length, surface, and solidity, are the same as those of the

United States. The same is true of the various systems of weight. The units of Liquid and Dry Measure differ from those of the United States, and also some of the denominations, as shown in the following Tables.

WINE MEASURE.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.
36 gallons = 1 tierce.
1½ tierces = 1 hogshead.
2 hogsheads = 1 pipe, butt, or puncheon.

BEER MEASURE.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.
9 gallons = 1 firkin.
2 firkins = 1 kilderkin.
3 kilderkins = 1 hogshead.
2 hogsheads = 1 butt.
2 butts = 1 tun.

DRY MEASURE.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.
2 gallons = 1 peck.
4 pecks = 1 bushel.
4 bushels = 1 coomb.
2 coombs = 1 quarter.
5 quarters = 1 wey.
2 weys = 1 last.

The pottle is ½ a gallon; the strike 2 bushels. The hogshead, pipe, and puncheon differ also in the case of different wines, or other spirituous liquors.

TABLES OF WEIGHTS AND MEASURES.

I. MEASURES OF LENGTH, OR EXTENSION.

I. LINEAR, OR LONG MEASURE.

205. Linear, or **Long Measure,** is used in measuring lines, or distances.

The *Standard Unit* is the *Yard,* which is subdivided into feet and inches.

As has been shown, this is the *fundamental unit* from which all the units, both of *measures* and *weights,* are derived.

TABLE.

12	Inches (in.)	make	1 Foot	ft.
3	Feet	"	1 Yard	yd.
5½	Yards	"	1 Rod	rd.
40	Rods	"	1 Furlong	fur.
8	Furlongs, or 320 rods	"	1 Mile	mi.
1.15	Statute Miles	"	1 Geographical, or Nautical Mile.	
3	Geographical Miles	"	1 League	lea.
69.168 60	Statute Miles, or Geographical Miles	"	1 Degree of latitude on a Meridian, or of longitude on the Equator.	
360	Degrees	"	1 A circumference of the Earth.	

The following denominations are also in use:—3 barley-corns = 1 inch, used by shoemakers in measuring the length of the foot, a barley-corn, or ⅓ of an inch, after a certain length, constituting a *size* or *number* of the shoe; 4 inches = 1 hand, used in measuring the height of horses; 9 inches = 1 span; 3 feet = 1 pace; 6 feet = 1 fathom, used in measuring depths at sea; 1 knot = 1 geographical mile, used by sailors.

NOTES.—1. The *Inch* is sometimes subdivided by artificers duodecimally, or into 12ths, but generally into halves, quarters, eighths, sixteenths, and thirty-seconds.

Decimal subdivisions of the *foot* and *inch* are used by civil and mechanical engineers, and others.

2. A geographical or nautical mile is one minute of the earth's circumference, or $\frac{1}{60}$ of a degree, measured on the equator; but a degree on a meridian varies from 68.72 miles, at the equator, to 69.34 in the polar regions.

3. Literally, a degree has no linear measure; a degree of longitude, as such, does not signify *distance*, but defines an *angle*. But when applied to a portion of a circle of definite dimensions, it may properly be considered as having length.

II. SURVEYORS' LINEAR MEASURE.

206. This measure is used by surveyors in measuring lines and distances.

The unit is the *Chain*, generally known as *Gunter's Chain*, which is 4 rods, or 66 feet long, and consists of 100 links.

TABLE.

7.92 Inches (in.)	make	1 Link	l.
25 Links	"	1 Rod, or pole	rd.
80 Chains	"	1 Mile	mi.

NOTES.—1. Links are written decimally, as hundredths of a chain.

2. In measuring lines for other purposes than the survey of lands, engineers commonly use chains with links 10 inches or a foot long.

II. MEASURES OF SURFACE.

I. SQUARE MEASURE.

207. **Square Measure** is used in computing surfaces or areas; as of land, boards, etc.

A **Surface** is that which has length and breadth, without thickness.

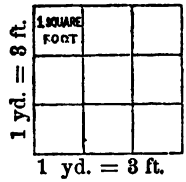
Area is expressed by the product of these two dimensions; hence, the basis of the measure is a *linear* unit.

A square inch is a square whose sides are each one inch long; a square foot, a square whose sides are each one foot long, etc.

TABLE.

144	Square Inches (sq. in.)	make	1 Square Foot	sq. ft.
9	Square Feet	"	1 Square Yard	sq. yd.
30¼	Square Yards	"	1 Square Rod	sq. rd.
40	Square Rods	"	1 Square Rood	R.
4	Roods	"	1 Acre	A.
640	Acres	"	1 Square Mile	sq. mi.

A **Square** is a figure having four equal sides and four right angles. Its area is the space, or surface, included within the bounding lines.



Let this diagram represent a square yard, divided by the cross lines into square feet. Each side is 1 yard or 3 feet long. The square of this dimension is the area; $1^2 = 1$, or $3^2 = 9$. Considering the dimensions in feet, we observe that there are three rows of small squares, each containing three square feet; and 3 times 3 square feet = 9 square feet. The area is expressed by the number of times it contains the *measuring unit.*

Any two adjacent rows of these squares form a parallelogram, 3 feet long, and 2 feet wide, and the area is 3 × 2, or 6 square feet. Hence

To find the area of any rectangular figure, *multiply the length by the breadth*

II. SURVEYORS' SQUARE MEASURE.

208. This measure is used by surveyors in computing the area or contents of land.

TABLE.

625	Square Links (sq. l.)	make	1 Pole	P.
16	Poles	"	1 Square Chain	sq. ch.
10	Square Chains	"	1 Acre	A.
640	Acres	"	1 Square Mile	sq. mi.

NOTE.—In the Western States a square mile is called in the surveys of public lands a *Section,* and 160 acres, a *Quarter Section.* 36 square miles, or sections, constitute a *Township.*

III. MEASURES OF SOLIDITY OR CAPACITY.

I. CUBIC MEASURE.

209. Cubic, or **Solid Measure**, is used in computing the contents of solids in body or space.

A solid, in mathematics, is a body, volume, or space, which has all the geometrical dimensions; viz., length, breadth, and thickness.

Solidity or Capacity is expressed by the product of these three dimensions; hence, the units of this measure are formed from the *linear* unit.

A cubic inch is a cube whose edge, or the length of one of its sides, is 1 inch; a cubic foot, a cube whose edge is 1 foot, or 12 inches = 12 × 12 × 12 = 1728 cubic inches, etc

TABLE.

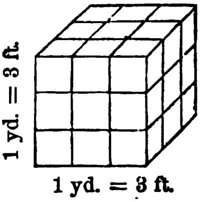
1728	Cubic Inches (cu. in.).........	make	1 Cubic Foot....	cu. ft.
27	Cubic Feet....................	"	1 Cubic Yard...	cu. yd.
16	Cubic Feet....................	"	1 Cord Foot....	cd. ft.
8 128	Cord Feet, or Cubic Feet...	"	1 Cord.........	C.
50 40	Cubic Feet of Square Timber, or Feet of Round Timber, as commonly measured......	"	1 Ton..........	T.
24¾	Cubic Feet....................	"	1 Perch of Stone or Masonry..	Pch.

NOTES.—1. A cord of wood is a pile 8 feet long, 4 feet wide, and 4 feet high, or its equivalent. A cord foot is 1 foot in length of such a pile.

2. The ordinary rule for computing the contents of round timber, gives the measure too small; so that what is called the Ton of 40 feet, really contains a fraction over 50 cubic feet. See Measurement of Lumber, page

210. A **Cube** is a solid having six equal square sides, or faces.

211 A **Parallelopipedon** is a solid having six rectangular plane faces, every opposite two of which are equal and parallel.



Let this figure represent a cubic yard. Each of the six sides is a square yard, containing 9 square feet of surface. A section of the cube, 1 foot thick, cut off by a plane parallel with one of the sides, may, therefore, be divided into 9 small cubes, each containing 1 cubic foot. And, since the cube is three feet thick, it must contain 3 times 9, or 27 cubic feet. Hence, 27 is the product of the three dimensions, or the cube of the linear measure of one edge; $3^3 = 3 \times 3 \times 3 = 27$. The solidity is expressed by the number of times it contains the measuring unit; it is 1 cubic yard, or 27 cubic feet.

Two of these sections form a *parallelopipedon*, 3 feet long, 3 feet wide, and 2 feet thick; and, by the same reasoning, we show that the contents equal $3 \times 3 \times 2 = 18$ cubic feet. Hence,

To find the solid contents of a cube or parallelopipedon, ***multiply together the length, breadth, and thickness.***

II. Liquid Measure.

212. **Liquid,** or **Wine Measure,** is used for measuring liquids of all kinds; such as liquors, molasses, oil, water, milk, etc.

213. The *Standard Unit* is the *Gallon,* containing 231 cubic inches. It is equal in volume to 8.3388 lbs. Avoirdupois, or 10.134 lbs. Troy of distilled water.

Table.

4	Gills (gi.)	make	1 Pint	pt.
2	Pints	"	1 Quart	qt.
4	Quarts	"	1 Gallon	gal.
31½	Gallons	"	1 Barrel	bbl.
2	Barrels	"	1 Hogshead	hhd.
2	Hogsheads	"	1 Pipe	P.
2	Pipes	"	1 Tun	T.

Notes.—1. There are also casks called Tierces, and Puncheons: 42 gals. make 1 Tierce; 2 Tierces, 1 Puncheon.

2. In Massachusetts a barrel is 32 gallons.

3. In commerce, barrels, hogsheads, pipes, etc., are merely the names of casks of about the capacity given in the table; but all kinds of liquids are usually sold by the gallon. The contents are found by gauging the casks, or are estimated by their gauged capacity.

4. The Imperial Gallon of Great Britain contains 277.274 cubic inches, and is therefore equal to about 1.2 of our wine gallons.

5. The old Table of Ale or Beer Measure is practically obsolete.

III. APOTHECARIES' FLUID MEASURE.

214. This Measure is used by apothecaries and physicians in compounding medicines.

The *Unit* of this System is called a *Minim,* and is equal to about .95 of a grain of pure water, at 60° F. Practically it is a drop of water.

TABLE.

60	Minims (♏)	make	1	Fluidrachm	fʒ.
8	Fluidrachms	"	1	Fluidounce	f℥.
16	Fluidounces	"	1	Pint	O.
8	Pints	"	1	Gallon	Cong.

NOTE.—O stands for the Latin *Octarius*, a Pint; C, for *Congius*, a Gallon

IV. DRY MEASURE.

215. Dry Measure is used in measuring grain, fruit, vegetables, salt, and other articles not liquid.

The *Standard Unit* is the *Bushel* of 2150.42 cubic inches, or a cylindrical measure 18½ inches in diameter, and 8 inches deep. It is equal in volume to 77.6274 lbs. Avoirdupois, or 94.339 lbs. Troy of distilled water.

TABLE.

2	Pints (pt.)	make	1	Quart	qt.
8	Quarts	"	1	Peck	pk.
4	Pecks	"	1	Bushel	bush.
8	Bushels	"	1	Quarter	qr.
36	Bushels	"	1	Chaldron	ch.

NOTES.—1. The Standard Imperial Bushel of Great Britain contains 2218.192 cubic inches; so that 32 Imperial Bushels are equal to about 33 United States Bushels.

2. The English Quarter of grain is 8 bushels.

3. The English Quarter of Wheat is equal to a quarter of a ton of 2240 lbs. = 560 lbs., or 70 lbs. to the bushel, while our bushel consists of 60 lbs.; so that our wheat bushel by weight is just $\frac{6}{7}$ of the English; or $9\frac{1}{3}$ United States bushels are equal to one quarter.

IV. MEASURES OF WEIGHT.

I. TROY WEIGHT.

216. Troy Weight is used in weighing gold, silver, and jewels; also, articles employed in philosophical experiments, etc.

The *Standard Unit* is the *Pound,* which is equal to the weight of 22.8156 cubic inches of distilled water.

TABLE.

24 Grains (gr.)	make	1 Pennyweight	pwt.
20 Pennyweights	"	1 Ounce	oz.
12 Ounces	"	1 Pound	lb.

NOTE.—The Troy Pound contains 5760 grains; and the Avoirdupois Pound, 7000 grains. Therefore, 1 lb. Troy = $\frac{5760}{7000} = \frac{144}{175}$ lb. Avoirdupois. An ounce Troy is $\frac{5760}{12} = 480$ grains, and an ounce Avoirdupois is $\frac{7000}{16} = 437\frac{1}{2}$ grains.

217. COMPARISON OF WEIGHTS, AND MEASURES OF CAPACITY, BY TROY WEIGHT.

1 cu. in. distilled water = 252.458 grains = $.5259\frac{13}{24}$ oz.
5760 grains = 1 Pound Troy.
7000 grains = 1 Pound Avoirdupois.
58372.2 grs. (10.134 lbs. Troy) = 231 cu. in. dist. water, 1 U. S. Gallon.
543391.89 grs. (94.339 lbs. Troy) = 2152.405 cu. in. dist. water, 1 U. S. Bush.
70000 grs. (10 lbs. Avoir.) = 277.274 cu. in. dist. water, 1 Imp. Gal.
560000 grs. (80 lbs. Avoir.) = 2218.192 cu. in. dist. water, 1 Imp. Bush.

II. DIAMOND WEIGHT.

218. Diamond Weight is used in weighing Diamonds and Precious Stones.

TABLE.

4 Quarters	make	1 Grain.
4 Grains	"	1 Carat.

NOTES.—1. A diamond grain is equal to .8 of a grain Troy.

2. The word *Carat* used to denote the fineness of gold, does not signify weight, but a part of pure gold. The whole mass of gold regarded as consisting of 24 equal parts, is called as many *carats fine* as it contains 24th parts of pure metal. Thus, gold 18 carats fine consists of 18 parts of pure gold, and 6 parts of alloy.

III. APOTHECARIES' WEIGHT.

219. Apothecaries' Weight is used by apothecaries and physicians in compounding medicines. Drugs and medicines are, however, bought and sold, in gross, by Avoirdupois weight.

The pound, ounce, and grain correspond with the same denominations in Troy weight. The subdivisions, scruples and drams, are peculiar to this weight.

TABLE.

20 Grains (gr. xx)	make	1 Scruple	sc., or ℈.
8 Scruples (℈iij)	"	1 Dram	dr., or ʒ.
8 Drams (ʒviij)	"	1 Ounce	oz., or ℥.
12 Ounces (℥xij)	"	1 Pound	lb., or ℔.

IV. AVOIRDUPOIS WEIGHT.

220. Avoirdupois Weight is used in weighing all common articles of merchandise, and the base metals.

The *Standard Pound* is 7000 grains Troy, or the weight of 27.7274 cubic inches of distilled water.

TABLE.

16 Drams (dr.)	make	1 Ounce	oz.
16 Ounces	"	1 Pound	lb.
25 Pounds	"	1 quarter	qr.
4 Quarters	"	1 Hundred-weight	cwt.
20 Hundred-weight	"	1 Ton	T.

NOTES.—1. The ton of 2240 pounds, or *Long Ton*, as it is now sometimes called, was formerly in general use. It is still the standard at the United States Custom-Houses, and in some branches of business, particularly the wholesale coal trade.

2. In freights, a ton (by the rules of the New York Chamber of Commerce) consists of 8 barrels of flour; 22 bushels of grain, pease, or beans, in casks; 36 bushels of grain in bulk, or European salt; 31 bushels West India salt; 6 barrels of beef, pork, tallow, or pickled fish; 200 gallons, wine measure, of oil, wine, brandy, or other liquors; 29 bushels of sea-coal; 40 cubic feet of square timber, oak or pine plank, cotton, wool, and other bale goods; 2000 lbs. of bar or pig iron, ashes, and all other heavy goods.

TABLE OF POUNDS IN A BUSHEL.

221. In accordance with custom and statutory regulations, grain, seeds, and sometimes vegetables, are now usually bought and sold, in quantity, by weight, or so many pounds to the nominal bushel. Though the laws of the different States are not uniform in this respect, the following are the prevailing standards:—

Wheat	60	lbs.	Buckwheat	42	lbs.
Wheat Bran	20	"	Beans	60	"
Indian Corn	56	"	Clover Seed	60	"
Indian Corn Meal	50	"	Timothy Seed	60	"
Indian Corn in ear	68	"	Blue Grass Seed	14	"
Rye	56	"	Flax Seed	56	"
Rye Meal	50	"	Hemp Seed	44	"
Barley	48	"	Potatoes	60	"
Oats	32	"	Onions	57	"

NOTE.—Efforts have been made in various cities to introduce the *Cental System*, or the buying and selling of grain and other commodities by the hundred pounds, instead of the bushel; but want of concerted action, or indisposition to change existing systems, has thus far prevented its general adoption.

TIME MEASURE.

222. Time is a part of duration, and is measured by the diurnal revolution of the earth on its axis, and its annual revolution around the sun.

The natural unit is the *Day*, or period in which the earth revolves once on its axis. This period is subdivided into hours, minutes, and seconds; and its multiples form weeks, months, and years.

TABLE.

60	Seconds (sec.)	make	1 Minute	m.
60	Minutes	"	1 Hour	h.
24	Hours	"	1 Day	da.
7	Days	"	1 Week	wk.
365 12	Days, or Calendar Months	"	1 Common Year	yr.
366	Days	"	1 Leap Year	yr.
100	Years	"	1 Century.	

223. The year is divided into Calendar Months as follows:—

SEASONS.	NO. OF MO.	NAMES.	NO. OF DAYS.
Winter	1st	January	31.
	2d	February	28 or 29.
Spring	3d	March	31.
	4th	April	30.
	5th	May	31.
Summer	6th	June	30.
	7th	July	31.
	8th	August	31.
Autumn	9th	September	30.
	10th	October	31.
	11th	November	30.
Winter	12th	December	31.

224. The number of days in each month may be easily remembered by the following lines:—

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save February, which alone
Hath twenty-eight; and this, in fine,
One year in four hath twenty-nine."

225. METHOD OF RECKONING COMMON AND LEAP-YEARS.

An exact *Tropical*, or *Solar Year*, according to Laplace, is 365 da., 5h., 48 m., and 49.7 sec. of mean solar time.

The *Common*, or *Civil Year*, consists of 365 days for three years in succession, and then the fourth, except as hereafter stated, is reckoned 366 days, one day being added for the excess of the solar year over 365 days. This day is added to the month of February, which then has 29 days, and the year is called *Bissextile*, or *Leap-Year.*

But, the addition of one day every fourth year produces a slight error in the calendar, because the yearly excess of 5 h., 48 m., 49.7 sec. is thus reckoned 6 hours. The difference is 11 m., 10.3 sec., which in 100 years amounts to 18 h., 37 m., 10 sec. To correct this error, every centennial year, except as mentioned below, is called a common year, instead of a leap-year, which it would be according to the above rule.

This again creates an error the other way, of 5 h., 22 m., 50 sec., which in 400 years amounts to 21 h., 31 m., 20 sec. Hence, for a further correction, every centennial year which is divisible by 400 is called a leap-year.

With these corrections, the time gained in the calendar in 4000 years will be 24 h., 46 m., 40 sec. For this reason, Sir John Herschel suggested that the year 4000 A.D. be reckoned a common year.

The following rule for determining the leap-year is thus established :—

Every year whose number is exactly divisible by 4, *except the centennial years which are not exactly divisible by* 400, *is a leap-year ; all the other years are common years.*

For example, 1872, 1876, 1880, etc., will be leap years; 1900 will be a common year, and 2000 the next centennial leap-year.

NOTES.—1. The civil year of 365 days and 6 hours was established by a decree of Julius Cæsar, the Roman emperor, made in the year 46 B.C., and hence is sometimes called the *Juilan Year.*

2. In 1582 the error arising from reckoning the year too long by a small fraction, had amounted to 10 days; that is, the date in the civil year was so much *behind* the solar year. To correct this error Pope Gregory decreed that 10 days should be stricken from the calendar, by calling the 4th day of October, 1582, the 14th day. The calendar so established was called the *Gregorian Calendar*, and is the one now used.

3. The period during which time was reckoned by the Julian Calendar, is called the *Julian Period.*

4. The Gregorian Calendar was not adopted in England until 1752, when the British Parliament decreed that 11 days should be stricken from the calendar, by calling the 3d day of September in that year the 14th.

5. Dates by the old calendar were afterward called *Old Style* (O. S.), and by the new, *New Style* (N. S.)

6. The Russians still use the Old Style; and as the difference is now 12

days, their dates are 12 days behind ours. So, their 1st day of January is our 13th.

7. A month, in most business transactions, especially in computing interest, is reckoned thirty days.

8. A so-called "lunar month" is 4 weeks, or 28 days. A mean lunation, or real lunar month, is approximately 29½ days, or more exactly, 29.53059 mean solar days. 235 lunations, nearly, measure a period of 19 years, called a *Lunar* or *Metonic Cycle.*

9. A *Sidereal Day* is the period in which a star apparently completes a revolution in the heavens, being the exact interval of time in which the earth revolves on its axis.

10. The *Civil Day* commences and ends at midnight, and the *Astronomical Day*, at noon of the civil day.

11. A *Solar Day* is the interval of time between two successive passages of the sun across the meridian of any place.

12. The solar days are of unequal length, on account of the unequal orbital motion of the earth and the obliquity of the ecliptic.

13. A *Mean Solar Day* is the average length of all the solar days in the year.

14. *Apparent Time* is the true solar time regulated by the apparent motion of the sun.

15. *Mean,* or *Clock Time,* is time regulated by the average or mean, instead of the unequal or apparent, motion of the sun.

16. The difference between sun-time and clock-time is called the *Equation of Time.*

17. When, therefore, it is 12 o'clock mean time, the sun may be either a little past or a little behind the meridian, and is said to be "fast," or "slow."

18. The sun is slowest about February 10th, when it is about fourteen and a half minutes behind mean time. It is fastest about the 1st of November, when it is sixteen and a quarter minutes in advance of the clock. Mean and apparent time coincide four times in the year; namely, April 15th, June 15th, September 1st, and December 24th.

19. Almanacs give the equation of time under the heading "sun slow," or "sun fast," and by adding or subtracting the same, as the case may require, apparent time, as shown by a dial or "sun mark," may be readily changed to mean time.

CIRCULAR MEASURE.

226. Circular, or **Angular Measure,** is used in astronomy, navigation, geography, and surveying, for reckoning latitude and longitude, measuring angles, etc.

The Unit is the *Degree,* which is $\frac{1}{360}$ part of the circumference of a circle.

TABLE.

60	Seconds (″)	make	1 Minute	′.
60	Minutes	“	1 Degree	°.
30	Degrees	“	1 Sign	s.
12	Signs, or 360°	“	1 Circle	c.

NOTES.—1. All circles, great and small, contain the same number of degrees. Therefore, a degree is not a measure of space, or extension. It measures an angle whose vertex is the centre of the circle.

2. An *Arc* of a circle is any part of its circumference; a quadrant is one-fourth; a sextant, one-sixth; a sign, one-twelfth.

MISCELLANEOUS TABLES.

12	Units, or things	make	1 Dozen	doz.
12	Dozen	“	1 Gross	gro.
12	Gross, or 144 Dozen	“	1 Great Gross	gr. gro.
20	Units, or things	“	1 Score.	
14	Pounds	“	1 Stone.	
21½	Stones	“	1 Pig of iron or lead.	
8	Pigs	“	1 Fother.	
100	Pounds	“	1 Quintal of fish.	
196	Pounds (7 old quarters)	“	1 Barrel of flour.	
200	Pounds	“	1 Barrel of beef or pork.	

227. BOOKS AND PAPER.

Designation of different Sizes of Paper.

Double Imperial	32 by 44 in.	Imperial	22 by 32 in.
Double Super Royal	27 by 42 “	Super Royal	21 by 27 “
Double Medium	23 by 26 “	Royal	19 by 24 “
Double Medium	24 by 37½ “	Medium	18½ by 23½ “
Double Medium	25 by 38 “	Demy	17 by 22 “
Royal and Half	25 by 29 “	Folio Post	16 by 21 “
Imperial and Half	26 by 32 “	Foolscap	14 by 17 “

Crown, 15 by 20 inches.

Paper.

24 Sheets	make	1 Quire.
20 Quires	"	1 Ream.
2 Reams	"	1 Bundle.
5 Bundles	"	1 Bale.

Note.—A sheet folded in 2 leaves is called a *folio;* in 4 leaves, a *quarto*, or 4*to;* in 8 leaves, an *octavo*, or 8*vo;* in 12 leaves, a 12*mo;* in 18 leaves, an 18*mo;* in 24 leaves, a 24*mo;* and in 32 leaves, a 32*mo*.

English Money.

228. English, or **Sterling Money,** is the currency of England.

Table.

4 Farthings (far. or qr.)	make	1 Penny	d.
12 Pence	"	1 Shilling	s.
20 Shillings	"	1 Pound or Sovereign	£.
21 Shillings	"	1 Guinea.	
5 Shillings	"	1 Crown.	

Notes.—1. The coins are,—*Gold*, the sovereign, half-sovereign, and guinea;—*Silver*, the crown, half-crown, florin (2*s*.), shilling, sixpence, fourpenny piece, and threepenny piece;—*Copper*, the penny, halfpenny, and farthing. The guinea, crown, and half-crown are not now coined.

2. The intrinsic par value of the Pound, or Sovereign, in United States money, as estimated at the Mint, and proclaimed by the Secretary of the Treasury, January 1st, 1875, under an Act of Congress passed March 3d, 1873, is $4.866½.—See an abstract of this act, page 329.

3. In round numbers, where no accurate calculation is required, a pound is often called $5.

French Money.

229. French Money, like our own, is a decimal currency, the franc being the unit, and centimes and millimes, decimals.

Table.

10 Millimes	make	1 Centime	ct.
100 Centimes	"	1 Franc	fr.

NOTES.—1. The coins are,—*Gold*, 100, 40, 20, and 10 franc pieces;—*Silver*, 5, 2, 1, ½, and ¼ franc pieces. 1 franc = 100 centimes; ½ franc = 50 centimes; ¼ franc = 25 centimes. The *Copper* coins are,—1 centime; 1 sou = 5 centimes; 2 sous = 10 centimes.

2. The par value of the 20 franc piece, or louis, is $3.872; of the franc, 0.191\frac{7}{12}$, when compared with the pound sterling, at $4.84.

For Tables and explanations of other Foreign Currencies, see pages 233, and the article on Exchange.

MEASURES OF HEAT.

230. Temperature, or **Intensity of Heat,** is measured by instruments called Thermometers. Of these, the principal ones in common use are Fahrenheit's, the Thermomĉtre Centigrade, and Réaumur's. In England and America, Fahrenheit's is most frequently used, though the Thermométre Centigrade, owing to its convenient decimal scale, is coming into general use among scientific men throughout the world, and has been wholly adopted by several European countries. A comparison of the three different scales is exhibited in the following

TABLE.

STANDARD POINTS.	DEGREES ON THE SCALE.		
	Fahrenheit.	*Centigrade.*	*Réaumur.*
Boiling point of water under one atmosphere....................	212°	100°	80°
Melting point of ice	32°	0°	0°

231. These different scales are readily reducible to each other. Thus, let F°, C°, R°, represent any number of degrees Fahrenheit, Centigrade, or Réaumur. Then to reduce

Fahrenheit to Centigrade, $\frac{5}{9}(F^\circ - 32) = C^\circ$.
Centigrade to Fahrenheit, $\frac{9}{5}\,C^\circ + 32 = F^\circ$.
Fahrenheit to Réaumur, $\frac{4}{9}(F^\circ - 32) = R^\circ$.
Réaumur to Fahrenheit, $\frac{9}{4}\,R^\circ + 32 = F^\circ$.
Centigrade to Réaumur, $\frac{4}{5}\,C^\circ = R^\circ$.
Réaumur to Centigrade, $\frac{5}{4}\,R^\circ = C^\circ$.

NOTE.—On the scale of Fahrenheit, the *freezing point* is,—fresh water, 32°; sea-water, 28°; strong wine, 20°; mercury, 39° below zero. The *boiling point* is,—quicksilver, 660°; linseed oil, 600°; oil of turpentine, 560°; water, 212°; alcohol, 174°. The *melting point* is,—cast iron, 20577°; gold, 5237°; silver, 4717°; Swedish copper, 4587°; brass, 3807°; zinc, 700°; lead, 594°; bismuth, 476°; tin, 442; sulphur, 226°; beeswax, 142°; spermaceti, 112°; tallow, 92°.

The mean temperature of the human blood is 99.5°. The heat of common fire is 790°; red heat fully visible in daylight, 1077°; iron red-hot in twilight, 884°; in the dark, 752°.

EXAMPLES.

1. Fresh water freezes at 32° F.: what is the freezing point on the Centigrade thermometer?

SOLUTION: $(32 - 32) \times \frac{5}{9} = 0$, *Ans.*

2. The boiling point of water on the scale of Fahrenheit is 212°: what is it on the Centigrade?

SOLUTION: $(212 - 32) \times \frac{5}{9} = 100$, *Ans.*

3. Reduce $-39°$ F, the freezing point of mercury, to R.

Ans. $31\frac{5}{9}°$ below 0, called $-32°$.

4. When Réaumur's thermometer marks the temperature 10° below zero, what is the temperature by C and F?

Ans. C, $12\frac{1}{2}°$ below zero; F, $9\frac{1}{2}°$ above zero.

REDUCTION OF COMPOUND NUMBERS.

232. **Reduction** is the process of changing a number from one denomination to another, without altering its value.

233. The Reduction of Compound Numbers is of two kinds, *Descending* and *Ascending.*

234. **Reduction Descending** consists in changing a number of one denomination to another of a *lower* denomination; thus, £5 = 100*s.* = 1200*d.*

235. **Reduction Ascending** consists in changing a number of one denomination to another of a *higher* donomination; thus, 1200*d.* = 100*s.* = £5.

REDUCTION DESCENDING.

CASE I.

236. To reduce the higher denominations of a compound number to a lower.

EXAMPLE. Reduce 8 mi. 3 fur. 10 rd. 1 yd. 2 ft. 6 in. to inches.

OPERATION.

```
     7 mi. 3 fur. 10 rd. 1 yd. 2 ft. 6 in.
     8
     —
    59 fur.
    40
  ————
  2370 rd.
    5½
 —————
 11851
  1185
 —————
 13036 yd.
     3
 —————
 39110 ft.
    12
——————
469326 in., Ans.
```

ANALYSIS.—Since 8 furlongs make 1 mile, 7 miles and 3 furlongs are equal to 8 × 7 + 3 = 59 furlongs. Then, since there are 40 rods in a furlong, 59 furlongs equal 40 times 59 rods, or 2360 rods; and, adding the 10 rods in the given number, gives 2370 rods as the value of 59 fur. 10 rd. Continuing the reduction in this manner, we obtain for a final result 469326 inches, the value of the given number in its lowest denomination. Hence we deduce the following

RULE.—*Multiply the number of the highest denomination by that number which will reduce it to the next lower denomination in the scale, and to the product add the given number in the multiplicand, if any, of that lower denomination. Proceed in like manner with the result, and so on with the successive results, till the given number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

1. Reduce 45 A. 3 R. 25 sq. rd. to rods.
2. Reduce 2 hhd. 1 bbl. 23 gall. 3 qt. to pints.
3. Reduce 125 bush. 3 pk. 5 qt. to quarts.

4. How many feet in 75 mi. 5 fur.?

5. Required the number of pounds in a hogshead of sugar, weighing 18 cwt. 3 qr. 14 lbs.

6. Reduce £95 10*s.* 6*d.* to pence.

7. How many grains are there in 55 lb. 7 oz. 13 pwt. 16 gr., Troy weight?

8. How many pence are there in £1?

9. Reduce 12 ℔. 9 ℥. 2 ʒ. to scruples.

10. Reduce 3 yr. 13 wk. and 7 da. to days, reckoning 365 days to a year.

REDUCTION ASCENDING.

CASE I.

237. To reduce a denominate number to a compound number of higher denominations.

EXAMPLE. In 1268*d.* how many pounds, shillings, and pence?

OPERATION.

12)1268*d.*

20)105*s.* . . . 8*d.* rem.

£5 . . . . 5*s.* rem.

£5 5*s.* 8*d.*, *Ans.*

ANALYSIS.—We first divide by 12, because 12 pence make a shilling, and obtain 105*s.* with 8*d.* remaining. Next, we divide the 105*s.* by 20, because 20*s.* make £1, and obtain for a quotient £5, with 5*s.* remaining. £5 written with the remainders in succession, is the required compound number. Hence we derive the following

RULE.—I. *Divide the given number by the number which it takes of that denomination to make one of the next higher denomination.*

II. *Divide the quotient in like manner; and so proceed to the highest denomination required. The last quotient, with the successive remainders, written in reverse order, will be the required compound number.*

EXAMPLES FOR PRACTICE.

1. Reduce 7320 square rods to acres.

2. Reduce 725 qts. to hogsheads.

3. How many bushels of corn are there in 4029 qts.?

4. Reduce 399300 feet to miles.

5. Find the quantity in higher denominations of 1889 pounds of sugar.

6. How many pounds, shillings, and pence, are there in 10926 pence?

7. Reduce 320488 grains Troy to pounds.

8. Change 240 pence to the highest denomination in the scale of English money.

9. Reduce 3678 scruples to pounds.

10. How many common years, weeks, and days, are there in 8309 days?

REDUCTION OF DENOMINATE FRACTIONS.

238. A **Denominate Fraction** is a fraction of a denominate number; as $\frac{3}{4}$ of a ton; .25 of a mile.

CASE I.

239. To reduce a denominate fraction to integers or lower denominations.

EXAMPLE. Reduce $\frac{7}{9}$ of a pound Troy to integers of lower denominations.

OPERATION.

```
   7
  12
  ——
9)84(9 oz. 6 pwt. 16 gr.
  81
  ——
   3
  20
  ——
9)60
  54
  ——
   6
  24
  ———
9)144
```

ANALYSIS.—Since 12 oz. = 1 lb., $\frac{7}{9}$ of a pound = $\frac{7}{9} \times 12 = \frac{84}{9}$ of an oz. = 9 oz. and 3 ninths rem. $\frac{3}{9}$ of an oz. = $\frac{3}{9} \times 20 = \frac{60}{9}$ of a pwt. = 6 pwt. and 6 ninths rem. $\frac{6}{9}$ of a pwt. = $\frac{6}{9} \times 24 = \frac{144}{9}$ of a gr. = 16 grains.

The several quotients form the answer. Hence we derive the following

RULE.—I. *Multiply the numerator of the fraction by the proper number to reduce it to the next lower denomination, and divide the product by the denominator.*

II. *Proceed in like manner with the remainder, if there be any, and so continue to the lowest denomination required. The several quotients in order will be the answer.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{5}{19}$ of a ton to integers of lower denominations.
2. What is the value of $\frac{9}{13}$ of a mile?
3. What is the value of $\frac{4}{11}$ of a year (365 days)?
4. What is the value of $\frac{25}{33}$ of an acre?
5. Find the value of $\frac{4}{5}$ of a hogshead.
6. Find the value of $\frac{37}{56}$ of a bushel.
7. Find the value of £$\frac{17}{20}$.
8. Reduce .875 of a ton to integers of lower denominations.

OPERATION.

$$\begin{array}{r} .875 \\ 20 \\ \hline 17.500 \text{ cwt.} \\ 4 \\ \hline 2.0 \text{ qr.} \end{array}$$

Ans. 17 cwt. 2 qr.

NOTE.—No separate rule is required for the reduction of decimals. Multiply according to the rule above given and point off as in decimal fractions. The integers in the several products, written in order, will constitute the answer.

9. What is the value of .75 of a mile?
10. What is the value of .625 of an acre?
11. What is the value of £.125?
12. What is the value of $\frac{7}{8}$ of a week?
13. Reduce .46$\frac{3}{4}$ leagues to integers of lower denominations.
14. Reduce to integers of lower denominations, $\frac{7}{8}$ of a cubic foot; 3.75 tons; .5 of an acre; $.\dot{3}$ of 1 lb. Troy; .95 lb. apothecaries' weight; $\frac{4}{5}$ of a gal. apothecaries' fluid measure.

CASE II.

240. To reduce one denominate fraction to another of a higher or lower denomination.

EXAMPLE 1. Reduce $\frac{1}{356}$ of a rod to the fraction of an inch.

OPERATION.

$$\frac{1}{\underset{178}{\cancel{356}}} \times \frac{33}{\cancel{2}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{99}{178}, \textit{ Ans.}$$

Or,

$$\begin{array}{r|l} \cancel{356} & 1 \\ 178\ \cancel{2} & 33 \\ 1 & \cancel{12}\ 3 \\ \hline 178 & 99 \end{array} = \frac{99}{178}, \textit{ Ans.}$$

ANALYSIS.—The multipliers required to reduce rods to inches are $16\frac{1}{2}$ and 12; hence, $\frac{1}{356} \times 16\frac{1}{2} \times 12 = \frac{99}{178}$, which is the fraction of an inch equal to $\frac{1}{356}$ of a rod.

EXAMPLE 2. Reduce $\frac{8}{13}$ of a pound Avoirdupois to the fraction of a ton.

OPERATION.

$$\frac{\cancel{8}}{13} \times \frac{1}{25} \times \frac{1}{\cancel{4}} \times \frac{1}{\underset{10}{\cancel{20}}} = \frac{1}{3250}, \textit{ Ans.}$$

Or,

$$\begin{array}{r|l} 13 & \cancel{8} \\ 25 & \\ \cancel{4} & \\ 10\ \cancel{20} & \\ \hline 3250 & 1 \end{array} = \frac{1}{3250}, \textit{ Ans.}$$

ANALYSIS.—The several divisors required to reduce pounds to tons are 25, 4, and 20; hence, $\frac{8}{13} \div \overline{25 \times 4 \times 20} = \frac{8}{26000} = \frac{1}{3250}$, which is the fraction of a ton equal to $\frac{8}{13}$ of a pound. From these examples and illustrations we deduce the following

RULE.—*Multiply or divide the given fraction by the proper numbers to reduce it to the required denomination, according to the rules for Reduction Descending and Ascending.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{7}{314}$ of a gallon to the fraction of a pint.
2. Reduce $\frac{16}{4289}$ of a week to the fraction of an hour.
3. Reduce $\frac{11}{12}$ of an ounce Troy to the fraction of a pound.

4. Reduce $\frac{6}{17}$ of a square rod to the fraction of an acre.

5. $\frac{2}{13}$ of a pound Avoirdupois is $\frac{1}{4}$ of how many ounces?

6. What fraction of an hour is $\frac{3}{8}$ of 15 seconds?

CASE III.

241. To reduce a compound number to a fraction of a given denomination.

EXAMPLES. What part of £1 is 15 shillings? *Ans.* $\frac{15}{20}$

NOTE.—The unit is £1 = 20*s.*; and 15*s.* = $\frac{15}{20}$ of 20*s.* = £$\frac{15}{20}$.

2. Reduce 32 rd. 8 ft. 6 in. to the fraction of a furlong.

OPERATION.

32 rd. 8 ft. 6 in. = 6438 inches.
1 fur. = 7920 inches.
$\frac{6438}{7920} = \frac{1073}{1320}$, *Ans.*

ANALYSIS.—It is evident that to find what part one number is of another, both must be reduced to the same denomination. Having reduced the given number to inches, the lowest denomination in it, and the unit, 1 furlong, to the same denomination, the question is, what part of 7920 inches is 6438 inches? It is $\frac{6438}{7920} = \frac{1073}{1320}$. Therefore, 32 rd. 8 ft. 6 in. are $\frac{1073}{1320}$ of a furlong. Hence the following

RULE.—*Reduce the given number to the lowest denomination it contains for the numerator, and a unit of the required denomination to the same denomination for the denominator of the required fraction.*

EXAMPLES FOR PRACTICE.

1. What part of £1 is 13*s.* 6½*d.*?

2. Reduce 9 oz. 11 pwt. 14 gr. Troy to the fraction of a pound.

3 Reduce 3 cd. ft. 13 cu. ft. to the fraction of a cord.

4. What part of 3 bushels is 3 pk. 7 qt. 1½ pt.?

NOTE.—When the given number contains a fraction, reduce the whole, and also the unit, to the same fractional denomination.

5. What part of 5 weeks is 6 da. 23 h. 40 min.?

6 From a barrel containing 31½ gallons of wine, 4 gall. 3 qt. 1 pt. were drawn: what part of the contents remained?

CASE IV.

242. **To reduce a compound number to a decimal of a given denomination.**

EXAMPLE. Reduce 4 cwt. 3 qr. 12 lbs. 14 oz. to the decimal of a ton.

OPERATION.

$$\begin{array}{r|l} 16 & 14 \\ \hline 25 & 12.875 \\ \hline 4 & 3.515 \\ \hline 20 & 4.87875 \\ \hline & .2439375 \end{array}$$

ANALYSIS.—14 oz. = $\frac{14}{16}$ of a pound, and this fraction reduced to a decimal is, .875, which annexed to 12 lbs. gives 12.875 lbs. as the value of 12 lbs. 14 oz. Then, dividing 12.875 by 25, the number of pounds in a quarter, and annexing the quotient to 3 qrs., we have 3.515 qrs. In like manner, dividing by 4, and 20, we obtain .2439375, the decimal of a ton which is equivalent to the given number. Hence the following

RULE.—*Divide the number of the lowest denomination by that number which will reduce it to the next higher denomination in the scale, and annex the quotient to the number of the next higher denomination. Proceed in the same manner with the result, and so on till the required decimal is obtained.*

EXAMPLES FOR PRACTICE.

1. Reduce £12 19*s.* 0¾*d.* to the decimal of a pound.
2. Reduce 15 pwt. 18 gr. Troy to the decimal of a pound.
3. Reduce 36 rd. 13 ft. 9 in. to the decimal of a furlong.
4. Reduce 116 rd. to the decimal of a mile.
5. Reduce 2 A. 2 R. 3 sq. ft. to the decimal of an acre.
6. Reduce 4 links 5 in. to the decimal of a chain.
7. Reduce 4 cd. ft. 9 cu. ft. to the decimal of a cord.
8. Reduce 213 da. 13 h. 24 m. 36 sec. to the decimal of a year of 365 days.

9. Reduce 5 da. 13⅜ h. 10 sec. to the decimal of a week.

10. Reduce 21 bu. 2 pk. 5⅛ qt. to the decimal of a bushel.

11. Reduce 125 lb. to the decimal of a ton.

12. Reduce 4 wk. 5 da. 10 h. 25 m. 56 sec. to the decimal of a week.

13. Reduce 14° 25′ 25″ to the decimal of a sign.

14. Reduce 50 sec. to the decimal of a year of 365 days.

SHORT METHODS.

CASE I.

243. To reduce shillings, pence, and farthings to the decimal of a pound.

RULE.—I. *Divide the number of shillings by* 2, *and write the quotient as tenths. If there be an odd shilling, write it as* 5 *hundredths. Or, multiply the number of shillings by* 5, *and write the product as hundredths.*

II. *Write the number of farthings in the given pence and farthings as thousandths, adding* 1 *if the number exceed* 12, *and* 2 *if it exceed* 36.

III. *The sum of these will be the required decimal, approximately correct.*

EXAMPLE. Reduce 15*s.* 6¾*d.* to the decimal of a pound.

OPERATION.

15*s.* = .75 of a pound.
27 far. = .028 "
.778+, *Ans.*

EXPLANATION.—Since shillings are twentieths, 15*s.* = $\frac{15}{20}$ of a pound = 1.5 ÷ 2 = .75; or $\frac{15}{20} = \frac{75}{100}$. And in 1 pound there are 960 farthings, and 960 increased by $\frac{1}{24}$ of itself = 1000; that is, by adding 1 for every 24 farthings, the sum will be the exact number of thousandths of a pound they equal. 27 farthings = £.028⅛, and we call the decimal .028. The result is sufficiently accurate for most practical purposes. The true answer is, .778125.

CASE II.

244. To reduce hundred-weights, quarters, and pounds to the decimal of a ton.

RULE.—I. *Divide the number of pounds by 2, writing the quotient as thousandths, and the remainder for an odd pound as 5 ten-thousandths.*

II. *Divide the number of hundred-weights, with the quarters annexed decimally, by 2, and write the quotient as tenths, hundredths, etc.*

III. *The sum of these will be the required decimal.*

EXAMPLE. Reduce 13 cwt. 3 qr. 18 lb. to the decimal of a ton.

OPERATION.

$$\begin{array}{lrl}
18\text{ lb.} & = .009 & \text{of a ton.} \\
\left.\begin{array}{r}3\text{ qr.}\\ 13\text{ cwt.}\end{array}\right\} = 13.75\text{ cwt.} & = .6875 & \text{“ “} \\
 & \overline{.6965,} & \textit{Ans.}
\end{array}$$

EXPLANATION.—2 lb. $= \frac{2}{2000} = .001$ of a ton; and 2 cwt. $= \frac{2}{20} = .1$ of a ton. Hence, dividing the pounds by 2 reduces them to thousandths of a ton; and dividing the hundred-weights by 2 reduces them to tenths of a ton.

NOTE.—Hundred-weights alone, or with quarters annexed decimally, may also be reduced to the decimal of a ton, by multiplying by 5, and placing the decimal point before the product.

EXAMPLE. Reduce 19 cwt. to the decimal of a ton.

$19 \times 5 = 95.$ *Ans.* .95 of a ton.

2. Reduce 15 cwt. 3 qr. to the decimal of a ton.

$15.75 \times 5 = .7875.$ *Ans.* .7875.

EXAMPLES FOR PRACTICE.

1. Reduce 13*s*. 11½*d*. to the decimal of a pound.
2. Reduce 17 cwt. to the decimal of a ton.
3. Reduce 12 cwt. 2 qr. to the decimal of a ton.

4. Find the cost of 6 T. 13 cwt. of coal, at $7 per ton.

5. Reduce 18*s.* 10¾*d.* to the decimal of a pound.

6. Reduce 14 cwt. 3 qr. 12 lb. to the decimal of a ton.

7. Reduce 21 cwt. 2 qr. to the decimal of a ton.

ADDITION OF COMPOUND NUMBERS.

245. The operations of Addition, Subtraction, Multiplication, and Division in Compound Numbers, are the same in principle as like operations in simple numbers, except that in passing from one denomination to another, we borrow and carry, when necessary, according to a *varying* instead of a *uniform* scale.

The principles will be sufficiently explained under the examples, to render special rules unnecessary.

EXAMPLE. Add 3 fur. 17 rd. 12 ft. 6 in.; 7 fur. 13 rd. 8 ft. 10 in.; 2 fur. 7 ft. 11 in.; 6 fur. 18 rd.; 5 fur. 23 rd. 15 ft. 7 in.

OPERATION.

	fur.	rd.	ft.	in.
	3	17	12	6
	7	13	8	10
	2	0	7	11
	6	18	0	0
	5	23	15	7
3	0	33	11	10

ANALYSIS.—Having written the given numbers so that those of the same denomination stand in the same column, we commence at the right, and find the sum of the inches to be 34 = 2 ft. 10 in. Hence, we write the 10 inches under the column of inches, and carry the 2 feet to the column of feet. The sum of the feet, with 2 carried, is 44 = 2 rd. 11 ft. Writing the 11 under the column of feet, we carry 2 to the column of rods, which then foots up 73 = 1 fur. 33 rd. Then 1 carried to the column of furlongs, makes the sum of that column 24 = 3 mi. As there is no remainder, we write 0 under the column of furlongs, and place the 3 miles to the left in its proper order. This completes the work, and the whole sum is 3 mi. 33 rd. 11 ft. 10 in.

NOTES.—1. If a fraction occur in any division, it must be reduced to integers of lower denominations, and the result added to the other part of the sum.

2. Fractions occurring in the higher denominations should first be reduced to integers.

Examples for Practice.

Find the sum of the following compound numbers:—

1. Add 13 T. 17 cwt. 3 qr. 22 lb.; 11 cwt. 2 qr. 24 lb.; and 156 T. 9 cwt. 1 qr. 17$\frac{1}{5}$ lb.

2. Add 13 cd. 6 cd. ft.; 56 cd. 7 cd. ft. 11 cu. ft.; 22 cd. 3 cd. ft.; and 1 cd. 13 cu. ft.

(3)

£	s.	d.
35	11	6
275	4	4
36	8	10
264	3	2

(4)

A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.
3	2	17	18	7	112
19	3	20	6	4	24
32	1	29	25	3	47
56	3	31	9	2	108

(5)

lb.	oz.	pwt.	gr.
30	8	14	11$\frac{1}{3}$
184	11	19	23
204	6	9	10
18	11	16	5

(6)

T.	cwt.	qr.	lb.	oz.
15	14	3	20	8
40	5	2	11	9
6	18	0	12	0
34	15	1	24	4

7. What is the sum of 12 bush. 2$\frac{1}{2}$ pk.; 3$\frac{4}{5}$ bush. 2 pk. 4$\frac{1}{2}$ qt.; 3 pk. 2 qt.; 1 bush. 2 pk. 7 qt.?

8. Add £120 11*s.* 3*d.*; £175 10*s.* 11$\frac{3}{4}$*d.*; £7 14*s.* 10$\frac{1}{2}$*d.*; 12*s.* 0$\frac{3}{4}$*d.*

9. What is the sum of 215 A. 2 R. 39 rd. 200 sq. ft.; 17 A. 3 R. 20 rd. 19 sq. ft.; 145 A. 3 R. 36 rd. 114 sq. ft. 112 sq. in.?

10. Add 10 S. 20° 30′ 40″; 9 S. 19° 35′ 48″; 5 S. 52″; 7 S. 1′.

11. A farmer sold three loads of hay, at $15.50 per ton. The first load weighed 1 T. 3 cwt.; the second, 17 cwt. 3 qrs.; the third, 1$\frac{1}{8}$ T.: how much did he receive for the whole?

12. To $\frac{4}{7}$ of a mile add $\frac{3}{4}$ of a rod.

13. Add £$\frac{1}{4}$ to $\frac{3}{4}$ of a shilling.

SUBTRACTION OF COMPOUND NUMBERS.

Example. From 65 lb. 10 oz. 9 pwt. 15 gr. take 54 lb. 11 oz. 16 pwt. 12 gr.

OPERATION.

lb.	oz.	pwt.	gr.
65	10	9	15
54	11	16	12
10	10	13	3

Analysis.—Having written the subtrahend under the minuend, pounds under pounds, ounces under ounces, etc., we begin at the right to subtract. 12 gr. from 15 gr. leaves 3 gr., which we write underneath. 16 pwt. cannot be taken from 9 pwt.; therefore, we borrow one (1 oz.) from the next higher denomination, reduce it to pennyweights, add it to the 9 pwt., and from the sum subtract 16 pwt.; 1 oz. = 20 pwt.; 20 + 9 = 29; and 29 − 16 = 13. We must now carry or add the 1 borrowed to 11 oz., or consider it subtracted from the 10 oz. Passing to the next denomination, we find we must borrow again, for we cannot take 12 oz. from 10 oz. We, therefore, add 1 lb. = 12 oz. to the 10 oz., and subtract 12 oz. from the sum; 12 + 10 = 22; and 22 − 12 = 10. Finally, we carry 1 to 54, and subtract 55 lb. from 65 lb. The complete result is 10 lb. 10 oz. 13 pwt. 3 gr.

Examples for Practice.

1. From 14 rd. 12 ft. 9 in. take 11 rd. 15 ft. 11 in.

2. Subtract 2 T. 13 cwt. 23 lb. from 17 T.

3. Bought 18 cords of wood, and sold 6 cords and 3 cord feet of the same: how much remained?

4. From 1 great gross subtract 4½ doz.

5. A farmer had 450 bush. 3 pk. of wheat, of which he sold at one time 122 bush. 2 pk., and at another, 37½ bush.: how much had he left?

6. A gentleman owned three tracts of land. The first contained 140 A. 3 R. 14 rd.; the second, 275 A. 20 rd.; the third, 75 A. 2 R. He sold from the first tract, 40 A. 3 R.; from the second, 25 A. 2 R.; and from the third, 1 R. 30 rd.: how much remained in all?

7. The longitude of New York is 74° 1′ 6″ west from Greenwich, and that of St. Louis is 90° 15′ 16″ west: what is the difference of longitude between the two places?

8. The latitude of St. Paul's Cathedral, in London, is 51° 31 north, and that of the City Hall, New York, is 40° 42′ 43″ north: required the difference of latitude between the two places.

9. From $\frac{1}{2}$ a ton take $\frac{7}{8}$ cwt.

10. From 27 days take $\frac{4}{5}$ of an hour.

COMPUTING TIME.

246.—I. To find the interval of time between two dates.

Example. What is the difference in time between January 20th, 1856, and April 12th, 1871?

OPERATION.

yr.	mo.	da.
1871	4	12
1856	1	20
15	2	22

Explanation.—Representing the months by their numbers instead of their names, the proposition is simply to find the difference between two compound numbers. The later date forms the minuend, and the earlier the subtrahend. We therefore subtract as in compound numbers, allowing 30 days for a month, and 12 months for a year.

This rule will not give the exact number of days in the interval, except when the months *passed* average 30 days each. But, when it is required, the error can be corrected as shown below.

Example. Find by this method the number of days from January 17th to August 28th, 1871.

OPERATION.

mo.	da.	
8	28	
1	17	
7	11	= 221 days.
Add for correction		2 "
Correct Ans.		223

Note.—7 mo. 11 da. = 221 days, reckoning 30 days to the month. But the excess over 30 is,—for January, 1; March, 1; May, 1; July, 1 = 4; and the deficiency for February is 2; 4 − 2 = 2. Therefore, we must add 2 to 221, which makes it 223, the exact number of days in the interval. The 7 months *passed* contain, 31 + 28 + 31 + 30 + 31 + 30 + 31 = 212 days; and 212 ÷ 30 = $7\frac{2}{30}$; that is, the 2 remainder shows the excess in the number of days over the number found by the rule; in other words, the months *average* 30 days each, with 2 over.

247.—II. Practical Method of finding the number of days between two dates within a short period.

Example. What is the number of days from March 18th to September 25th in any year?

13 days remaining in March.
30 " in April,
31 " " May,
30 " " June,
31 " " July,
31 " " August,
25 " " September to the 25th inclusive.
―――
191 days, *Ans.*

Note.—The legal period of time within which an act is to be done, or a contract performed, is always computed *from* the date of the instrument, order, or contract, or from a day otherwise specified therein; that is, that day is *excluded*, and the time begins with the following day. For example, a promissory note dated September 5th, payable in 13 days after date, including grace, is due September 18th; 5 + 13 = 18. The day of the date is excluded, and the day of maturity included.

248.—III. To find the time a Promissory Note or Draft has to run from any given date.

Example. A promissory note dated January 8th, 1871, payable in 90 days, was discounted March 16th: what time had it to run from the day of discount?

OPERATION.

23
28 93
16 67
― ―
67 26 days, *Ans.*

Explanation.—By subtracting the time the note *has run* from its date, from the time it *had to run*, we obtain the answer directly. 31 − 8 = 23, the number of days *after* its date in January. There are 28 days in February, and, *including* the discount day, 16 days have passed in March. 23 + 28 + 16 = 67; 93, the whole time, including 3 days' grace, less 67, equals 26, *Ans.*

Notes.—1. It is the custom with banks in several States to include both the day of discount and the day of maturity, in reckoning the discount on commercial paper.

2. When the time in promissory notes and drafts is expressed in months, it is always construed to mean calendar months, with reference to the maturity of the paper; but banks, even in that case, usually compute the discount by days.

249. The number of days between any two dates, within a year, may also be readily ascertained by the use of the following

TABLE,

Showing the number of days from any day of one month to the corresponding day of any other month within one year.

FROM ANY DAY OF	TO THE CORRESPONDING DAY OF											
	JAN.	FEB.	MAR.	APR.	MAY.	JUNE.	JULY.	AUG.	SEPT.	OCT.	NOV.	DEC.
January	365	31	59	90	120	151	181	212	243	273	304	334
February....	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June........	214	245	273	304	334	365	30	61	92	122	153	183
July........	184	215	243	274	304	335	365	31	62	92	123	153
August......	153	184	212	243	273	304	334	365	31	61	92	122
September...	122	153	181	212	242	273	303	334	365	30	61	91
October.....	92	123	151	182	212	243	273	304	335	365	31	61
November...	61	92	120	151	181	212	242	273	304	334	365	30
December...	31	62	90	121	151	182	212	243	274	304	335	365

ILLUSTRATION.—How many days from March 13 to July 23? In the column of months on the left, we find *March*, and opposite, in the column of figures headed *July*, stands 122, which is the number of days from any day in March to the corresponding day in July. Therefore, from the 13th of March to the 13th of July it is 122 days; and to July 23d, it is 10 days more; 122 + 10 = 132, *Ans.*

If the later date be an earlier day of the month than the earlier date, subtract from the number found in the table the requisite number to give the correct result. In a leap-year, add or subtract one, as the case may require, for the 29th day of February.

Another very convenient time-table will be found on page 293

EXAMPLES FOR PRACTICE.

1. The American civil war was begun April 11, 1861, by the attack on Fort Sumter, and closed with the surrender of General Lee, April 9, 1865: how long did the war continue?

2. The war between England and the American colonies was commenced April 19, 1775; the Declaration of Independence was signed July 4, 1776, and peace was restored January 20, 1783: how long before the signing of the Declaration did the war commence? and how long did it continue?

3. When will a note dated March 11, payable three months after date, fall due? If made payable in ninety days, when will it become due?

4. A note at sixty days, dated August 15, is discounted September 9: for how many days should discount be charged, not including the day of discount?

5. Shakespeare was born April 23, 1564: how long from that time to the present?

6. Marco Bozzaris, the Grecian patriot, made his celebrated attack upon the camp of a Turkish pacha, August 20, 1823. The independence of Greece was acknowledged September 14, 1829: what is the difference between these two dates?

7. Suppose a draft payable 30 days after sight, and accepted July 6, 1871, to be discounted July 20: how many days' discount should be charged, including in the time the day of discount?

MULTIPLICATION OF COMPOUND NUMBERS.

250. EXAMPLE. Multiply 16 T. 14 cwt. 23 lb. 8 oz. by 7.

OPERATION.

T.	cwt.	lb.	oz.
16	14	23	8
			7
116	19	64	8

ANALYSIS.—Beginning with the lowest denomination, we multiply the numbers separately in order, and carry according to the scale of the multiplicand, as in addition. Thus, 7 times 8 oz. = 56 oz. = 3 lb. 8 oz.; and, writing the 8 oz. underneath, we multiply the next number, and to the product add the 3 lb.; and so continue to the highest denomination.

251. When the multiplier is large, and is a composite number, we may multiply successively by its component factors.

Example. What will 42 yards of broadcloth cost, at 17*s.* 9*d.* per yard?

OPERATION.

	s.	*d.*
	17	9
		7
6	4	3
		6
£37	5	6, *Ans.*

Analysis.—42 = 7 × 6. Therefore, we multiply first by one of these factors, and the product thus obtained by the other.

252. So, when the multiplier is a prime number, we may multiply by the component factors of any part of it, and then by the other part, and add the products.

Example. What is the value of 59 tons of iron, at £15 13*s.* 6*d.* per ton?

OPERATION.

£	*s.*	*d.*	
15	13	6	
		8	
125	8	0	= value of 8 tons.
		7	
877	16	0	= value of 56 tons.
47	0	6	= value of 3 tons.
£924	16	6	= value of 59 tons.

Analysis.—59 is a prime number, and is equal to the composite number 56, plus the prime number 3; or 8 × 7 + 3 = 59. We may, therefore, find the value of 56 tons by multiplying by the component factors of 56, and of 3 tons by multiplying the price of one ton by 3. The sum of these is the value of 59 tons. We might find the value of 60 tons by multiplying by 10 and 6, and then subtract the price of one ton.

Examples for Practice.

1. Multiply 3 mi. 7 fur. 27 rd. 12 ft. by 42.

2. What quantity of wood is there in 17 piles, each containing 3 cords 75 cubic feet?

3. Multiply 14 lb. 8 oz. 16 pwt. by 29.

4. Multiply 47 A. 3 R. 14 P. 12 sq. yd. by 53.

5. Multiply £13 11*s.* 9*d.* by 14⅔. *Ans.* £199 5*s.* 8*d.*

NOTE.—The multiplicand may be reduced to pence, or the shillings and pence to the decimal of a pound, before multiplying; or, it may be multiplied as it stands, by the whole number, and then by the fraction.

6. What is the length of a fence which incloses a square field, each side of which is 37 rd. 7 yd. 2½ ft. long?

7. The tropical or solar year is exactly 365 da. 5 h. 48 m. 49.7 sec. What time is lost in the calendar in 4 years by reckoning 365 days as one year? What time will be gained or lost in 100 years, if every fourth year be reckoned 366 days?

Ans. Lost in 4 years, 23 h. 15 m. 18.8 sec.
Gained in one hundred years, 18 h. 37 m. 10 sec.

8. Bought of Markland & Wells, London, 2 cases Worcestershire sauce, pints, each 5 doz., at 22*s.* per doz.; 3 cases do., ½ pints, each 10 doz., at 12*s.* per doz. The charges added were, for 5 cases, 17*s.* 6*d.*; commission, 11*s.* 5*d.*; consul's certificate, 11*s.* 6*d.*; and a discount was allowed of £6 12*s.* 3*d.*: what was the total amount of the invoice?

9. Multiply 17 T. 5 cwt. 3 qr. 13 lb. by 18.

10. Multiply 9 lb. 3 ℥. 4 ʒ. 2 ℈. by 7.

11. Multiply 112° 15′ 15″ by 40.

12. How many pounds Troy are equal to 100 lbs. Avoirdupois?

DIVISION OF COMPOUND NUMBERS.

253. EXAMPLE. Divide 57 mi. 3 fur. 36 rd. 14 ft. 6 in. by 6.

OPERATION.

mi.	fur.	rd.	ft.	in.
6)57	3	36	14	6
9	4	26	2	5

ANALYSIS.—Commencing with the highest denomination, we divide 57 mi. by 6, and get 9 as the quotient, with 3 remainder. Writing the 9 under the number divided, we reduce the remainder to the next lower denomination; 3 mi. = 24 fur. To this we add the 3 in the dividend, and divide their sum, obtaining the quotient 4, with 3 remaining. Continuing the division in this way to the lowest denomination, the entire quotient is found to be 9 mi. 4 fur. 26 rd. 2 ft. 5 in.

254. When the divisor is large, the work may be performed by long division; or, if it is a composite number, by dividing by its factors in succession.

Example. Divide £437 10*s.* 9*d.* equally among 32 persons.

OPERATION.

£. *s.* *d.*
32)437 10 9(£13 13*s.* 5*d.* 2$\frac{1}{8}$ *far.*
32
117
96
21 £ rem.
20
430*s.*
32
110
96
14*s.* rem.
12
177*d.*
160
17*d.* rem.
4
68 *far.*
64
4 *far.* rem.

£. *s.* *d.*
Or, 8)437 10 9
4)54 13 10$\frac{1}{8}$
£13 13 5 2$\frac{1}{8}$ *far.*

255. When the divisor is a mixed number, both dividend and divisor must be reduced to the same denomination as the fraction of the divisor, before dividing, But, in multiplying the dividend by the denominator of the fraction, it is not necessary to carry, as in other cases. Each denominate number may be multiplied as a simple number.

Example. Divide £151 19*s.* 11$\frac{1}{4}$*d.* by 3$\frac{1}{2}$.

OPERATION.

£.	s.	d.	far.
3$\frac{1}{2}$)151	19	11	1
			2
7)302	38	22	2
£43	8	6	2$\frac{4}{7}$

The same, by the ordinary method of multiplication.

£.	s.	d.	far.
3$\frac{1}{2}$)151	19	11	1
			2
7)303	19	10	2
£43	8	6	2$\frac{4}{7}$

Examples for Practice.

1. Divide 37 T. 1400 lb. 13 oz. by 8.

2. Divide 17 lb. 11 ℥. 4 ʒ. 2 ℈. 7 gr. by 17.

3. Divide 18 A. 2 R. 7 rd. 3 yd. 3 ft. by 6.

4. Divide 372 da. 21 h. 13$\frac{4}{7}$ m. by 75.

5. If 1777 acres of land be equally divided between 7 persons, how much will each receive?

6 Divide 54 lb. 10 oz. 19 pwt. 23 gr. by 9$\frac{1}{3}$.

7. How many cords of wood are there in a pile 24 feet long, 6 feet wide, and 4 feet 6 inches high?

8. Divide 242 cu. ft. 6 cu. in. by $\frac{3}{8}$.

9. Divide 48 d. 11 h. 30 m. 10 sec. by the decimal .5.

Miscellaneous Examples in Compound Numbers.

1. Reduce 1 mile to feet.

2. How many statute miles are there in the circumference of the earth?

3. Reduce 76 ch. 14.5 l. to inches.

4. A certain tract of land is 75 ch. 3 l. long, and 18 ch. 20 l. wide: how many acres does it contain?

5. How many square feet are there in a board 14 feet long and 18 inches wide?

6. How many bricks 8 in. long, 4 in. wide, and 2 in. thick, will it take to pave a yard 16 ft. 8 in. long by 8 ft. 4 in. wide? *Ans.* 625.

7. What is the value of $.\dot{3}$ of 12 cwt.?

8. Divide 17 d. 3 h. 30 m. by $\frac{3}{7}$.

9. Divide 13 S. 15° 40′ by $4\frac{1}{2}$.

10. Suppose the temperature on a certain day at St. Petersburg to be 8° R.: what is it on the scale of Fahrenheit? *Ans.* 50°.

11. If the Fahrenheit thermometer marks the temperature at 13° below zero, what will be the corresponding degree on Thermometre Centigrade? *Ans.* 25° below zero.

12. Suppose a clock to mark the correct time at 10 o'clock A.M. on Monday, and gain 2 seconds per hour: how much too fast will it be the next Saturday at 7 o'clock P.M.? *Ans.* 4 min. 18 sec.

13. What is the difference between .025 of a ton and .6 of a pound?

14. Reduce 13*s.* 6*d.* to the decimal of £1.

15. Reduce 7 oz. 12 pwt. 16 gr. to the decimal of a pound.

16. Reduce .16 of a year of 365 days to integers of lower denominations.

17. If a horse consume $3\frac{1}{2}$ bushels of oats per week: how many bushels will he require from the 9th day of November to the 15th day of April in a leap-year? *Ans.* 79.

18. A grocer sold 2 cwt. 3 qr. of sugar, at retail, and weighed it on scales which gave $\frac{1}{2}$ oz. short of the true weight on every pound: of how much money did he defraud his customers, reckoning the sugar at 15 cts. per pound? *Ans.* \$1.33$\frac{2}{31}$.

19. If the wheel of a carriage be 15 ft. 6 in. in circumference, how many revolutions will it make in running a mile?

20. The velocity of sound is 1142 feet per second: how far distant is a cloud from which a flash of lightning comes 12 seconds before the thunder is heard?

THE METRIC SYSTEM.

256. The **Metric System** of Weights and Measures, first adopted by France, was in 1866, by Act of Congress, authorized, or made lawful in the United States. It has also been legalized in Great Britain, Spain, Belgium, Portugal, Holland, Austria, Greece, Italy, Turkey, Norway, Sweden, Switzerland, and in some of these countries exclusively adopted; and, also in Mexico, Guatemala, Venezuela, Ecuador, United States of Colombia, Brazil, Chili, San Salvador, and the Argentine Republic.

257. The cosmopolitan character of the system, and the importance of a uniform standard of weights and measures among the civilized nations, will, it is believed, in time, lead to its universal use over a large portion of the globe.

258. The **Metre**, or unit of length, on which the system is based, and from which its name is derived, is, or was originally intended to be, a ten-millionth part of a quadrant of the earth's circumference, or the distance from either pole to the equator. It is equal to 39.37079 United States inches.

This distance was computed from a very accurate survey of the length of an arc of the meridian between Dunkirk and Barcelona, made by MM. Delambre and Mechain, and other surveys and measurements, and the metre derived from this natural base was marked by two very fine parallel lines on a platinum bar, at the temperature of the maximum density of water (or 4° Centigrade nearly), which was deposited in the national archives of France.

Investigations made since the metre was thus fixed, seem to show that it is not absolutely correct, when referred to the natural standard, being a minute fraction too small. For the standard, therefore, as now established, reference must be had to the bar on which it is preserved.

259. The **Unit of Area** is called the **Are**, and consists of a square whose side is 10 metres; that is, it contains 100 square metres.

260. The **Unit of Solidity** is called the **Stere**, and is a cube whose edge is one metre.

261. The **Unit of Capacity** is called the **Litre**, and is equal in volume to a cube whose edge is one-tenth of a metre.

262. The **Unit of Weight** is called the **Gramme**, and is the weight of a cube of distilled water (weighed in a vacuum, 4° C., or 39.2° F.) whose edge is one-hundredth of a metre.

NOMENCLATURE OF THE SYSTEM.

263. From the standard units thus established, are derived their multiples and submultiples, which are named systematically to express units of a higher or lower order upon a decimal scale. The names of the derivative units are formed by prefixing to the names of the standard units:—

1st. For the multiples, or integers of higher orders, the Greek numerals,

Deca, 10,
Hecto, 100, abbreviated from *hecaton*,
Kilo, 1000, " " *kilioi*,
Myria, 10000.

2d. For the submultiples, or decimals, the Latin ordinals,

Deci, 10th, abbreviated from *decimus*,
Centi, 100th, " " *centesimus*,
Milli, 1000th, " " *millesimus*.

Hence, decametre signifies 10 metres; decagramme, 10 grammes; hectometre, 100 metres; hectogramme, 100 grammes, etc.; and, decimetre signifies $\frac{1}{10}$ of a metre; centimetre, $\frac{1}{100}$ of a metre, and so on, by a uniform decimal system.

TABLES.

264. The following are the principal tables of weights and measures. The standard units are printed in small capitals; the denominations above are integers; those below, decimals.

Linear Measure.

		1 Millimetre	=	.03937079	inch.
10 Millimetres	=	1 Centimetre	=	.3937079	“
10 Centimetres	=	1 Decimetre	=	3.937079	inches.
10 Decimetres	=	1 Metre	=	3.2808992	feet.
10 Metres	=	1 Decametre	=	32.808992	“
10 Decametres	=	1 Hectometre	=	19.927817	rods.
10 Hectometres	=	1 Kilometre	=	.6213824	mile.
10 Kilometres	=	1 Myriametre	=	6.213824	miles.

The *Metre*, which is the *unit*, is equal to 39.37079 inches = 3.2808992 feet = 1.0936331 yards, United States measure. The *Hectometre* is 20 rods, nearly; the *Kilometre*, about ⅝ of a mile.

265.

Square Measure.

1 Centiare		(1 Square Metre)	=	1550.0591	sq. inches.
100 Centiares	=	1 Are	=	119.6034	sq. yards.
100 Ares	=	1 Hectare	=	2.47114	acres.

The *Are*, which is 100 *square metres*, or a square decametre, is the *unit*, and is equal to 119.6034 square yards, United States measure. This measure is used in measuring lands, etc. The lower denominations are not used.

266.

Solid Measure.

		1 Decistere	=	6102.705151	cu. inches.
10 Decisteres	=	1 Stere	=	35.31658	cu. feet.
10 Steres	=	1 Decastere	=	353.1658	“ “

The *Stere*, which is a cubic metre, is the *unit*, and is equal to 35.31658 cubic feet, United States measure, = 1.308 cubic yards = .2759 cord. The lower denominations are seldom used.

267.

Measures of Weight.

		1 Milligramme	=	.0154324874	grain.
10 Milligrammes	=	1 Centigramme	=	.154324874	“
10 Centigrammes	=	1 Decigramme	=	1.54324874	grains.
10 Decigrammes	=	1 Gramme	=	15.4324874	“
10 Grammes	=	1 Decagramme	=	.3527398	oz. Avoir.
10 Decagrammes	=	1 Hectogramme	=	3.527398	oz. “
10 Hectogrammes	=	1 Kilogramme	=	2.20462125	lbs. “
10 Kilogrammes	=	1 Myriagramme	=	22.0462125	“ “
10 Myriagr's or 100 Kilogrammes	=	1 Quintal	=	220.462125	“ “
10 Quintals or 1000 Kilogrammes	=	1 Tonneau	=	2204.62125	“ “

The *Gramme* is the unit, and is equal to 15.43234874 grains, or a little more than ½ a dram Avoirdupois.

32 grammes are very nearly equal to 1 ounce Troy. 15 grammes are taken as ½ oz. at the United States Post-Offices. A *kilogramme*, a common weight in trade, is a small fraction over 2½ lbs. Avoirdupois. A *Millier*, or *Tonneau*, is about 204 lbs. more than a *short ton*, and 36 lbs. less than a *long ton.*

268. MEASURES OF CAPACITY.

		1 Millilitre	=	.0610270515	cu. inch.
10 Millilitres	=	1 Centilitre	=	.610270515	" "
10 Centilitres	=	1 Decilitre	=	6.10270515	cu. inches.
10 Decilitres	=	1 LITRE	=	61.0270515	" "
10 Litres	=	1 Decalitre	=	661.0270515	" "
10 Decalitres	=	1 Hectolitre	=	6610.270515	" "
10 Hectolitres	=	1 Kilolitre	=	6102.70515	" "
10 Kilolitres	=	1 Myrialitre	=	61027.0515	" "

The *Litre*, or cube of a decimetre, is the *unit* of capacity, both of Liquid and Dry Measure. It contains 61.027 + cubic inches, and is equal to 1.05673 quarts, United States wine measure, and .9081 quart dry measure. The *Hectolitre* is equal to 100 Litres, or 2.8379 bushels dry measure.

EXAMPLES.

1. Write 4 metres and 5 centimetres. *Ans.* 4.05 m.

2. Write 3 grammes, 6 decigrammes, and 8 centigrammes. *Ans.* 3.68 g.

3. Write 17 litres and 7 decilitres. *Ans.* 17.7 l.

4. Write 85 ares and 15 centiares. *Ans.* 85.15 a.

5. Reduce 375 centimetres to metres. *Ans.* 3.75 m.

6. Reduce 50 kilogrammes to grammes. *Ans.* 50000 g.

7. Reduce 7 hectometres and 36 centimetres to metres. *Ans.* 700.36 m.

8. Required the cost of 30 metres and 5 decimetres of silk, at 10 francs per metre. *Ans.* 305 francs.

9. Bought 4 decares of land, at $75 per are: what was the cost? *Ans.* $3000.

LONGITUDE AND TIME.

269. The **Longitude** of a place on the globe is its distance in degrees, either east or west, from a given meridian, measured on the equator, or a parallel circle.

270. Longitude is reckoned from a meridian which is taken arbitrarily. The English and Americans (usually) reckon from that of the Royal observatory at Greenwich; the French from Paris; and in other countries different meridians are chosen, chiefly those which pass through their capitals. The meridian of Washington is sometimes used in this country.

271. The meridian thus selected as the base is called the *prime* or *first* meridian, and its longitude is marked 0°. Places east of this are in East Longitude, and places west, in West Longitude.

272. The diurnal revolution of the earth on its axis from west to east, causes the *apparent* motion of the sun from east to west, round the earth, or over 360 degrees of longitude, every 24 hours. Hence, the difference of time between any two places may be determined from their difference of longitude, and *vice versa.* The sun passes over

360°	of	longitude	in	24 hours,
15°	"	"	"	1 "
15′	"	"	"	1 minute,
15″	"	"	"	1 second,
1°	"	"	"	4 minutes,
1′	"	"	"	4 seconds.

Hence,

273. To find the difference of time between two places, from their difference of longitude.

RULE.—*Divide the difference of longitude, expressed in degrees, minutes, and seconds, by* 15; *the quotient will be the difference* *of time in hours, minutes, and seconds: or, multiply the difference of longitude in degrees and minutes, by* 4; *and the product will be the difference of time in minutes and seconds.*

And,

274. To find the difference of longitude between two places, from their difference of time.

RULE.—*Multiply the difference of time, expressed in hours, minutes, and seconds, by* 15; *the product will be the difference of longitude, in degrees, minutes, and seconds;*

Or, *divide the difference of time, in minutes, and seconds, by* 4, *and the quotient will be the difference of longitude, in degrees and minutes.*

NOTES.—1. In the following problems the meridian of Greenwich is taken as the first meridian.

2. Longitudes are usually reckoned to 180°, and no further. Thus, a motion in longitude of 185° east, will bring the traveller into 175° west longitude.

3. As longitude is either east or west, the difference of longitude between two places, one east and the other west, will be the sum of their longitudes, when that sum does not exceed 180°; but when it is greater than 180°, it will be the difference between their sum and 360°.

275. Since the sun apparently moves from east to west, the time at any given instant, at one place, will be *earlier* than the time at any place east of it, and later than the time at any place west of it. For example, when it is noon here, it is *past* noon everywhere east, and *before* noon everywhere west of our meridian. Therefore, if the difference of time between two places be *added* to the time at the *westerly place*, the sum will be the time at the easterly place; and, if this difference be *subtracted* from the time at the *easterly* place, the remainder will be the time at the westerly place.

At sea, the time at Greenwich, or other place from which longitude is reckoned, is kept by a chronometer. The navigator, by an astronomical observation, determines the time at his ship, and comparing it with that marked by his chronometer, ascertains the longitude of his ship.

EXAMPLES.

1. The longitude of the City Hall, New York, is 74° 1′ 6″ west, and of Independence Hall, Philadelphia, 75° 9′ 54″ west: what is the difference of time between the two places?

$$\begin{array}{rrrrl} \text{SOLUTION:} & 75^\circ & 9' & 54'' & \\ & 74^\circ & 1' & 6'' & \\ \hline 15) & 1^\circ & 8' & 48'', & \text{difference of longitude.} \\ \hline & & & 4 \text{ m. } 35\frac{1}{5} \text{ sec.,} & \text{difference of time.} \end{array}$$

2. The difference of time between London (St. Paul's Cathedral), and New York (City Hall), is 4 h. 55 m. 41$\frac{1}{5}$ sec.: what is the difference of longitude?

$$\begin{array}{rrrrl} & \text{h.} & \text{m.} & \text{sec.} & \\ \text{SOLUTION:} & 4 & 55 & 41\frac{1}{5} & \\ & & & 15 & \\ \hline & 73^\circ & 55' & 28'' & \text{difference of longitude.} \end{array}$$

3. Boston is 70° 3′ 58″, and Philadelphia 75° 9′ 54″ west longitude: when it is noon at Philadelphia, what is the time at Boston? *Ans.* 20 m. 23$\frac{11}{15}$ sec. past 12 o'clock, M.

4. The longitude of San Francisco (Point Boneta Light) is 122° 31′ west, and of Melbourne, Australia, 144° 59′ east longitude: what is the difference of time? *Ans.* 6 h. 10 m.

5. A gentleman travelling from Halifax, Nova Scotia, to Charleston, S. C., finds his watch 1 h. 5 m. 8 sec. faster than the time at Charleston. If his watch gives the correct time at Halifax, which is in longitude 63° 35′ west, what is the longitude of Charleston?

6. The longitude of Cincinnati, O., is 84° 24′ west, and of St. Louis, Mo., 90° 15′ 16″ west: what is the time at St. Louis when it is half-past 5 o'clock A.M. at Cincinnati?

7. Pekin, China, is 116° 27′ 30″ east, and Washington, 77° 1′ 30″ west longitude. At 10 o'clock A.M., Washington time, what is the hour at Pekin? *Ans.* 9 h. 6 m. 8 sec. P.M.

DUODECIMALS.

276. Duodecimals are denominate fractions of a foot, formed by dividing the unit into 12 equal parts, called inches or primes, and these again into 12ths, called seconds, and so on indefinitely.

277. In their use the denominators are omitted and the numerators are treated as Compound Numbers. The scale of increase and decrease is uniformly 12.

278. The denominations are marked by indices; thus, 1 inch, or prime, is written 1′; 1 second, 1″; 1 third, 1‴, etc.

279. The unit value of the different orders is as follows:—

1′ (inch or prime) = $\frac{1}{12}$ of a foot, or 1 in. linear measure.
1″ (second) or $\frac{1}{12}$ of $\frac{1}{12}$ = $\frac{1}{144}$ of a foot, or 1 in. square measure.
1‴ (third) or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ = $\frac{1}{1728}$ of a foot, or 1 in. cubic measure

Table.

12 fourths (′′′′)	make	1 third	1‴
12 thirds	"	1 second	1″
12 seconds	"	1 prime	1′
12 primes	"	1 foot	ft.

280. Duodecimals are employed chiefly by artificers in computing the superficial and solid contents of their work.

Note.—The word *duodecimal* is derived from the Latin word *duodecim*, which signifies twelve.

ADDITION AND SUBTRACTION.

281. Duodecimals are added and subtracted in the same manner as Compound Numbers.

Examples.

1. Add 7 ft. 3′ 8″ 5‴, 9 ft. 4′ 11″ 3‴, 16 ft. 9′ 10″.
Ans. 33 ft. 6′ 5″ 8‴.

2. What is the sum of 13 ft. 2′ 3″ 4‴, 21 ft. 11′ 9″ 10‴, and 11 ft. 9′ 0″ 6‴?

3. From 19 ft. 6′ 9″ 3‴, take 14 ft. 5′ 11″ 5‴.

4. From 43 ft. 5′, take 27 ft. 6′ 4″.

5. Add 37 yds. 2 ft. 3′, 14 yds. 1 ft. 11′ 10″, and 2 ft. 9′.

6. From 15 ft. take 9′ 3″.

MULTIPLICATION OF DUODECIMALS.

282. The **Multiplication of Duodecimals** is similar to that of Compound Numbers.

283. The Denomination of the product of any two or more factors is indicated by the sum of their indices, for,

$$\begin{aligned}
1' &\times 1 = \tfrac{1}{12} \times 1 = \tfrac{1}{12} = 1' \\
1'' &\times 1 = \tfrac{1}{144} \times 1 = \tfrac{1}{144} = 1'' \\
1' &\times 1' = \tfrac{1}{12} \times \tfrac{1}{12} = \tfrac{1}{144} = 1'' \\
1'' &\times 1' = \tfrac{1}{144} \times \tfrac{1}{12} = \tfrac{1}{1728} = 1'''
\end{aligned}$$

NOTE.—Scientifically speaking, it is absurd to say that one concrete number can be multiplied by another; as, 2 *feet* by 3 *inches*. Yet, as a convenient mode of expression, we say, feet multiplied by primes produce primes; inches by inches, seconds, etc. These expressions mean that units multiplied by 12ths produce 12ths, and 12ths by 12ths, 144ths, etc.

284. The definition of Duodecimals as *Denominate Fractions* clearly describes their character; and, from a knowledge of common fractions, the denomination of any product may be easily determined, as shown above.

EXAMPLES.

1. Multiply 14 ft. 6 in. by 7 ft. 3 in.

OPERATION.

14 ft.	6′	
7 ft.	3′	
3 ft.	7′	6″
101 ft.	6′	
105 ft.	1′	6″

ANALYSIS.—Beginning at the right, 6′ × 3′ $(\tfrac{6}{12} \times \tfrac{3}{12} = \tfrac{18}{144})$ = 18″ = 1′ 6″. We write the 6″ one place to the right, and carry the 1′ to the next product. Then, $\overline{14 \times 3'} + 1' = 43' = 3$ ft. 7′, which we write in their order. Next 6′ × 7 = 42′ = 3 ft. 6′. Writing the 6′, we carry the 3. Finally, $\overline{14 \times 7} + 3 = 101$. The sum of these partial products is the required product, which is 105 ft. 1′ 6″.

EXAMPLES FOR PRACTICE.

1. What is the product of 37 ft. 9 in. multiplied by 8 ft. 10 in.?

2. What is the area of a marble slab 6 ft. 7′ 3″ long, and 2 ft. 4′ 9″ wide?

3. How many square feet in 8 boards, each 14 ft. 6 in. long, and 1 ft. 3 in. wide?

4. A certain room is 20 ft. 6 in. long, 18 ft. 6 in. wide, and 9 ft. 6 in. high. What will it cost to plaster the walls and ceiling, at 25 cents per square yard?

5. Multiply 14 yds. 3 ft. 2′ by 6 yds. 2 ft. 3′.

6. How many cords in a pile of wood 42 feet 6 inches long, 4 feet wide, and 5 feet 6 inches high?

CONTRACTED METHOD.

285. A product practically correct to a given denomination may be obtained by the following method.

EXAMPLE.

1. Multiply 14 ft. 2′ 4″ 9‴ by 3 ft. 5′ 6″ 3‴, extending the product only to seconds.

OPERATION.

	14 ft.	2′	4″	9‴
3‴	6″	5′	3 ft.	
	42 ft.	7′	2″	
	5 ft.	11′	0″	
		7′	1″	
			3″	
	49 ft.	1′	6 — ″ +	

ANALYSIS.—We write the multiplier in reversed order, with the integer, 3, under seconds in the multiplicand, the denomination to which the multiplication is to be extended. We then multiply 4″ by 3, and carry 2″, which results from multiplying 9‴ by 3, the remainder 3‴ being rejected. Next, we multiply each term of the multiplicand to the left in order by 3, and carry in the usual way. Then we multiply by 5′, rejecting all the terms of the multiplicand to the right, but carrying 2″ because 5′ × 4″ = 20‴ = 1″ 8‴, which is nearer 2″ than 1″. $\overline{14 \times 6''}$ +

1″ (carried) = 85″ = 7′ 1″. Lastly, 14 × 3‴ = 42‴ = 3″+. The sum of these partial products is the required product approximately correct. It will be seen by an examination of this process that all the factors which would produce fourths are rejected, those which would give thirds are only considered in carrying, and all the higher terms are multiplied together in regular order. The answer is correct within a second. The true product is 49 ft. 1′ 6″ 8‴ 3⁗ 2′⁗.

EXAMPLES FOR PRACTICE.

1. Multiply 9 ft. 2′ 6″ 7‴ by 3 ft. 5′ 8″ 10‴, extending the product only to seconds.

2. Multiply 17 ft. 4′ 3″ 2‴ by 6 ft. 8′, to find the product correct to primes.

3. Find the cubical contents of a block of marble 6 ft. 3 in. long, 2 ft. 4 in. wide, and 2 feet thick, correct within an inch.

DIVISION OF DUODECIMALS.

286. Since Division is the reverse of Multiplication, the denomination of the quotient arising from dividing one duodecimal by another may be readily determined by *subtracting* the number of indices in the divisor from the number of the dividend. Thus 6″ ÷ 3′ = 2′. The quotient is greater than the dividend, for $\frac{6}{144} \div \frac{3}{12} = \frac{6}{144} \times \frac{12}{3} = \frac{2}{12} = 2'$.

EXAMPLE.

1. Divide 13 ft. 1′ 6″ by 5 ft. 3′.

OPERATION.

$$\begin{array}{r|llll}
5\text{ ft. }3') & 13\text{ ft.} & 1' & 6''(2\text{ ft. }6' \\
 & 10\text{ ft.} & 6' & \\
\hline
 & 2\text{ ft.} & 7' & 6'' \\
 & 2\text{ ft.} & 7' & 6'' \\
\hline
\end{array}$$

ANALYSIS.—First, 5 is contained in 13 2 times with a remainder; therefore, we infer that 13 ft. 1′ will contain 5 ft. 3′ 2 times, and we place 2 in the quotient, and multiply the whole divisor by it, obtaining the product 10 ft. 6′. Subtracting this from 13 ft. 1′, and bringing down the next term, we have 2 ft. 7′ 6″ to be divided. Now since 5 is greater than 2, we reduce the 2 ft. to primes (mentally) and add 7′, making 31′. Then 5 is contained in 31 6 times, and we place 6′ in the quotient, and multiply as before. The product is 2 ft. 7′ 6″, and there is no remainder. Hence, 2 ft. 6′ is the quotient.

EXAMPLES FOR PRACTICE.

1. Divide 34 ft. 7′ 7″ 6‴ by 4 ft. 9′.

2. There are 8 ft. 5′ 3″ in the surface of a marble slab, the length of which is 3 ft. 9′: what is its width?

3. A block of granite is 3 ft. 2′ wide, 2 ft. 11′ thick, and contains 135 cu. ft. 10′ 2″ 1‴: what is its length?

4. Divide 48 ft. 5′ 1″ 1‴ 6⁗ by 3 ft. 4′ 5″.

5. Divide 108 ft. 9′ 9″ 9‴ 6⁗ by 4 ft. 3′ 2″.

6. How many bricks, 8 inches long, 4 inches wide, and 2 inches thick, are there in a wall 24 feet long, 12 feet high, and 1½ feet thick?

7. How many square yards of carpeting will it require to cover a floor 18 ft. 6′ long, and 12 ft. 4′ wide?

8. A certain garden is inclosed by a brick wall 15 inches thick and 8 feet high. The inclosure, inside of the wall, is 50 yards long and 30 yards wide: how many cubic feet are in the wall? *Ans.* 4850 cubic feet.

9. Required the cost, at $30 per thousand, of 4 boards, measuring as follows: one, 14 ft. 10′ by 1 ft. 3′; one, 18 ft. 3′ by 1 ft. 4′; one, 16 ft. 8′ by 1 ft. 6′; one, 14 ft. by 9′.

10. Required the thickness of a block of granite, in the form of a parallelopipedon, one of whose surfaces contains 25 ft. 10′ 4″, and whose solid contents are 64 ft. 2′ 10″.

11. Divide 18 ft. 3′ 11″ 3‴ by 7.

12. Three men own two piles of wood in equal shares. The first is 8 ft. 6′ long, 3 ft. 4′ high, and 4 feet wide; the second is 10 ft. 5′ long, 4 ft. 8′ high, and 4 ft. wide. Required each one's share.

13. How many cords of wood are there in a pile of wood 15 feet long, 4 feet wide, and 6 feet 6 inches high?

14. Required the cost, at 20 cents per square yard, of plastering a room, the length of which is 30 ft. 6 in., the width 22 ft. 4 in., and height of ceiling 9 ft. 3 in.

PERCENTAGE.

287. Percentage is a charge, allowance, or estimate, of a certain number of units on every hundred in a given number or quantity.

288. The term **Per Cent.** is a contraction of the Latin words *per centum*, and signifies *by the hundred.* Therefore, any per cent. of a number is so many hundredths of it; 1 per cent. is $\frac{1}{100}$; 2 per cent., $\frac{2}{100}$, etc. 3 per cent. of $150 is $\frac{3}{100}$ of $150, or $4.50; and 5 per cent. of 80 bushels is $\frac{5}{100}$ of 80 bushels, or 4 bushels.

289. The **Rate per Cent.** is the number of units per hundred taken or considered.

290. The **Rate per Unit** is the rate on each unit denoted by the rate per cent. expressed decimally as hundredths, or by an equivalent fraction.

NOTE.—The rate per cent., as such, can be expressed only by a whole or mixed number, or a fraction; but, as a *multiplier*, or as the *rate per unit*, it is usually written as a decimal, and therefore it is said that the rate may be expressed decimally.

The words *per cent.* are represented by the sign %· thus, 5 per cent. is written 5%.

291. The mode of writing different rates per cent. decimally is shown by the following examples:

1%	is written	.01	$\frac{1}{2}$%	is written	.00$\frac{1}{2}$, or .005
5%	"	.05	$\frac{1}{4}$%	"	.00$\frac{1}{4}$, or .0025
10%	"	.10	5$\frac{1}{2}$%	"	.05$\frac{1}{2}$, or .055
50%	"	.50	7$\frac{3}{10}$%	"	.07$\frac{3}{10}$, or .073
100%	"	1.00	18$\frac{3}{4}$%	"	.18$\frac{3}{4}$, or .1875

292. A given per cent. of a number may be an aliquot part of it; in which case, to compute the percentage, it is often simpler to take the required fractional part of the number than to multiply by the rate.

50%	of a number is	$\frac{1}{2}$	of it.	16$\frac{2}{3}$%	of a number is	$\frac{1}{6}$	of it.
25%	" "	$\frac{1}{4}$	"	6$\frac{1}{4}$%	" "	$\frac{1}{16}$	"
20%	" "	$\frac{1}{5}$	"	5%	" "	$\frac{1}{20}$	"
33$\frac{1}{3}$%	" "	$\frac{1}{3}$	"	2$\frac{1}{2}$%	" "	$\frac{1}{40}$	"

EXERCISES IN NOTATION AND NUMERATION.

1. Express decimally, 4%, 7%, 9$\frac{1}{2}$%, 5$\frac{3}{4}$%, 10%, 12$\frac{1}{2}$%.
2. Express decimally, $\frac{1}{4}$%, $\frac{1}{2}$%, $\frac{3}{4}$%, $\frac{7}{8}$%, $\frac{1}{10}$%, $\frac{4}{5}$%.
3. What per cent. of a number is .75 of it?
4. What per cent. of a number is .5 of it?
5. What per cent. of a number is .005 of it?
6. What per cent. of a number is .0075 of it?
7. What per cent. of a number is $\frac{1}{3}$ of it?
8. What per cent. of a number is the whole of it?
9. What per cent. of 7 is 7?
10. What per cent. of a number is 1$\frac{1}{4}$ times itself?
11. What per cent. of 50 is 100?
12. Write $\frac{1}{2}$ of 1%; $\frac{7}{8}$ of 1%, .3%; .75%; 185%.
13. What rate per cent. of a number is .08$\frac{1}{2}$ of it?
14. What per cent. of a number is .125 of it?

PROBLEMS IN PERCENTAGE.

293. In the solution of problems in percentage five things are to be considered; namely, the *Base, Rate, Percentage, Amount,* and *Difference.*

1. The **Base** is the number upon which the percentage is computed.

2. The **Rate** is the number of hundredths of the base given or required.

3. The **Percentage** is the number obtained by taking any given per cent. of the base.

4. The **Amount** is the base increased by the percentage.

5. The **Difference** is the base diminished by the percentage.

NOTE.—In all problems in this book, understand the simple term rate to mean the rate expressed decimally.

294. From the principles of percentage we derive the following Formulas, by which it will be seen that, when any two of these elements or things are given, a third can be found:

The Base × *The Rate* = *The Percentage.*
The Percentage ÷ *The Base* = *The Rate.*
The Percentage ÷ *The Rate* = *The Base.*

CASE I.

295. To compute percentage.

EXAMPLE 1. What is 15 per cent. of 780?

SOLUTION.

$$\begin{array}{r} 780 \\ .15 \\ \hline 117.00 \end{array}$$ *Ans.*

ANALYSIS.—The percentage is the base multiplied by the rate. 15% of 780 is 15 hundredths of it; hence, 780 × .15 = 117, the percentage. Hence the

RULE.—*Multiply the base by the rate.*

NOTES.—1. When the *Amount* is required, add the percentage to the base, or multiply by 1 plus the rate.

2. When the difference is required, subtract the percentage from the base, or multiply by 1 minus the rate.

Example 2. A trader bought a horse for $250 and sold him at a profit of 20%: for how much did he sell him?

SOLUTION.

$250	Or, $250
.20	1.20
50.00, Per centage gained.	$300.00, *Ans.*
250, Cost.	
$300, Amount, *Ans.*	

Example 3. A merchant sold a quantity of goods which cost him $480, at a loss of 5%: for how much did he sell the goods?

SOLUTION.

$480	Or, 480
.05	.95
24.00, Percentage lost.	2400
480, Cost.	4320
$456, Difference, *Ans.*	$456.00, *Ans.*

Examples for Practice.

1. What is 6 per cent. of $400? *Ans.* $24.
2. What is 10 per cent. of 5000 men?
3. What is 8 per cent. of 37 tons? *Ans.* 2.96 tons.
4. What is 40% of 350 bushels?
5. What is $66\frac{2}{3}$% of 980 men?
6. What is $2\frac{1}{2}$% of £750?
7. What is $\frac{1}{4}$% of $87.50? $\frac{1}{8}$%? $\frac{3}{4}$%?
8. What is .1 per cent. of $189.75?
9. What is 100 per cent. of .12?
10. What is $\frac{1}{2}$% of $\frac{3}{8}$? $\frac{1}{4}$% of $\frac{7}{12}$? $\frac{1}{8}$% of .75?
11. What is 5% of 40%? *Ans.* 2%.

12. What is the amount of $500 increased by 5% ?

13. What is the difference of $1000 diminished by 10% ?

14. What is 115% of 375?

15. What is 95% of $3700?

16. Bought 100 bbls. flour @ $7.50 per bbl., and sold it at 10% profit: what amount did it bring?

17. Sold a quantity of merchandise that cost $1670, at a loss of 3%: for what amount did I sell it?

18. If 5% be deducted from a bill of goods amounting to $140.50, what is the percentage allowed?

19. A man failed who owed $25000, and paid his creditors 25 cents on a dollar. How much did he pay?

CASE II.

296. The percentage and rate being given, to find the base.

EXAMPLE. 24 is 6% of what number?

OPERATION.

.06)24.00

400, *Ans.*

ANALYSIS.—The percentage is found by multiplying the base by the rate; consequently the percentage, 24, divided by the rate, .06, gives the base. Or, we may reason thus: if 24 is 6%, 1% is ⅙ of 24, or 4; and 100% is 4 × 100 = 400. Hence the

RULE.—*Divide the percentage by the rate.*

NOTE.—When the *amount* is given, the number of per cent. which it is of the base will be more than 100, and 1 plus the rate will be the proper divisor. If the *difference* be given, the divisor will be 1 minus the rate.

EXAMPLES FOR PRACTICE.

2. $326.25 is 87% of what number?

3. What number diminished by 6% of itself will be equal to 564?

4. What number increased by 25% of itself will be equal to 250?

5. If the percentage be 72, and the rate 9%, what is the base?

6. If the amount be $795, and the rate 6%, what is the base?

7. If the difference be $315.25, and the rate 3%, what is the base?

8. 104060 is 110% of a certain number: what is that number?

9. $1.25 is $\frac{1}{8}$% of what sum of money?

10. $\frac{5}{8}$ is $\frac{1}{2}$% of what number?

11. Paid $45 premium on a policy of insurance, and the rate was 1$\frac{1}{2}$%: what was the amount of the policy?

12. A house was sold at an advance of 5% on the cost, for $13000: what was the cost?

13. Loaned for one year a certain sum of money, at 6% per annum, and received at the end of the year, for principal and interest, $715.50: required the amount loaned.

14. Sold a horse for $106.40, which was 112% of his cost: what was the cost?

15. .0028 is $\frac{1}{4}$% of what number?

CASE III.

297. The percentage and base being given, to find the rate.

EXAMPLE. What per cent. of 850 is 51?

SOLUTION.

850)51.00(.06 = 6%, *Ans.*
5100
——

ANALYSIS.—The percentage, 51, is the product of the base, 850, by the rate. Therefore, 51 ÷ 850 = the rate. Or, we may reason thus: 51 is $\frac{51}{850}$ of 850, and the question is, How many hundredths are equal to $\frac{51}{850}$? The answer is found by reducing $\frac{51}{850}$ to hundredths, which is done by annexing two decimal ciphers to the numerator and dividing by the denominator. Hence the

RULE.—*Divide the percentage by the base.*

Note.—The rate per cent. is found directly as a whole or mixed number or fraction, by multiplying the percentage by 100, and dividing by the base.

Examples for Practice.

1. The base is $600, and the percentage $30: what is the rate?

2. What per cent. of 1000 men is 200 men?

3. What per cent. of £1 is 15 shillings?

4. What per cent. of $\frac{3}{4}$ is $\frac{1}{8}$?

5. If $15 be paid for the use of $200 for one year, what is the rate per cent.?

6. What per cent. of 10 is .1?

7. What per cent. of 40 is 10% of 50?

8. My profit on the sale of a quantity of goods which cost me $1050, was $125: required the gain per cent.

9. If I pay $36 for the use of $200 for 4 years, what is the rate per annum?

10. A broker bought 10 shares of railroad stock, at $90 per share, and sold it at $105 per share: what per cent. profit on his investment did he make?

11. What is 10 per cent. of $\frac{3}{8}$?

12. What per cent. of $100 is 5 mills?

PERCENTAGE ON STERLING MONEY.

298. The usual method of computing percentage on Sterling Money is, to reduce the shillings, pence, and farthings to the decimal of a pound, and then multiply by the rate. The following method is simpler and shorter.

Rule.—*Multiply the given sum, as in Compound Numbers, by the rate expressed as a whole number, and divide the product by* 100.

Example. Find 5% of £112 14*s.* 6*d.*

OPERATION.			Or,		
£.	s.	d.	£.	s.	d.
112	14	6	112	14	6
		5			5
£5.63	12	6	£5.60	70	30
20			20		
s. 12.72			*s.* 12.70		
12			12		
d. 8.70			*d.* 8.70		
4			4		
far. 2.80			*far.* 2.80		

Ans. £5 12*s.* 8½*d.*+.

Explanation.—After multiplying by the rate, point off two figures from the right of the pounds, reduce the decimal to shillings, add the shillings, and again point off two figures; and so continue till the division is complete. The percentage will be found on the left of the points, in pounds, shillings, pence, etc.

Case IV.

299. To find the percentage when the rate is an aliquot part of the base.

Rule.—*Divide the base by the denominator of the fraction which denotes the aliquot part.*

Example 1. Find 5% of $2750; also 2½%.

SOLUTION AT 5%.	SOLUTION AT 2½%.
$5\% = \frac{1}{20}$. 2|0)$275|0	$2\frac{1}{2}\% = \frac{1}{40}$. 4|0)$275|0
$137.50, *Ans.*	$68.75, *Ans.*

Example 2. What is 2½% of £165 12*s.* 6*d.*?

SOLUTION. 4|0)£16|5 12*s.* 6*d.*

£4 2*s.* 9*d.*+, *Ans.*

EXAMPLES FOR PRACTICE.

1. Find 2½% of $4000. Of $840. Of $175.

2. What is 5% of £456 9*s.* 6*d.*?

3. Find 12½% of $75.50. Of $1840.

4. Find 25% of £395. Of £800.

5. What is 33⅓% of 975 tons?

6. What is 16⅔% of $455.50?

MISCELLANEOUS PROBLEMS.

1. What is 25 per cent. of 1000?

2. 50 is 10 per cent. of what number?

3. What is the amount of $30 increased 5%?

4. What per cent. of 125 is 20?

5. If 40% of itself be deducted from 800, what will the remainder be?

6. If the amount be 63, and the rate 5%, what is the base?

7. If the difference be 194, and the rate 3%, what is the base?

8. A man having an income of $4500 per year, spends 20% of it for house rent, and 40% of it for general expenses: how much does he pay for house rent? how much for general expenses? and what per cent. of his whole income does he spend?

9. Sold a horse for $25 less than I paid for him, and lost 10% of the cost: what did the horse cost me? *Ans.* $250.

10. What per cent. of a number is ¾ of it?

11. A man sold a house for 10 per cent. less than it cost him. If he had sold it for $5175 he would have gained 15 per cent.: for how much did he sell it? *Ans.* $4050.

12. 38500 is 1000 per cent. greater than another number: what is that other number?

13. What per cent. is 10% of 50%? *Ans.* 5%.

14. A merchant lost 5% of his capital by the failure of one debtor, and 4½% of the remainder by the failure of another, and then had $7258 left: what was his capital?

Ans. $8000.

15. A gentleman having a certain amount of money, deposited 90% of ¾ of it in bank, and then had $1625 on hand: what was the amount of his money?

16. Bought 100 shares of railroad stock at 95% of its par value, $100 per share: what did it cost?

17. What per cent. of 5 kilogrammes is 50 grammes?

Ans. 1%.

18. What is 5 per cent. of 3 hectares? *Ans.* 15 ares.

19. The sum of two numbers is 300, which is 150% of the greater number: what is the less number?

INTEREST.

300. Interest is the sum paid for the use of money. It formerly meant the *use* of money.

301. The **Principal** is the sum, or base, on which the interest is computed.

302. The **Rate** is the number of units per hundred, or per cent. of the principal paid for its use for any given time, usually *per annum*.

303. The **Amount** is the sum of the principal and interest.

304. Simple Interest is interest upon the principal only.

305. Compound Interest is interest upon both principal and interest, compounded at certain periods of time when the interest becomes due and remains unpaid; as, annually, semi-annually, and quarterly.

306. Annual Interest is simple interest on interest payable annually and overdue, as well as on the principal.

307. Legal Interest is the rate allowed by law, and varies in the different States.

308. The taking of any more than the legal rate is, in most of the States, an offence called *usury*, which is prohibited under various penalties.

309. In many States a certain rate is fixed by law, to be allowed on all contracts when no particular rate is specified, while a higher rate may be taken by special agreement between the parties.

TABLE.

Showing the legal rates of interest, and the rates allowed by special agreement, in the several States and Territories of the Union.

STATES.	LEGAL RATE.	BY SPECIAL AGREEMENT.	STATE.	LEGAL RATE.	BY SPECIAL AGREEMENT.
Alabama	8%		Mississippi	8%	10%
Alaska (Ter.)	—	—	Missouri	6%	10%
Arizona (Ter.)	—	—	Montana (Ter.)	10%	Any rate.
Arkansas	6%	Any rate.	Nebraska	10%	Any rate.
California	10%	Any rate.	New Hampshire	6%	
Colorado (Ter.)	10%	Any rate.	Nevada	10%	Any rate.
Connecticut	6%		New Jersey	6%	7%*
Dakota (Ter.)	—	—	New York	7%	
Delaware	6%		New Mexico (Ter.)	6%	
Dist. of Columbia	6%		North Carolina	6%	
Florida	6%	8%	Ohio	6%	
Georgia	7%		Oregon	10%	Any rate.
Idaho (Ter.)	10%		Pennsylvania	6%	7%†
Illinois	6%	10%	Rhode Island	6%	
Indiana	6%		South Carolina	7%	
Iowa	6%	10%	Tennessee	6%	
Kansas	7%	12%	Texas	8%	12%
Kentucky	6%		Utah (Ter.)	10%	Any rate.
Louisiana	5%	8%	Vermont	6%	
Maine	6%		Virginia	6%	
Maryland	6%		West Virginia	6%	
Massachusetts	6%	Any rate.	Washington (Ter.)	—	—
Michigan	7%	10%	Wisconsin	7%	12%
Minnesota	7%	12%	Wyoming (Ter.)	—	—

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* In certain localities only, and under a proviso.

† On balances in the hands of commission merchants, etc.

310. Interest is usually reckoned upon the basis of 360 days to the year, and 30 days to the month. This practice is not strictly correct, although sanctioned by almost universal usage, and in some States by legal enactments. Thus, the interest of $500 for 18 days, at 6 per cent., computed by this method, is $\frac{18}{360}$ of $30, the interest of $500 for one year, or $1.50. If 365 days be reckoned a year, it will be $\frac{18}{365}$ of $30, or 1.47\frac{69}{73}$, which is $\frac{5}{365}$ or $\frac{1}{73}$ less than $1.50, the amount found by reckoning 360 days to the year.

Case I.

311. To compute interest for years and months.

It is obvious, from the fact that interest is a certain per cent. of the principal per annum, that, if the principal be multiplied by the rate expressed as hundredths, the product will be the interest for one year. This product, multiplied by the number of years, will be the interest for any given time. Hence the following

Rule.—I. *Multiply the principal by the rate per cent., and the resulting product by the number of years and fraction of a year; or, take aliquot parts of one year's interest for the months.*

II. *Or, multiply the rate per cent. by the time expressed in years, and the principal by the product thus obtained.*

Examples.

1. What is the interest of $750, for 4 years and 6 months, at 6 per cent.?

$$750 \times .06 \times 4\tfrac{1}{2} = 202.50$$

$$\text{Or, } .06 \times 4\tfrac{1}{2} \times 750 = 202.50$$

Ans. 202,50.

2. What is the interest of $475, for 3 years, at 5 per cent.?

3. Find the interest of $184.25, for 4 years and 3 months, at 7 per cent.

4. Find the interest of $240.50, for 1 year and 6 months, at 8 per cent.

5. What is the amount of $3416.40, for 2 years and 10 months, at 6 per cent. ?

6. What is the amount of $716.40, for 3 years, at 9 per cent. ?

7. What is the amount of $1840.37, for 2 years and 3 months, at 7 per cent. ?

CASE II.

312. To compute interest for months only.

At 6 per cent. for one year, or 12 months, the rate is *one half per cent. a month,* for 6 is one half of 12. Therefore, when the time is given in months, one-half of the number of months, expressed as hundredths, is the *rate per unit* for that time. Hence the

RULE.—*Multiply the principal by half the number of months, expressed as hundredths; the product will be the interest at* 6 *per cent.*

NOTE.—For any other rate than 6 per cent., increase or diminish the interest so found by as many sixths of itself as the given rate is greater or less than 6%. Thus, for 8%, add $\frac{2}{6}$, or $\frac{1}{3}$; for 7%, add $\frac{1}{6}$; for 5%, subtract $\frac{1}{6}$.

EXAMPLES.

1. What is the interest of $640, for 10 months, at 6 per cent. ? *Ans.* $32.

SOLUTION. $640 \times .05 = 32.$

2. What is the interest of $375.48, for 9 months, at 6 per cent. ? *Ans.* $16.90.

SOLUTION. $375.48 \times .04\frac{1}{2}$, or $.045 = 16.8966.$

3. Find the interest of $1137.50, for 14 months, at 7 per cent.

4. Find the interest of $19.75, for 11 months, at 8 per cent.

5. What is the amount of $3264.50, for 18 months, at 6 per cent. ?

6. What is the amount of $244.60, for 28 months, at 7 per cent.?

7. What is the interest of $200, for 7 months, at 6 per cent.?

8. What is the amount of $1000, for 6 months, at 7 per cent.?

9. Find the interest of $189.75, for 7 months, at 6%.

10. Find the interest of $130.50, for 8 months, at 7%.

11. Find the interest of $240.18, for 6 months, at 9%.

CASE III.

313. To compute interest for days only.

As shown by Case II, the interest of $1, for one month, at 6 per cent., is one half a cent, or 5 mills. Hence, for 6 days ($\frac{1}{5}$ of 30 days), it is $\frac{1}{5}$ of 5 mills, or 1 mill, and for 1 day, $\frac{1}{6}$ of a mill. And, since mills are thousandths of a dollar, when the time is given in days, one sixth of the number of days, expressed as thousandths, is the *rate per unit* for that time. Hence the following

RULE.—*Multiply the principal by one sixth of the number of days, expressed as thousandths; the product will be the interest at* 6 *per cent.*

NOTE.—When it is more convenient, to avoid fractions in the multiplier, multiply by the number of days, and divide by 6. The result will be the interest in mills. Therefore, point off three decimal places, in addition to the number in the multiplicand.

EXAMPLES.

1. Find the interest of $945, for 18 days, at 6 per cent.

SOLUTION. 945
18 ÷ 6 = 3
$2.835, *Ans.*

Or, 945
18
6)17010
$2.835, *Ans.*

2. What is the interest of $164.50, for 11 days, at 6 per cent.?

SOLUTION.

$$\begin{array}{r} 164.50 \\ .011 \\ \hline 6)180950 \\ \hline .30158+ \end{array} \quad \textit{Ans.}\ \$0.30+.$$

NOTE.—The number of days taken as a multiplier may be expressed as thousandths, for we thus divide one of the factors by 1000, instead of the product. Or, we may explain it thus; the interest of $1 is as many sixths of a mill as there are days; hence $\frac{.011}{6}$ is the rate per unit.

3. Find the interest of $800, for 24 days, at 6 per cent.

4. Find the interest of $1965, for 37 days, at 7 per cent.

5. What is the interest of $175.50, for 21 days, at 6 per cent.?

6. What is the interest of $500, for 41 days, at 7 per cent.?

7. Find the interest of $250, for 6 days, at 6 per cent.

8. Find the interest of $2100, for one day, at 6 per cent.

Find the interest of—

9. $375.50, for 6 days, at 6%.
10. $975.80, for 9 days, at 7%.
11. $3750, for 3 days, at 5%.
12. $742.37, for 21 da., at 6%.
13. $87.50, for 27 da., at 1%.
14. $1000, for 6 days, at 1%.
15. $240.80, for 60 days, at 6%.
16. $855.55, for 5 days, at 5%.
17. $999.99, for 9 days, at 9%.
18. $25.75, for 10 days, at 12%.
19. $445.92, for 15 days, at 7%.
20. $115.65, for 24 days, at 6%.

CASE IV.

314. To compute interest at any rate per cent. for years, months, and days, by taking aliquot parts.

RULE.—I. *Multiply the principal by the rate per cent., to find the interest for one year.*

II. *Multiply this interest by the number of years, and take aliquot parts for the months and days.*

EXAMPLES.

1. What is the interest of $1560 for 3 years, 5 months, and 10 days, at 8 per cent.?

	$1560			
	.08			
	$124.80,	Interest for	1 year.	
	3			
	$374.40,	"	"	3 years.
$\frac{1}{3}$ Interest for 1 year,	41.60,	"	"	4 months.
$\frac{1}{4}$ " 4 months,	10.40,	"	"	1 month.
$\frac{1}{3}$ " 1 month,	3.46,	"	"	10 days.
	$429.86			*Ans.* $429.86.

2. Required the interest of $1785.60, for 9 months and 13 days, at 5 per cent.

3. Find the interest of $1000, for 2 years, 8 months, and 15 days, at 7 per cent.

4. What is the interest of $3450, from Nov. 10th, 1869, to Dec. 15th, 1871, at $7\frac{3}{10}$ per cent.?

5. What is the interest of $2700, for 1 year, 11 months, and 6 days, at 7 per cent.?

6. Required the amount of $1350, from Jan. 12th, 1870, to Sept. 19th, 1871, at 6 per cent.

GENERAL PRACTICAL RULE.

315. When the rate is 6 per cent. per annum, the interest of $1,

For 12 months	= $0.06	and of any sum,		$\frac{6}{100}$	of the principal.
" 2 " ($\frac{1}{6}$ of 12)	.01,	"	"	$\frac{1}{100}$	"
" 1 " ($\frac{1}{2}$ of 2)	.005,	"	"	$\frac{1}{2}$ of $\frac{1}{100}$	"
" 6 days ($\frac{1}{5}$ of 30)	.001,	"	"	$\frac{1}{1000}$	"
" 1 day ($\frac{1}{6}$ of 6)	.000$\frac{1}{6}$	"	"	$\frac{1}{6}$ of $\frac{1}{1000}$	"

Hence,

316. To find the interest of any sum of money at 6 per cent.

For 2 MONTHS,.............*point off two figures from the right of the dollars for decimals.*

For 1 MONTH,..............*point off two figures, and divide by 2.*

For ANY NUMBER OF MONTHS, *point off two figures, and multiply by half the number of months; or multiply by the number of months, and divide by 2.*

For 15 DAYS,...............*point off two figures, and divide by 4.*

For 6 DAYS,....... ....*point off three figures.*

For 3 DAYS,................*point off three figures, and divide by 2.*

For 1 DAY,.................*point off three figures, and divide by 6.*

For ANY NUMBER OF DAYS,..*point off three figures, and multiply by one sixth of the number of days, or multiply by the number of days, and divide by 6.*

The interest of $1, or the rate per unit, at 6 per cent., may be found, on the same principle, and the interest on any sum of money computed by what is known as the

INSPECTION RULE.

RULE.—*Take one half the number of months for the* CENTS, *or* HUNDREDTHS, *and one sixth of the number of days for the* MILLS, *or* THOUSANDTHS. *Multiply the principal by the decimal so found, and the product will be the interest.*

An odd month will give 5 mills, or .005.

EXAMPLES.

1. Required the interest of $2460, for 2 months, at 6 per cent.

SOLUTION. Point off two figures. *Ans.* $24.60.

2. Required the interest of $1248.60, for 1 month, at 6 per cent.

SOLUTION. Point off two figures, and divide by 2.

2)$12.486
$6.243, *Ans.*

3. Find the interest of $1368, for 6 days, at 6 per cent.

SOLUTION. Point off three figures. *Ans.* $1.368.

4. Find the interest of $194, for 3 days, at 6 per cent.

SOLUTION. Point off three figures, and divide by 2.

2)$.194
$0.97, *Ans.*

5. Find the interest of $650, for 63 days, at 6 per cent.; also for 33 days.

SOLUTION, 1st.

$6.50
.325
$6.825, *Ans.*

2d.

2)$6.50
3.25
.325
$3.575, *Ans.*

6. What is the interest of $960, for 10 months and 18 days, at 6 per cent.?

$9.60
5
48.00
2.88 = .960 × 3
$50.88, *Ans.*

Or, by the Inspection Rule.

$960
.053
2880
4800
$50.880, *Ans.*

7. What is the interest of $427.50, for 3 days, at 6 per cent.?

8. Required the interest on a promissory note of $350, for 60 days, and 3 days' grace, at 7 per cent. *Ans.* $4.29.

9. Required the interest of $400, for 18 months and 6 days, at 8 per cent.

10. Find the interest of $875, at 5%, for 10 months.

11. Find the interest of $356.40, for 1 year, 4 months, and 12 days, at 6 per cent.

12. Find the interest of $962, for 37 days, at 7 per cent.

13. Find the amount of $1000, for 2 years and 21 days, at 6 per cent.

14. Find the amount of $150 from January 27 to July 16, at 7 per cent.

15. What is the interest of $1542, for 1 year and 3 months, at 8 per cent.?

16. What is the interest of $685, for 63 days, at 7 per cent.?

17. What is the interest of $250, for 90 days, at 6 per cent.?

18. Required the interest on a promissory note for $76, payable 30 days after date, with interest, at 7 per cent.

19. Find the interest of $643.50, for 15 days, at 6 per cent.

20. Find the interest of $198.25, for 2 months, at 6 per cent.

21. Find the interest, at 7%, of $130, for 10 days.

22. Find the amount of $10,000, at 5% interest, from March 16, 1871, to October 30, 1871.

23. Find the amount of $375, for 30 days, at 6 per cent.

24. Find the interest at 6%, of $136.50, for 45 days.

25. What is the interest of $600, for 70 days, at 6 per cent.?

26. What is the interest of $500, at 6%, for 29 days?

27. What is the interest of $100,000, for 1 day, at 6 per cent.?

METHOD BY CANCELLATION.

317. The interest of any sum of money, for any number of years, is the product of the *Principal*, *Time*, and *Rate*.

Months are 12ths of a year, and days, according to the usual mode of reckoning interest, are 360ths of a year.

Therefore, if the time is expressed in months, the interest of any sum of money for the given time will be the *product of the Principal, Time,* and *Rate, divided by* 12; and if in days, it will be the *product of the Principal, Time,* and *Rate, divided by* 360.

Hence, to compute interest for any given time, at any given rate, the operation may be expressed, and the work abridged by cancellation.

RULE.—*Draw a vertical line, and place the Principal, Time, and Rate, on the right. If the time is in months, place* 12 *on the left; if in days,* 360. *Cancel, multiply, and divide, and the result will be the interest.*

NOTE.—When the time is given in months and days, reduce the days to *tenths* of a month, by dividing by 3, and annex the decimal to the number of months; or reduce the months to days.

EXAMPLES.

1. Find the interest of $850, for 6 months, at 7 per cent.; and for 15 months, at 8 per cent.

SOLUTION, 7%.

$$\begin{array}{r|l} & \$\cancel{850}\ \ 425 \\ \cancel{12} & \quad\cancel{6} \\ & \ .07 \\ \hline & \$29.75,\ \textit{Ans.} \end{array}$$

SOLUTION, 8%.

$$\begin{array}{r|l} & \$850 \\ \cancel{12} & \quad\cancel{15}\ \ 5 \\ & \ .0\cancel{8}\ \ 2 \\ \hline & \$85.00,\ \textit{Ans.} \end{array}$$

2. What is the interest of $175, for 48 days, at 7 per cent.? Of $364.50, for 19 days, at 6 per cent.?

SOLUTION OF FIRST.

$$\begin{array}{r|l} \cancel{360} & \$\cancel{175}\ \ 35 \\ \cancel{15} & \quad\cancel{48}\ \ 2 \\ 3 & \ .07 \\ \hline 3 & 4.90 \\ & 1.63\frac{1}{3},\ \textit{Ans.} \end{array}$$

SOLUTION OF SECOND.

$$\begin{array}{r|l} \cancel{360} & \$3\ \cancel{4.50}\ \ 6.075 \\ & \quad 19 \\ & \quad \cancel{6} \\ & \ .0 \\ \hline & \$1.15425,\ \textit{Ans.} \end{array}$$

3. Find the interest of $436, for 9 months and 18 days, at 7 per cent. Of $290 for 7 months and 14 days, at 6 per cent.

FIRST.

$$\begin{array}{r|l} & \$436 \\ \cancel{12} & \ \cancel{9.6}\quad .8 \\ & \ .07 \\ \hline & \$24.416,\ \textit{Ans.} \end{array}$$

SECOND.

$$\begin{array}{r|l} & \$\cancel{290}\ \ 145 \\ \cancel{12} & \quad 7.4\frac{2}{3} \\ & \ .0\cancel{6} \\ \hline & \$10.826\frac{2}{3},\ \textit{Ans.} \end{array}$$

Examples for Practice.

1. At 7 per cent., what is the interest of $956.50, for 1 year and 3 months?

2. At 5 per cent., what is the interest of $462.50, for 3 months and 18 days?

3. At 10 per cent., what is the interest of $75, for 17 months?

4. Required the interest of $250 from July 1, 1871, to August 1, 1872, at 9 per cent.

5. What is the amount, July 8, 1871, of a promissory note dated September 13, 1870, payable on demand, with interest, at 7 per cent.?

GOVERNMENT RULE,

Or, Accurate Interest.

318. On United States Bonds, interest is computed upon the basis of 365 days to the year; that is, for any number of days less than a year, such a part of one year's interest is taken as the number of days is of 365. For example, the interest on any sum for 30 days is $\frac{30}{365}$ of the interest for one year. This is *Accurate Interest*, and is $\frac{1}{73}$ less than that obtained by the mercantile method. Hence,

319. To compute Accurate Interest we have the following

Rule.—*Find the interest for years by the common method, but for any fraction of a year, take such part of one year's interest as the number of days is* 365. Or,

Compute the interest by the common method, and subtract $\frac{1}{73}$ *part of itself.*

Note.—At $3\frac{65}{100}$ per cent., according to this rule, the interest on $100 for day is 1 cent. Twice $3\frac{65}{100}$% is $7\frac{3}{10}$%, the rate which the United States -thirty Treasury Notes bore from which they took their name. This nts a day on $100, and is very easily reckoned.

EXAMPLES.

1. Required the interest on five U. S. 5–20 Bonds of $1000 each, from Nov. 1, 1870, to March 18, 1871.

SOLUTION. From Nov. 1 to March 1, is 137 days.

$5000 × .06 = $300, one year's interest.
$300 × $\frac{137}{365}$ = $112.60+. *Ans.* $112.60.

2. Required the interest on $10000 U. S. 5–20 Bonds, from May 1 to Sept. 10.

3. What is the interest on three U. S. 10–40 Bonds, from Mar. 1 to Aug. 16, at 5 per cent.?

4. What is the interest on a $500 U. S. 10–40 Bond, from March 1 to April 15?

INTEREST AT $7\frac{3}{10}$ PER CENT. FOR DAYS.

320. To compute accurate interest on Notes, Bonds, etc., at the rate of $7\frac{3}{10}$%, for any number of days.

At $7\frac{3}{10}$ per cent. for 365 days, the interest of $1 for 1 day is $\frac{1}{365}$ of .073 = .0002, or $\frac{2}{10}$ of a mill; or, twice the number of days equals the number of tenths of a mill. Hence the

RULE.—*Multiply the principal by the number of days, and the product by* .0002. Or,

Multiply the principal, in dollars, by twice the number of days, and point off four figures for decimals.

EXAMPLES.

1. Required the interest on a United States 7–30 Treasury Note for $1000, for 38 days.

$1000 × 38 × .0002 = $7.60, *Ans.*

2. What is the interest on a mortgage for $560, for 1 year and 76 days, at $7\frac{3}{10}$ per cent. per annum?

INTEREST ON ENGLISH MONEY.

In England, interest is calculated upon the basis of 365 days to the year, or according to the rule already given for computing *accurate interest.* But months in any given time are usually reckoned as 12ths of a year. For days, such a part of one year's interest is taken as the number is of 365.

The legal rate is 5 per cent. Hence,

321. To compute interest on English money we have the following

RULE.—I. *Reduce the shillings, pence, and farthings, to the decimal of a pound.*

II. FOR YEARS:—*Multiply the principal by the rate, and the product by the number of years.*

III. FOR MONTHS:—*Take such fractional part of one year's interest as the number of months is of* 12, *or apply any of the foregoing rules.*

IV. FOR DAYS:—*Take such fractional part of one year's interest as the number expressing the time is of* 365; *or, when the rate is* 5 *per cent., multiply the principal by the number of days, divide the product by* 73, *and point off two decimals in the quotient.*

If the rate is 5%, we may take $\frac{1}{20}$ of the principal for 1 year's interest, or 1*s.* for every pound, 3*d.* for every 5*s.*, and 1 far. for every 5*d.*; for, $\frac{1}{20}$ of £1 = 1*s.*; $\frac{1}{20}$ of 1*s.* = $\frac{12}{20}$ = $\frac{3}{5}$*d.*; and $\frac{1}{20}$ of 1*d.* = $\frac{4}{20}$ = $\frac{1}{5}$ far.

Hence, at this rate, we may readily find the interest for one year, by inspection.

And, to find the interest on any number of *pounds*, for any number of *years*, multiply the principal by the number of years, and the product will be the interest in *shillings;* and to find it for any number of *months*, multiply by the number of months; the product will be the interest in *pence.*

EXAMPLES.

1. What is the interest of £50 10*s.* 6*d.*, for 3 years, at 5 per cent.?

SOLUTION. £50 10*s.* 6*d.* = £50.525
£50.525 × .15 = £7.57875

And, reducing the decimal, we have £7 11*s.* 6¾*d.*+, *Ans.*

By inspection—

The interest of £50, for 1 yr., is 50*s.* = £2 10*s.*
" " " 10*s.* " " " " ⅗ of 10 = 6*d.*
" " " 6*d.* " " " " ⅕ of 6 = 1⅕*f.*

Interest for 1 year, £2 10*s.* 6¼*d.*+
3

Interest for 3 years, £7 11*s.* 6¾*d.*+

Or, we may multiply by the rate and time, without reduction, and divide by 100.

£50 10*s.* 6*d.*
5

250 50 30
3

£7.50 150 90
20

s. 11.50
12

d. 6.90 *Ans.*, as before, £7 11*s.* 6¾*d.* +.

This is unquestionably the simplest of the three methods.

2. Find the interest of £150, for 2 years and 6 months, at 5 per cent.

3. Find the interest of £60 8*s.* 5*d.*, for 3 years and 4 months, at 5 per cent.

4. Find the interest of £250 10*s.*, for 33 days, at 5 per cent.

5. What is the amount of £500 8*s.* 6*d.*, for 4 years and 6 months, at 5 per cent.?

6. What is the amount of £350 10*s.* 3*d.*, for 6 years, 9 months, and 12 days, at 5 per cent.?

PROBLEMS IN INTEREST.

PROBLEM I.

322. To find the Principal, the Interest, Time, and Rate being given.

EXAMPLE 1. What sum of money, at 6% interest, will gain $480 in 5 years?

OPERATION.

$.06 \times 5 = .30)480.00$

1600, *Ans.* $1600.

ANALYSIS.—Since the interest of $1 for 5 years, at 6%, is 30 cents, it will take as many dollars to gain $480 as .30 is contained times in 480 = 1600. Hence the

RULE.—*Divide the given interest by the interest of* $1 *for the given time, at the given rate.*

2. What principal, at 6%, is sufficient to gain $24.75 in 18 months? *Ans.* $275.

3. What sum of money put at interest at 7%, for 3 years and 6 months, will gain $61.81?

4. Required the principal that will produce $75.80, in 9 months and 21 days, at 8 per cent.

5. What sum of money, at 5 per cent. simple interest, will yield $275.40 in 3 years and 4 months?

PROBLEM II.

323. To find the Principal, the Amount, Time, and Rate being given.

EXAMPLE 1.—What sum of money, at 6% interest, will amount to $408.75, in 18 months?

OPERATION.

1.09)408.75(375
327
817
763
545
545

Ans. $375.

ANALYSIS.—Since $1 will, in the given time, amount to $1.09, it is evident that $408.75 is 109 per cent. of the required principal; and 408.75 ÷ 1.09 = 375. Hence the

RULE.—*Divide the given amount by the amount of* $1 *for the given time, at the given rate.*

2. What principal will, in 3 years, 8 months, and 6 days, at 6%, amount to $4541.25?

3. What sum invested at 7%, for 2 years and 9 months, will amount to $3000?

4. What sum invested January 1, 1871, at 5%, will amount to $5000 January 1, 1875?

PROBLEM III.

324. To find the Time, the Principal, Interest, and Rate being given.

EXAMPLE 1.—How long will it take $400 to gain $30.40 at 6% per annum?

OPERATION.

400 × .06 = 24)30.40(1.26⅔
24
64
48
160
144
16

Ans. 1.26⅔ years = 1 yr. 3 mos. 6 d.

Analysis.—$400, at 6%, gains $24 per annum. Therefore, it will take it as many years to gain $30.40 as 24 is contained times in 30.40 = 1.26⅔. Hence the

Rule.—*Divide the given interest by the interest of the principal for one year, at the given rate; the quotient will be the time in years.*

2. In what time will $3750 amount to $4541.25, at 6% per annum?

3. In what time will $10000 produce $1600, at 6% per annum?

4. How long will it take any sum of money to double itself, at 5% per annum? At 6%? At 7%? At 8%?

Note.—A question of this kind is solved by dividing 100 by the rate per cent.; for the principal must gain 100 per cent. of itself, which will require as many years as the number of per cent. gained per annum is contained times in 100. For example, at 5%, it will take 100 ÷ 5 = 20 years; at 6%, 100 ÷ 6 = 16⅔ years.

PROBLEM IV.

325. **To find the Rate,** the Principal, Time, and Interest being given.

Example 1.—If I pay $52.50 for the use of $750 for 14 months, what is the rate per cent.?

OPERATION.

```
     750
     .07
  ------
  6)52.50
  ------
   8.75)52.50(6
        52.50      Ans. 6%.
        -----
```

Analysis.—The interest of $750 for 14 months, at 1%, is $8.75. Therefore, the rate must be as many times 1, as 8.75 is contained times in 52.50 = 6. Hence the

Rule.—*Divide the given interest by the interest of the principal for the given time, at* 1 *per cent.; the quotient will be the required rate.*

2. If I receive $36.40 for the use of $400 for six months and 18 days, what is the rate per annum?

COMPOUND INTEREST.

326. Compound Interest is reckoned upon the principal and interest added together annually, semi-annually, quarterly, etc.

EXAMPLES.

1. What is the compound interest of $1000 for 3 years, at 6%, payable annually?

$1000.............Principal.
60..............Interest for the first year.

1060..............Amount, principal for the second year.
63.60............Interest for the second year.

1123.60...........Amount, principal for the third year.
67.416..........Interest for the third year.

1191.016..........Amount at the end of three years.
1000. ....Deduct the first principal.

$191.016 ....Compound Interest for 3 years, *Ans.*

2. What will be the amount of $1000 in 2 years and 3 months, at 4% compound interest, payable semi-annually?

$1000 ....Principal.
40............ Interest for the first 6 months.

1040................Amount.
41.60...........Interest for the second 6 months.

1081.60............Amount.
43.264...........Interest for the third 6 months.

1124.864...........Amount.
44.99456.........Interest for the fourth 6 months.

1169.85856.........Amount.
23.3971712.......Interest for 3 months.

1193.2557312.......Amount at the end of 2 years and 3 months.

327. Instead of making these laborious calculations, Tables, like the following, are generally used in computing Compound Interest.

TABLE,

Showing the amount of $1, *or* £1, *at* 2½, 3, 3½, 4, 5, 6, 7, *and* 8 *per cent., compound interest, for any number of years, from* 1 *to* 40.

Years	2½ per ct.	3 per cent.	3½ per ct.	4 per cent.	5 per cent.	6 per cent.	7 per ceut.	8 per ceut.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000	1.070000	1.080000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600	1.144900	1.166400
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477	1.310796	1.360489
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226	4.402552	1.469328
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630	1.605782	1.713824
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479	1.838459	1.999005
10	1.280085	1.343916	1.410599	1.480244	1.628885	1.790848	1.967151	2.158925
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299	2.104852	2.331639
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197	2.252192	2.518170
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928	2.409845	2.719624
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904	2.578534	2.937194
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558	2.759032	3.172169
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352	2.952164	3.425943
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773	3.158815	3.700018
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339	3.379932	3.996020
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600	3.616528	4.315701
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136	3.869685	4.660957
21	1.679582	1.860295	2.059431	2.278768	2.785963	3.399564	4.140562	5.033834
22	1.721571	1.916103	2.131512	2.369919	2.925261	3.603537	4.430402	5.436540
23	1.764611	1.973587	2.206114	2.461716	3.071524	3.819750	4.740530	5.871464
24	1.808726	2.032794	2.283328	2.563304	3.225100	4.048935	5.072367	6.341181
25	1.853944	2.093778	2.363245	2.665836	3.386355	4.291871	5.427433	6.848475
26	1.900293	2.156591	2.445959	2.772470	3.555673	4.549383	5.807353	7.396353
27	1.947800	2.221289	2.531567	2.883369	3.733456	4.822346	6.213868	7.988062
28	1.996495	2.287928	2.620172	2.998703	3.920129	5.111687	6.648838	8.627106
29	2.046407	2.356566	2.711878	3.118651	4.116136	5.418388	7.114257	9.317275
30	2.097568	2.427262	2.806794	3.243398	4.321942	5.743491	7.612255	10.062657
31	2.150007	2.500080	2.905031	3.373133	4.538040	6.088101	8.145113	10.867669
32	2.203757	2.575083	3.006708	3.508059	4.764942	6.453387	8.715271	11.737083
33	2.258851	2.652335	3.111942	3.648381	5.003189	6.840590	9.325340	12.676050
34	3.315322	2.731905	3.220860	3.794316	5.253348	7.251025	9.978114	13.690134
35	2.373205	2.813862	3.333590	3.946089	5.516015	7.686087	10.676582	14.785344
36	2.432535	2.898278	3.450266	4.103933	5.791816	8.147252	11.423942	15.968172
37	2.493349	2.985227	3.571025	4.268090	6.081407	8.636087	12.223618	17.245626
38	2.555682	3.074783	3.696011	4.438813	6.385477	9.154252	13.079271	18.625276
39	2.619574	3.167027	3.825372	4.616366	6.704751	9.703508	13.994820	20.115298
40	2.685064	3.262038	3.959260	4.801021	7.039989	10.285718	14.974458	21.724522

328. To find the amount of any principal for any number of years by the use of the Table.

Rule.—*Multiply the amount of* $1, *or* £1, *for the given time, at the given rate, by the principal.*

Notes.—1. To find the interest, subtract the principal from the amount.

2. The interest for additional months and days may be reckoned on the amount of $1, found in the table for the years, *before* multiplying by the principal, or on the amount of the principal.

3. When the interest is to be compounded semi-annually, quarterly, etc., take the rate for one interval, and find the amount for the number of intervals, as if they were so many years. For example, the amount of $1 at 6% compound interest per annum, payable semi-annually, for three years, is the same as the amount of $1, at 3% per annum, compounded annually for six years.

Examples.

1. What is the amount of $100 for 10 years, at 6% compound interest?

2. What is the compound interest of $436, for 5 years and 6 months, at 5 per cent.?

3. What is the amount of $800, for 17 years and 10 months, at 7% compound interest?

ANNUAL INTEREST.

329. Annual Interest is simple interest on the principal and on the interest overdue on promissory notes, or other contracts containing the words, "with interest payable annually," or, "with annual interest."

Interest upon the interest on such contracts is allowed by the courts in some of the States, in the nature of damages for the non-payment of the interest when it falls due; otherwise it cannot be considered legal.

Examples.

(1) **$1000.** New York, March 1, 1871.

Three years after date, for value received, I promise to pay John Walton, or order, One Thousand Dollars, with interest at seven per cent., payable annually.

Henry S. Lane.

Suppose nothing to be paid on this note till its maturity, what will be the amount then due?

SOLUTION.

Principal		$1000
Interest on same, for 3 years		210
Amount of Principal and Interest		$1210
Interest on $70 for 2 years	$9.80	
Interest on $70 for 1 year	4.90	14.70
Amount at maturity		$1224.70

Note.—The interest due at the end of the first year, $70, draws interest for two years, and that due at the end of the second year, for one year. But this interest is equal to the interest of $70 for 2 + 1 = 3 years. Therefore, the work may be shortened by computing the interest on the annual interest, for the sum of the years, in one operation, as shown in the solution of the following question.

2. What will $3000 amount to in 10 years and 6 months, annual interest, at 6 per cent.?

SOLUTION.

Principal	$3000.00	
$3000 × .06 × 10½ =	1890.00,	interest for 10½ years.
$180 (int. for 1 yr.) × .06 × 50 =	540.00,	interest on 1 year's int. for 9½ + 8½ + 7½ + 6½ + 5½ + 4½ + 3½ + 2½ + 1½ + ½ = 50 years.
Amount for 10½ years	$5430.00, *Ans.*	

3. What is the difference between annual interest and compound interest on $1500, for 4 years, at 6 per cent.?

Ans. $1.31+.

Note.—Annual and compound interest are the same up to the end of the second year.

4. Find the annual interest and the amount of $750, for 8 years, at 6% per annum.

5. Required the annual interest of $1500, for 7 years, at 6 per cent.

6. Find the amount of $475, at 7% annual interest, for 8½ years.

PARTIAL PAYMENTS.

330. When **Partial Payments** have been made on Notes, Bonds, and other obligations, the following rule for computing the interest, based on a decision of Chancellor Kent (1 John. C. C., 13), has been adopted by the United States Supreme Court, and by the courts in most of the States, and is, therefore, known as the

UNITED STATES RULE.

I. *Compute the interest on the principal to the time of the first payment, and if this payment exceed the interest then due, add the interest to the principal, and from the sum subtract the payment; the remainder will form a new principal, with which proceed as before.*

II. *But if the payment be less than the interest, compute the interest on the principal to the time when the sum of the payments shall first equal or exceed the interest due; add the interest to the principal, and from the sum subtract the sum of the payments, and treat the remainder as a new principal.*

EXAMPLES.

(1) **$4000.** NEW YORK, June 1, 1869.

Two years after date, I promise to pay William Claflin, or order, Four Thousand Dollars, for value received, with interest.

RICHARD PAYWELL.

On this note were indorsed the following payments:

September 15, 1869, received ($450) Four Hundred and Fifty Dollars.
December 15, 1869, received ($50) Fifty Dollars.
March 3, 1870, received ($500) Five Hundred Dollars.
January 1, 1871, received ($1000) One Thousand Dollars.

What remained due June 4, 1871?

OPERATION.

Principal on interest from June 1, 1869	$4000
Interest to September 15, 1869, 3 mos. 14 d.	80.89
Amount	$4080.89
Less first payment	450.00
Remainder for a new principal	$3630.89
The Interest from September 15 to December 15, 1869, is $63.44, which exceeds the payment.	
Interest from September 15, 1869, to March 3, 1870, 5 mos. 16 d.	117.20
Amount	$3748.09
Less the sum of the second and third payments	550.00
Remainder for a new principal	$3198.09
Interest from March 1, 1870, to January 1, 1871	186.55
Amount	$3384.64
Less payment January 1, 1870	1000.00
	$2384.64
Interest from January 1 to June 4, 1870	70.94
Balance due June 4, 1871	$2455.58

(2) **$1500.** PHILADELPHIA, January 1, 1870.

One year after date, we promise to pay Stevens & White, or order, Fifteen Hundred Dollars, value received, without defalcation.

J. D. COLEMAN & Co.

The following payments were made on this note:—March 16, 1870, $100; June 13, 1870, $400; September 1, 1870, $200.

What was due January 1, 1871, interest at 6 per cent.?

(3) **$3500.** BOSTON, March 15, 1870.

For value received, we jointly and severally promise to pay Wm. Smith, or order, Three Thousand Five Hundred Dollars, on demand, with interest.

JAMES MANNING,
HENRY SCHOFIELD.

Indorsed as follows:—June 1, 1870, received $800; September 1, 1870, $100; Jan. 1, 1871, $1560; March 1, 1871, $300.

What was due May 15, 1871, interest at 6 per cent.?

(4) **$1340.** CINCINNATI, July 15, 1870.

Eighteen months after date, I promise to pay Walter Cummings, or order, Thirteen Hundred and Forty Dollars, value received, with interest at six per cent. per annum.

W. H. BONNER.

Indorsed as follows:—Oct. 15th, 1870, received $30; Jan. 1st, 1871, $440; Mar. 18th, 1871, $150; Oct. 1st, 1871, $95.

What was due on this note at its maturity?

331. In Connecticut, the following rule prevails, having been adopted by the Supreme Court, March Term, 1784. (See Kirby's Reports, 49, note.)

CONNECTICUT RULE.

I. *Compute the interest to the time of the first payment; if that be one year, or more, from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after-payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, till all the payments are absorbed; provided the time between one payment and another be one year or more.*

II. *But if any payments be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation, for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid up to the end of the year, add it to the sum paid, and deduct that sum from the principal and interest added as above.*

III. *If any payments be made of less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.*

EXAMPLE.

1. A Bond, bearing date, New Haven, January 1st, 1868, was executed and delivered to Hiram Smith by John Comstock, for

the sum of $5000, payable four years after date, with interest at 6 per cent. per annum.

On this bond the following payments were made:—Feb. 1st, 1869, $1000; July 1st, 1869, $500; Mar. 1st, 1870, $100; July 16th, 1870, $800.

Required the amount due Jan. 1st, 1872.

OPERATION.

Principal		$5000
Interest from Jan. 1st, 1868, to Feb. 1st, 1869		325
Amount		$5325
Less the first payment		1000
Balance due		$4325
Interest for one year		259.50
Amount due Feb. 1st, 1870		$4584.50
Less amount of second payment from July 1st, 1869, to Feb. 1st, 1870		517.50
Balance due		$4067.00
Interest for one year		244.02
Amount due Feb. 1st, 1871		$4311.02
Less amount of fourth payment from July 16th, 1870, to Feb. 1st, 1871	$826	
And third payment, without interest	100	
		926.
Balance due		$3385.02
Interest to Jan. 1st, 1872		186.18
Balance due Jan. 1st, 1872		$3571.20

NOTE.—The former "Vermont Rule" was changed by an Act of the Legislature, passed in 1866, so that it is now substantially the same as the United States Rule. Payments must be applied; "first, to liquidate the interest that has accrued at the time of such payment; and secondly, to the extinguishment of the principal." The same act allows annual interest, and directs how it shall be computed on partial payments.

332. In New Hampshire, the courts allow interest on *annual interest*, in the nature of *damages* for its detention, from the time it becomes due, till paid. See 1st N. H. Reports, 179.

NEW HAMPSHIRE RULE.

I. *Find the amount of the principal for one year, and deduct from it the amount of each payment for that year, from the time it was made up to the end of the year; the remainder will be a new principal, with which proceed as before.*

II. *If the settlement occur less than a year from the last annual term of interest, make the last term of interest a part of a year accordingly.*

333. Merchants, in computing interest on accounts which are required to be settled annually, semi-annually, quarterly, etc., on which partial payments have been made, generally adopt the following rule, which is, therefore, called the

MERCHANTS' RULE.

I. *Find the amount of the principal from the time interest commenced to the time of settlement.*

II. *Find the amount of each payment from the time it was made to the time of settlement.*

III. *Subtract the sum of the amounts of the payments from the amount of the principal; the remainder will be the sum due.*

NOTE.—Merchants often compute interest by the above rule, on notes running for one year only, or less, when partial payments have been made during the year.

EXAMPLE.

1. What was due on the following Note at the time of settlement, January 1st., 1871?

$560. RICHMOND, VA., Jan. 1, 1870.

For value received, I promise to pay J. Norcross & Co., or order, Five Hundred and Sixty Dollars, on demand, with interest at 6 per cent. per annum.

C. D. HENDRICKS.

Payments indorsed as follows:—Feb. 15th, received $50; June 1st. $100; Sept. 15th, $100.

SOLUTION.

Principal		$560
Interest on the same for one year		33.60
Amount of principal		$593.60
First payment, Feb. 15th, 1870	$50	
Interest on same to Jan. 1st, 1871	2.61	
Second payment, June 1st, 1870	100.00	
Interest on same to Jan. 1st, 1871	3.50	
Third payment, Sept. 15th, 1870	100.00	
Interest on same to Jan. 1st, 1871	1.77	
		257.88
Balance due at date of settlement		$335.72

PRESENT WORTH AND DISCOUNT.

334. **Discount** is an allowance, or deduction, made for the payment of a debt before it becomes due.

335. The **Present Worth** of a debt, payable at a future time, without interest, is such a sum as, being put at interest, at the specified rate, will amount to the given debt when it becomes due.

336. **True Discount** is the difference between the present worth and the amount of the debt.

337. **Commercial,** or **Bank Discount,** is simple interest on the principal for the given time, taken in advance.

338. True Discount is the interest of the *present worth* for the given time, and Bank Discount is the interest computed on the *principal.*

EXAMPLES.

1. What is the present worth of a debt of $1650, due 8 months hence, without interest, money being worth 6 per cent.?

What is the true discount?

SOLUTION.

$1650 ÷ 1.04 = $1586.53+, present worth.
1650 − 1586.53 = $63.47, true discount.

ANALYSIS.—Since $1 will amount to $1.04 in 8 months, $1650 is 104 per cent. of the sum which will amount to $1650 at the end of that time. Therefore, $1650 ÷ 1.04 = the present worth. The difference between the present worth and the principal is the discount. Hence,

To find the present worth and the discount of any sum of money due at a future time.

RULE.—I. *Divide the debt by the amount of* $1 *for the given time, at the given rate; the quotient will be the present worth.*

II. *Subtract the present worth from the given sum; the remainder will be the true discount.*

NOTE.—The discount may be found directly, by dividing the interest of the principal by the amount of $1 for the given time, at the given rate.

2. What is the true discount of $475.50, due 2 years and 9 months hence, at 7 per cent.? *Ans.* $76.76.

3. Required the present worth of a debt of $846.50, due in 3 years, money being worth 5 per cent.

4. If a bank discounts a note, for $500, due in 60 days, and deducts simple interest on the amount, at 6%, how much more will it receive than the true discount? *Ans.* $0.049+.

5. What is the present worth, July 6, 1871, of a debt of $1700, due in 10 months from May 1, 1871, considering money worth 6 per cent. per annum?

6. Bought a bill of goods amounting to $1450, on 6 months' credit. If money is worth ½% a month, how much will be gained by settling the bill immediately, at 5% off for cash?

7. Find the discount, also the present worth of $4362.50, due in 2 years and 4 months, at 7 per cent.

BANK DISCOUNT.

339. The rate of interest which may be taken by banks is, unless otherwise allowed in their charters, the legal rate of the State in which they are located, or transact their business.

340. Banks usually loan money upon such securities as are approved by the Directors, as mortgages of real estate, commercial paper, with one or more responsible indorsers, or collateral securities.

341. In reckoning discount on notes, the legal three days of grace are always added to the time expressed therein, and in some States, as Pennsylvania, Delaware, Maryland, Missouri, Louisiana, the District of Columbia, and perhaps others, by established usage, both the day of discount and the day of payment are included.

The argument in support of this custom is, that the bank directors usually meet on the morning of Discount Day, and that the persons for whom discounts are made, can receive their money as soon as the bank opens, whereas they are not bound to pay till the last moment of the last day of grace, and, consequently, they have the use of the money for both days inclusive.

There seems to be no express sanction for this custom in any of the above States; but the United States Supreme Court has held it to be legal in the District of Columbia, upon proof of the long-established usage of the banks of Washington and Georgetown, to demand payment of notes due them, on the fourth day after the time limited therein.

342. It is customary to compute the discount on notes running for a few months, by days, although the time is expressed in months on their face. Thus, in discounting on July 10, a note dated July 1, payable two months after date, it will be considered as having 56 days to run, excluding the day of discount, or 57 days, including that day.

343. **Bank Discount** is simple interest on the principal. Therefore, in computing it, *use any of the rules already given* *for computing simple interest. Deduct the amount from the principal, and the remainder will be the proceeds.*

EXAMPLES.

1. Required the discount and the proceeds of the following note, March 12th, 1871, according to the Pennsylvania rule, at 6 per cent.

$1000. PHILADELPHIA, March 12, 1871.

Sixty days after date, I promise to pay to the order of H. B. Claflin & Co., One Thousand Dollars, at the Farmers and Mechanics' National Bank value received, without defalcation.

MARTIN STEVENS

SOLUTION.	$10.00	
	.50	$1000
	.17	10.67
	$10.67, Discount.	$989.33, Proceeds.

2. Discounted at the Chemical Bank, New York, June 8th, 1871, the following note, at 7 per cent. Required the proceeds.

$500. NEW YORK, May 25, 1871.

Sixty days after date, we promise to pay Samuel Wilson, or order, Five Hundred Dollars, at the Chemical Bank, New York, for value received.

HUBBARD & ENGLISH.

SOLUTION.

Time of note,	63 days.	.500	$500.
Time elapsed,	14 "	8⅙	4.76
Time to maturity,	49 "	4.000	$495.24, Proceeds.
		.082	
		6)4.082	
		.680	
	Discount,	$4.762	

3. What is the difference between true and bank discount on $1000, for 63 days, at 6 per cent.?

4. For what amount must a note at 60 days be drawn, so that the proceeds, when it is discounted at a Philadelphia bank, will be $2500?

Note.—Find the proceeds of $1, by which divide $2500.

344. In discounting a note drawing interest, the discount is computed on the amount of principal and interest due at its maturity.

5. What is the bank discount, at 6%, of a note for $450, dated March 23, payable ninety days after date, with interest at 7 per cent., discounted April 25? What are the proceeds?

Ans. Discount, $4.58.
Proceeds, $453.56.

6. Find the discount and proceeds of the following note, discounted at Washington, March 18, 1871.

$1500. WASHINGTON, D. C., March 1, 1871.

Sixty days after date, I promise to pay James Sibley, or order, Fifteen Hundred Dollars, value received, with interest.

AARON DAVIS.

7. Find the proceeds of the following note, discounted at Chicago, June 7, 1871.

$1256.50. CHICAGO, April 1, 1871.

Ninety days after date, we jointly and severally promise to pay John Whittaker, or order, Twelve Hundred and Fifty-six, $\frac{50}{100}$ Dollars, for value received.

WILLIAM JONES
HENRY THOMAS.

8. What amount will be passed to the credit of George Barnes, who gets the following note discounted at the First National Bank, Detroit, Mich., July 7, 1871?

$413.75. CLEVELAND, Ohio, June 27, 1871.

Sixty days after date, I promise to pay to the order of George Barnes, Four Hundred and Thirteen, $\frac{75}{100}$ Dollars, at the Third National Bank, Cleveland, for value received.

BENJAMIN HOWARD.

COMMISSION AND BROKERAGE.

345. **Commission** is an allowance made to an agent, factor, or commission merchant, for buying and selling merchandise, or other property, investing money, collecting debts, and transacting other business. It is usually reckoned at a certain per cent. of the amount of property sold or purchased, or money involved in the transactions.

346. A **Commission Merchant** is a person engaged exclusively, or chiefly, in the business of buying and selling goods on commission. When goods are sent to him to be sold, he is called the *Consignee;* the goods, a *Consignment;* and the person sending them, the *Consignor*.

347. A **Broker**, in a general sense, is an agent employed to effect bargains and contracts, as a middle-man, or negotiator, between other persons, for a compensation called *Brokerage.* He may, or may not, have possession of the subject-matter of the negotiation, though usually he does not, and, in this respect, he differs from a commission merchant, who transacts business in his own name.

348. Stock, Money, and Exchange Brokers, generally combine commission transactions with their own business.

349. The rate of commission depends upon the nature of the business transacted. In certain branches of business, it is established by custom, or the rules of Boards of Trade. The usual commission for buying and selling merchandise is from 2½ to 5 per cent.

350. Brokerage is computed, like commission, by percentage. The rate for buying and selling stocks is fixed by the Stock Exchange, and is from ⅛ to ¼ per cent. of their *par value.*

351. Since commission, or brokerage, is percentage, all problems occurring under this head may be solved *by applying the rules for like cases in Percentage.*

EXAMPLES.

1. A commission merchant sells goods for another to the amount of $3650.75: what is his commission at 2½ per cent.?

2. A merchant in New York remits to his agent in Cincinnati $6642, for the purchase of merchandise, after deducting his commission of 2½ per cent. How much will the agent invest for his principal, and what will be his commission?

NOTE.—The agent evidently ought not to receive a commission on his commission. He is required to invest such a sum of money as will amount to $6642 when his commission is added; or, in other words, he has received $1.02½ for every dollar which he is to lay out. Therefore, the amount to be invested, will be found by dividing $6642 by 1.02½.

Ans., To be invested, $6480,
His commission, $162.

3. A commission merchant in Havana bought a quantity of sugar for a firm in New York, amounting to $49087.50: what was his commission, at 2½%?

4. Sent to my agent in Chicago $3000 to invest in wheat, after deducting his commission of 1½%: how much will he have to invest?

5. The commission of a real estate agent for selling a house amounted to $625 at 5 per cent.: for how much did he sell the house?

6. Winans & Co., Savannah, sell for Gardner & Jones, New York, a quantity of merchandise, amounting to $3842.50, and charge a commission of 2½ per cent. By instructions, they invest the proceeds in cotton for shipment to New York, after deducting a purchasing commission of 1½%: what amount do they invest?

7. My attorney collected a certain sum of money amounting to $756.25, and charged me 5% for his services: what did he charge, and how much did I receive?

8. A wool merchant in Philadelphia remitted to his correspondent in New York the proceeds of a consignment, amounting to $2453.75, less his commission of 2½%, per draft, which he purchased at the expense of the consignor, at 1½ per cent premium: what was the amount of the remittance?

9. Received and sold for John Rupp, St. Louis, 15 sacks of wool. The gross proceeds were $1433.78, and the charges were as follows: Freight, $104.03; Drayage, $1.50; Government tax on sales, ⅛%; and my Commission 5% on the sales: what were the net proceeds? *Ans.* $1254.78.

10. A broker purchased for a gentleman $10000 worth of U. S. Bonds, and charged ⅛% for his services: what was the amount of his brokerage?

11. A broker sold for a customer 100 shares of railroad stock at $85 per share, the par being $100: what was his brokerage at ¼ per cent.?

PROFIT AND LOSS.

352. Under the head of **Profit and Loss** are included computations relative to gains and losses in buying and selling merchandise and other property, and the increase and decrease of values.

353. Four cases occur, which involve an application of the principles of Percentage:

1st. *To find the amount of gain or loss,* when the cost and rate per cent. of gain or loss are given.

2d. *To find the rate per cent. of gain or loss,* when the cost and selling price are given.

3d. *To find the cost,* when the selling price and rate per cent. of gain or loss are given.

4th. *To find the selling price,* when the cost and rate per cent. of gain or loss are given.

354. The **Cost** is the base on which the percentage is reckoned.

355. The **Rate per Cent.** is the number of per cent. which the gain or loss is of the cost.

356. The **Selling Price** is the cost plus the gain, or minus the loss. It corresponds with the *Amount,* or *Difference,* as defined in Percentage.

357. The **Amount** of gain or loss is the difference between the cost and selling price.

358. Therefore, to solve any problem in Profit and Loss, *apply the proper rule, or rules, as given in Percentage.*

EXAMPLES.

1. Bought a house and lot for $12000, and sold the same for $13500: what was the gain per cent.?

2. Bought a quantity of merchandise amounting to $1000, and sold the same at 25% profit: required the proceeds.

3. I desire to sell a quantity of broadcloth, which cost me $6 per yard, so as to make 15% profit: at what price must I sell it?

4. Sold a horse for $150, and lost 25%: how much did the horse cost me?

5. A farmer bought a yoke of oxen for $150, which was 25% less than their real value, and sold them for 25% more than their real value: what was his gain per cent.? *Ans.* 66⅔%.

6. Sold flour at $10.45 per barrel, and thereby lost 5% of the cost: what was the cost per barrel?

7. What price must I ask for cloth which cost me $3.80 per yard, in order that I may deduct 5% from my asking price, and still make 25% profit?

8. A broker bought stock at 20% above par, and sold it at 20% below par: what per cent. of the cost did he lose?

Ans. 33⅓%.

9. Bought a cask of wine containing 42 gallons, at $2.50 per gallon, and 12 gallons have leaked out: at what price per gallon must I sell the remainder, to gain 4 per cent. on the first cost?

10. Bought 100 barrels of flour, at $8.10 per barrel, and sold 90 barrels for what the whole cost. If I sell the remaining 10 barrels at the same price, what will be my total gain, and my gain per cent.?

11. A merchant sold a piece of goods which he had marked 25% above cost, at 20% discount from his marked price, thinking he should still make a profit. Did he gain or lose?

12. A grocer sold raisins which cost him $4 per box, at $3.50 per box: how much was the loss per cent.?

13. Sold flour at $9 per barrel, which was 10% less than the original cost. What would have been the gain per cent. if it had been sold at $10.50?

14. A gentleman sold two house-lots at $1200 each. On one he gained 20%, and on the other he lost 20%. Did he gain or lose by the transaction, and how much?

15. A speculator bought 200 shares of railroad stock, at $96 per share. He sold 100 shares of the same at a gain of 5%, and the other 100 shares at a loss of 5%. Did he gain or lose by the operation, and how much?

16. What per cent. advance on the cost of goods must I ask, in order to deduct 10% from the asking price, and still make a profit of 10 per cent.?

17. A merchant purchased cloth at $3.50 per yard, less 5% for cash. What was the net cost per yard, and what per cent. must be added to the invoice price, to give a profit of 10 per cent.?

18. A firm, having failed in business, owes to A $1975, to B $4250, to C $1682, and to D $1140. Their assets, less expenses of settlement, amount to $2397.45. What per cent. of their indebtedness can they pay? What dividend will each creditor receive?

GOLD VALUE OF A DEPRECIATED CURRENCY.

359. To find the value of one dollar in currency, when gold is at a premium.

EXAMPLES.

1. When gold is quoted at 150, what is the value of a dollar "greenback?"

SOLUTION. $\frac{1.00}{1.50} = .66\frac{2}{3}$. Since $1.50 in currency is worth $1 in gold, the value of $1 in currency is $\frac{100}{150}$ of $1 in gold = 66⅔ cents, *Ans.* Hence the

RULE.—*Divide* $1 *by* 1 *plus the premium expressed decimally.*

NOTE.—The term *premium* is here used in the popular sense, though, scientifically speaking, the currency is at a *discount*, and not gold at a premium, for gold is the standard.

MONTHLY TABLE

OF THE HIGHEST QUOTATIONS OF GOLD,

From and including January, 1862, *the date of the suspension of specie payment, to August* 1, 1871.

DATE.	HIGHEST PRICES OF GOLD.									
	1862.	1863.	1864.	1865.	1866.	1867.	1868.	1869.	1870.	1871.
January	105	160	159⅞	234¾	144½	137⅞	142	136⅜	123¼	111¾
February	104⅝	172½	161	216⅜	141¼	140⅜	143⅞	136⅝	121¼	112⅜
March	102⅜	171¼	169¼	201	136⅜	140⅝	141⅝	132⅜	116⅝	111⅝
April	102¼	157⅞	184	153⅝	·129⅜	141⅞	140¼	134¼	115⅜	111¼
May	104⅜	154¼	194⅜	145⅝	141⅛	138⅞	140⅝	144¼	115⅛	112⅝
June	109⅜	148⅝	250	147⅝	167¾	138¾	141⅛	139⅝	114¼	113⅜
July	120⅛	145	285	146	155¾	140¼	145¼	137⅞	128¼	113¾
August	116⅜	129¼	261¼	145⅝	152¼	142⅝	150	136⅝	122	
September	124	143⅜	254⅜	145⅞	147⅜	146⅝	145⅜	162½	117¼	
October	137	156¼	227¾	149	154⅝	145⅝	141⅝	132	114¼	
November	133¼	154	260	148¾	148⅝	141⅜	137	128⅜	113⅜	
December	134	152⅝	243	148⅜	141¾	137⅞	136¼	124	111⅝	

By the foregoing table, it will be seen that the highest price of gold, from the first issue of paper money by the Government, to August 1, 1871, was in July, 1864, when it reached 285; that is, it took so many dollars in currency to buy 100 dollars in gold. With some fluctuations, paper has gradually appreciated until the present time (August, 1871), when the ruling price of gold is about 112.

2. The greatest depreciation in "greenbacks" occurred in July, 1864, when gold was quoted at 285. What was the value of one dollar in currency at that time? *Ans.* $35\frac{7}{57}$ cents.

3. When gold sells at 110 for currency, what is the discount on the currency? *Ans.* $9\frac{10}{11}$%.

INSURANCE.

360. **Insurance** is a contract by which one party, in consideration of a certain sum paid, engages to indemnify another for a loss which he may sustain by certain casualties.

361. The **Insurer,** or **Underwriter,** is the party who makes the contract, or takes the risk.

362. The **Premium** is the sum paid for the insurance.

363. The **Policy** is the written contract made by the insurer.

364. There are five kinds of insurance, namely, *Fire, Marine and Inland, Health, Accident, and Life Insurance.*

365. The Insurance Business is carried on chiefly by incorporated associations, called *Insurance Companies.*

366. Insurance Companies are divided into two classes,—*Stock Insurance Companies*, and *Mutual Insurance Companies.*

A **Stock Insurance Company** is a company established with a specified sum as *Capital.* This is divided into shares, and sold to individuals, which gives the company a "paid up" capital as a *Reserve Fund,* to cover all losses which may occur to the insured, the stockholders receiving all the profits as dividends on their capital invested.

367. A purely **Mutual Company,** on the other hand, is not only established without any *Capital*, but takes risks without *Cash Premiums*, except a small charge for the Policy, and sometimes a percentage on the "Premium Notes," to defray incidental expenses.

The persons insured become members of the company, and give notes for the premiums, and when a loss occurs, each member is "assessed" for his share, to discharge the liability of the company for the loss. The profits are distributed *pro rata* among the members at stated periods.

368. Some companies, called *Mixed Companies*, are conducted upon the Stock and Mutual plans combined.

FIRE INSURANCE.

369. Fire Insurance is insurance against loss by fire, on Buildings, Merchandise, Household Furniture, Rents, Leases, Manufactories, and other insurable interests.

370. A **Rate List** is a schedule of charges for premiums adopted by a company, in which buildings and other property are classified, and the rates given, according to the nature of the risks.

371. The charges are stated either at a certain rate per cent., or a certain number of cents on $100. In the latter case, the rate is *a fractional rate per cent.* Thus, 30 cents on $100 is $\frac{30}{100}$ or $\frac{3}{10}$ of 1% = .003; 75 cents on $100 is .75% = .0075; and $1.50 on $100 is 1½%.

A charge is also made for the Policy, and for Revenue Stamps.[*]

The Revenue Tax on Insurance Policies is as follows:—On a Life Insurance Policy, issued for a sum not exceeding $1000, 25 cents; exceeding $1000, and not exceeding $5000, 50 cents; exceeding $5000, $1.00:—On Marine, Inland, or Fire Insurance Policies, when the premium amounts to $10 or more, 25 cents; when it does not exceed $10, 10 cents.

372. Risks are usually taken on furniture and ordinary merchandise, at the same rates as are charged for insuring the houses or stores in which they are contained.

373. In computing Insurance, the rules of Percentage apply.

EXAMPLES.

1. Required the cost of insuring a house and out-buildings, for $3000, at a premium of 75 cents on $100 per annum, allowing $1 for the policy, and the amount required by law for revenue stamps.

SOLUTION.

$3000	
.0075	
$22.5000,	Amount of Premium,
1.00	Charge for Policy,
.25	Revenue Stamp.
$23.75, *Ans.*	

2. A merchant insures his store and goods for ⅔ of their value. The store is valued at $12000, and the goods at $18000. He pays at the rate of ⅞%, for 1 year, and $1.50 for the policy, and for the required revenue stamps. How much does the insurance cost him for one year?

3. A hotel and its contents, worth $80000, are insured for ⅔ their value, for 1 year, at 1¼% premium. A charge of $1.50 is made for the policy, and — cents for revenue stamps. Required the cost of the insurance.

4. Required the cost of insuring a manufactory, deemed "extra hazardous," for $60000 for 1 year, at 5%. Survey and policy, $5.50; stamp, — cents.

NOTE.—A survey is an examination of the premises, to determine the character of the risk.

5. What will it cost in cash to insure a frame dwelling-house, for $1000, in a Mutual Insurance Company, which takes for such a risk a premium note for 3 per cent., and charges 2½ per cent. in cash on the face of the note, 50 cents for the policy, and the cost of stamp? *Ans.*, $1.35.

6. A farmer gets his stone dwelling-house insured for $800, at 87½ cents on $100; policy, $1; stamp, 10 cents. What is the cost the first year?

MARINE AND INLAND INSURANCE.

374. Marine and Inland Insurance is insurance upon vessels of all kinds navigating the ocean, seas, lakes, and rivers, and upon their cargoes, and other insurable interests, against loss caused by the perils of navigation.

375. A **Valued,** or **Closed Policy,** is a policy in which a specified amount or value of the vessel or goods is insured, and inserted upon the making of the contract.

376. An **Open Policy** is a policy taken out for a certain amount, to remain open for a specified time, for the insertion of such amounts, not exceeding the amount named, as may be required to be insured within the specified time. When such a policy is issued, the company takes a note for the amount of the premium, which is held till the *policy is closed,* when a settlement is made.

377. The rates vary with the risks, taking into consideration the seaworthiness of the vessel, the distance to be traversed, and all the hazards.

EXAMPLES.

1. Shipped on board the *Ocean Wave,* for Liverpool, a quantity of goods valued at $4500. Required the amount of the insurance at 2%; $1.00 for the policy, and the cost of revenue stamps.

NOTE.—An insurance is sometimes desired to cover the value of the goods and the premium. In that case, to find the amount of the insurance, *divide the value of the goods by* 1 *minus the rate of premium.*

2. For what sum must a policy be issued to insure $10000 and the premium, at 3½%? and what amount of premium must be paid?

SOLUTION. $10000 ÷ .965 = $10362.69, Amt. to be insured.
$10362.69 × .03½ = $362.69, Amt. of premium.

Since the premium is 3½% of the amount to be insured, $10000 must be 96½% of the same. Hence $10000 ÷ .965 = $10362.69, *Ans.*

3. Required the cost of insuring a quantity of merchandise, worth $2850, from Philadelphia to Pittsburg, by canal or railroad, thence by steamboat to Cincinnati, at the rate of $\frac{3}{8}$%, including stamp, and $1 for policy.

LIFE INSURANCE.

378. **Life Insurance** is a contract which provides for the payment of a stipulated sum to the heirs of the insured, or other person, at his death, or to himself at the expiration of a specified term of years.

379. A policy payable at the death of the person insured, is called a *Life Policy;* one payable at the end of a certain period, an *Endowment Policy.*

380. An insurance upon a life is effected by the payment, in advance, of an annual premium during life, or a specified term of years, the rate of premium depending upon the age of the insured, at the time the policy is issued.

381. The rates of premium are based upon the *Expectation of Life* of the persons insured, and the *rate of interest* which may be obtained by investing the money received for premiums.

382. The **Expectation of Life** of a person, at any age, is the average duration of the lives of individuals after that age, as determined from the rates of mortality.

383. Several tables, showing the Expectation of Life, at different ages, have been constructed. The one most used in England is the *Carlisle Table*, prepared from data furnished by the Bills of Mortality in the town of Carlisle, from 1779 to 1787. The *Northampton Table*, based on observations on the deaths from 1735 to 1780 in the town of Northampton, is also used to some extent.

These tables, together with the *Wigglesworth Table,* prepared by Dr. Wigglesworth, from statistics of mortality in this country, are the basis of the premium tables adopted by many of the companies in the United States.

The ACTUARIES', or COMBINED EXPERIENCE TABLE, formed from the experience of seventeen prominent English Companies, and published in

1843, is now accepted as a standard and reliable guide by the majority of the American Companies.

THE AMERICAN TABLE, constructed from the experience of the New York Mutual Life Insurance Company, by Sheppard Homans, its Actuary, has been adopted by several companies, and by the State of New York for State purposes.

The CARLISLE and WIGGLESWORTH TABLES will be found on page 384.

The Actuaries' Table gives the Expectation of Life of persons under 30 years of age greater, and over that, less than these tables.

384. The following is a Table of Rates in

A NEW YORK LIFE INSURANCE CO.

No. 1.—ANNUAL PREMIUMS.

To secure $1000, payable as indicated, or at death, if prior.

Age.	At Death only.	In 35 Years.	In 30 Years.	In 25 Years.	In 20 Years.	In 15 Years.	In 10 Years	Age.
25	$19 89	$26 38	$30 61	$37 17	$47 68	$66 02	$103 91	25
26	20 40	26 57	30 80	37 34	47 82	66 15	104 03	26
27	20 93	26 83	31 02	37 52	47 98	66 29	104 16	27
28	21 48	27 11	31 25	37 72	48 15	66 44	104 29	28
29	22 07	27 42	31 50	37 92	48 33	66 60	104 43	29
30	22 70	27 76	31 78	38 16	48 53	66 77	104 58	30
31	23 35	28 13	32 09	38 41	48 74	66 96	104 75	31
32	24 05	28 54	32 43	38 69	48 97	67 16	104 92	32
33	24 78	28 98	32 79	38 98	49 22	67 36	105 11	33
34	25 56	29 46	33 19	39 31	49 49	67 60	105 31	34
35	26 38	30 00	33 63	39 68	49 79	67 85	105 53	35
36	27 25	30 58	34 11	40 07	50 11	68 12	105 75	36
37	28 17	31 22	34 64	40 50	50 47	68 41	106 00	37
38	29 15	31 93	35 23	40 98	50 86	68 73	106 28	38
39	30 19	32 70	35 88	41 52	51 30	69 09	106 58	39
40	31 30	33 55	36 59	42 10	51 78	69 49	106 90	40
41	32 47		37 38	42 75	52 31	69 92	107 26	41
42	33 72		38 24	43 47	52 89	70 40	107 65	42
43	35 05		39 19	44 26	53 54	70 92	108 08	43
44	36 46		40 23	45 12	54 25	71 50	108 55	44
45	37 97		41 37	46 08	55 04	72 14	109 07	45
46	39 58			47 15	55 91	72 86	109 65	46
47	41 30			48 32	56 89	73 66	110 30	47
48	43 13			49 61	57 96	74 54	111 01	48
49	45 09			51 04	59 15	75 51	111 81	49
50	47 18			52 60	60 45	76 59	112 68	50
51	49 40				61 90	77 77	113 64	51
52	51 78				63 48	79 07	114 70	52
53	54 31				65 22	80 51	115 86	53
54	57 02				67 14	82 09	117 14	54
55	59 91				69 24	83 82	118 54	55
56	63 00					85 73	120 09	56
57	66 29					87 84	121 78	57
58	69 82					90 15	123 64	58
59	73 60					92 70	125 70	59
60	77 63					95 50	127 96	60
61	81 96						130 45	61
62	86 58						133 19	62
63	91 54						136 20	63
64	96 86						139 52	64
65	102 55						143 16	65

The rates are about the same in all the companies. The premiums may be paid semi-annually, or quarterly; but, in that case, a percentage is added. Some companies have "participating," and "non-participating rates," the former entitling the policy-holders to a share in the profits. The foregoing table exhibits the rates of a Purely Mutual Company.

EXAMPLES.

1. A gentleman, at the age of 30, insures his life by taking a life policy for $3000, premiums payable annually: required the amount of the first payment, including $1 for the policy, and the cost of stamps. *Ans.* $69.60.

2. A person, aged 40, insures his life for $2000, payable in 15 years: what annual premium must he pay? And what will be the amount of the first payment, reckoning the policy $1, and adding the cost of stamps? *Ans.* to the last, $140.48.

3. What will be the amount of the first payment for a life policy for $10,000, including charges as above, the age of the insured being 45?

4. Required the annual premium on a policy payable in 25 years, taken by a person, aged 35, for the sum of $8000.

NOTE.—For the method of estimating the amount of premiums paid annually, for a given term of years, at compound interest, see *Annuities.*

TAXES.

385. A **Tax** is a sum of money assessed upon the person or property of an individual for the support of the government, and other public purposes.

386. A **Poll,** or **Capitation Tax,** is a tax levied upon the person of each male citizen liable to pay taxes, without regard to property.

NOTE.—The word *poll* means *head,* and hence is often used to signify a person who pays a poll-tax.

387. A tax assessed on property is called a *Property Tax*, and is reckoned at a certain rate per cent. on the estimated value of each person's property.

388. Taxable property is either *Real* or *Personal*.

389. Real Property, or **Real Estate,** consists of *fixed* property; as, lands and houses.

390. Personal Property consists of *movable* property; as, merchandise, cash, stocks, ships, etc.

391. For the purpose of assessing taxes, *Assessors* are appointed, to estimate the value of all the taxable property of the inhabitants of a city, district, or town; and if a poll-tax is to be levied, to make a list of the *polls*.

EXAMPLES.

1. A tax of $5860 is to be raised in a certain town. The town contains 450 polls, which are assessed $1.50 each. The real estate of the town, subject to taxation, is valued at $756500; and the personal property, at $10400. What will be the rate on $1, and what will be the tax of Aaron Brown, who pays for 1 poll, and whose property, real and personal, is valued at $3350?

SOLUTION.

$1.50	× 450	= $675,	Total poll-tax.
$5860	− $675	= $5185,	Amount to be assessed on property.
$756500	+ $10400	= $766900,	Amount of taxable property.
$5185	÷ $766900	= $0.006¾+,	Tax to be assessed on each dollar.
$3350	× .006¾	= $22.612,	Brown's property tax.
$22.61	+ $1.50	= $24.11,	Amount of Brown's tax.

From the above illustration we derive the following

RULE.—I. To find the rate of property-tax, *divide the sum to be raised, less the amount assessed on the polls, by the value of the taxable property, real and personal.*

II.—To find each person's tax, *multiply his taxable property by the rate, and to the product add his poll-tax.*

392. Having found the tax on $1, or the rate per unit, as above, assessors facilitate their business by preparing a tax table, as follows:

Table.

At the rate of 6¾ *mills on* $1.

PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.
$1	$0.006¾	$20	$0.135	$300	$20.25	$4,000	$27.
2	.0135	30	.1925	400	2.70	5,000	33.75
3	.019¼	40	.27	500	3.375	6,000	40.50
4	.027	50	.3375	600	4.05	7,000	47.25
5	.033¾	60	.405	700	4.725	8,000	54.00
6	.0405	70	.4725	800	5.40	9,000	60.75
7	.047¼	80	.54	900	6.075	10,000	67.75
8	.054	90	.6075	1,000	6.75	20,000	135.00
9	.060¾	100	.675	2,000	13.50	30,000	202.25
10	.0675	200	1.35	3,000	20.25	40,000	270.00

Examples for Practice.

2. The town of Hudson is to raise $6840 by taxes; the taxable property of the town is $862000; there are 540 polls, each taxed $1. What will be the tax on $1, and what is A's tax, he paying for 1 poll, and his property being valued at $1750?

3. In a certain village a schoolhouse is to be erected, at an expense of $3750. This amount is to be raised by a tax on the citizens of the district, whose property is valued at $22500. What will be the rate of taxation, to cover both the cost of the schoolhouse and the collector's commission of 3½ per cent.?

Note.—When the expense of collecting a tax is to be included, the net amount required, *divided by* 1 *minus the rate to be paid for collecting*, will be the gross amount to be assessed. Hence, in this case, $3750 ÷ .965 = $3886.01, gross amount of the tax.

4. Suppose it is required to raise by taxation, in a certain town, $7600, and that the taxable property of the town is $1558000: what will be the amount to be assessed upon property, if the cost of collection is estimated at 2½ per cent.? What will be the rate on $1, and how much will be the tax of A, whose property is valued at $2460?

CUSTOM-HOUSE BUSINESS.

393. A **Duty** is a sum of money required by the government to be paid on the importation of goods, for revenue purposes, and the protection of home industry.

394. Custom-Houses are branches of the Treasury Department, established by the General Government for the collection of duties, each being controlled by a Collector and Naval Officer (in small ports of entry by a collector alone), who are responsible for such collection to the Secretary of the Treasury.

395. Duties are *Specific*, *Ad valorem*, or *Combined.*

396. A **Specific Duty** is a rate of duty chargeable upon quantity, without regard to cost.

397. An **Ad valorem Duty** is a rate of duty chargeable upon the value of the goods at the last port of exportation.

398. A **Combined Duty** is a combination of specific and ad valorem duties, or a rate of duty assessed upon both quantity and value.

TARES, ETC.

399. Tare is an allowance, deducted before estimating the duties, for the weight of the box, cask, or bag, containing the goods.

400. The following is a Table of Tares fixed by Act of Congress, July 14, 1862: On all articles not included in the schedule, the actual tare only is allowed; or, where the tare is expressed in the invoice, it may be taken, with the consent of the collector and consignee, as the actual tare; but in no case shall there be any allowance for draft:—

Almonds, in bale, 2½%; in bags, 2%; in frails, 8%; Alum, in casks, 10%; coarse or ground, in sacks, 2 lbs. per sack; Barytes, 3%; Cheese, in

casks or tubs, 10%; Cassia, in mats, 9%; Coffee, Rio, in single bags, 1%; in double bags, 2%; all other kinds, actual tare; Cinnamon, in bales, 6%; Cocoa, in bags, 2%; in ceroons, 8%; Chiccory, in bags, 2%; Copperas, in casks, 10%; Currants, in casks, 10%; Hemp, Manilla, 4 lbs. per bale; Hamburg, Leghorn, and Trieste, 5 lbs. per bale; Indigo, in ceroons, 10%; Melade, in bags, 11%; Nails, in bags, 2%; in casks, 8%; Ochre, dry, in casks, 8%; in oil, in casks, 12%; Peruvian Bark, in ceroons, 10%; Paris White, in casks, 10%; Pepper, in bags, 2%; in double bags, 4%; Pimento, in bags, 2%; Raisins, in casks, 12%; in boxes, 25%; half-boxes, 27%; quarter-boxes, 29%; in frails, 4%; Rice, in bags, 2%; Spanish Brown, dry, in casks, 10%; in oil, in casks, 12%; Sugar, in hogsheads, 12½%; in tierces, 12%; in barrels, 10%; in boxes, 14%; in bags, 2%; in mats, 2½%; Salt, fine, in sacks, 3 lbs. per sack; Teas, China or Japan, invoice weight; all other kinds, actual tare; Tobacco, leaf, in bales, 10 lbs. per bale; in extra covers, 12 lbs. per bale; Whiting, in casks, 10%.

NOTE.—In reckoning tares, any fraction equal to, or exceeding half a pound, is called one pound, and any fraction less than half a pound is disregarded.

401. **Gross Weight** is the weight of the goods with the box, cask, bag, etc., containing them.

402. **Net Weight** is the weight of the goods after all deductions have been made.

DUTIABLE VALUE OF MERCHANDISE.

403. The **Dutiable Value of Merchandise,** under the present law, is the original cost, or actual wholesale price, in addition to commissions (in no case less than 2½%) and general charges incurred in the purchase, and also the cost of transportation, shipment, and transshipment, with all the expenses included, from the place of growth, production, or manufacture, whether by land or water, to the vessel in which shipment is made to the United States.

FOREIGN CURRENCIES.

404. The values of foreign currencies, to be used in estimating duties, are proclaimed annually under the Act of Congress passed March 3d, 1873. See page 329. The following is the circular issued January 1st, 1875.

CIRCULAR TO COLLECTORS OF CUSTOMS AND OTHERS.

1875.
Department No. 2.
Secretary's Office.

TREASURY DEPARTMENT,
Washington, D. C., January 1, 1875.

The first section of the Act of March 3, 1873, provides "that the value of foreign coin, as expressed in the money of account of the United States, shall be that of the pure metal of such coin of standard value," and that "the values of the standard coins in circulation, of the various nations of the world, shall be estimated annually by the Director of the Mint, and be proclaimed on the first day of January by the Secretary of the Treasury."

The estimate of values contained in the following Table has been made by the Director of the Mint, and is hereby proclaimed in compliance with the above-stated provisions of law:

COUNTRY.	MONETARY UNIT.	STANDARD.	VALUE IN U. S. MONEY.
Argentine Republic.	Peso fuerte	Gold	$1.00
Austria	Florin	Silver	.45,3
Belgium	Franc	Gold and silver	.19,3
Bolivia	Dollar	Gold and silver	.96,5
Brazil	Milreis of 1,000 reis	Gold	.54,5
British Possessions in North America	Dollar	Gold	1.00
Bogota	Peso	Gold	.91,2
Central America	Dollar	Silver	.91,8
Chili	Peso	Gold	.91,2
Cuba	Peso	Gold	.92,5
Denmark	Crown	Gold	.26,8
Ecuador	Dollar	Silver	.91,8
Egypt	Pound of 100 piastres	Gold	4.97,4
France	Franc	Gold and silver	.19,3
Great Britain	Pound Sterling	Gold	4.86,6½
Greece	Drachma	Gold and silver	.19,3
German Empire	Mark	Gold	.23,8
Hayti	Dollar	Silver	.95,2
Japan	Yen	Gold	.99,7
India	Rupee of 16 annas	Silver	.43,6
Italy	Lira	Gold and silver	.19,3
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	.99,8
Netherlands	Florin	Silver	.38,5
Norway	Crown	Gold	.26,8
Paraguay	Peso	Gold	1.00
Peru	Dollar	Silver	.91,8
Porto Rico	Peso	Gold	.92,5
Portugal	Milreis of 1,000 reis	Gold	1.08,4
Russia	Rouble of 100 copecks	Silver	.73,4
Sandwich Islands	Dollar	Gold	1.00
Spain	Peseta of 100 centimes	Gold and silver	.19,3
Sweden	Crown	Gold	.26,8
Switzerland	Franc	Gold and silver	.19,3
Tripoli	Mahbub of 20 piastres	Silver	.82,9
Tunis	Piastre of 16 caroubs	Silver	.11,8
Turkey	Piastre	Gold	.04,3
United States of Colombia	Peso	Silver	.91,8
Uruguay	Patacon	Gold	.94,9

B. H. BRISTOW, *Secretary of the Treasury.*

VALUE OF CURRENCIES BY USAGE WHEN A CONSULAR CERTIFICATE OF THE REAL VALUE OF EXCHANGE IS NOT ATTACHED TO THE INVOICE.

	$ cts.	FRACTIONAL PARTS OF THE CURRENCY.	
Banco Rix Dollar, Sweden	.39¾		
" " Norway	.39¾		
" " Denmark	.53		
Guilder, Brabant	.33¾		
Crown of Tuscany	1.05	20 Soldi	12 Denari
Curaçoa Guilder	.40	20 Stivers	12 Pfennings
Francisconi	1.06		
Kobang of Japan	1.38	4 Itzebou	1600 Seni
Leghorn Dollar, or Pezzo	.90$\frac{76}{100}$	20 Soldi	12 Denari
Livre of Catalonia	.53½	20 Sueldos	12 Dineros
Livre of Neufchatel	.26½	20 Sols	12 Dincros
Rix Mynth Dollar of Sweden.	.26½		
Rix Ral Thaler of Gottenburg	.27¾		
Swiss Livre	.27	100 Centimes	
Scudi of Malta	.40	12 Tair	20 Grani
Scudi, Roman	99@99½		
St. Gall Guilder	40$\frac{36}{100}$	60 Kreutzers	4 Pfennings
Rix Dollar of Batavia	.75		
Roman Dollar	1.05		
Rouble, paper, of Russia		100 Kopecks	Varies from 4 roubles 65 copecks, to to 4 roubles 8 copecks to the dollar
Tical of Spain	.61		
Turkish Piastre	.05	100 Aspers	
Current Mark	.28		
Florin of Prussia	.22¾		
Florin of Balse	.41		
Genoa Livre	.21		
Livre Tournois of France	.18½		

In all cases when the Consul's Certificate is not attached to the Invoice, a Bond for the production of one will be required.

ENTRIES.

405. Upon the arrival of any goods from a foreign port, the invoice and bill of lading must be presented at the Custom-House, and verified by oath or affirmation. The entry must then be made in due form, when the clerk charged with the estimating of duties will make his estimate upon the value, weight, gauge, or measurement, as may be specified in the invoice, and order the same to be weighed, gauged, or measured by the United States weighers, gaugers, or measurers, as the law contemplates, charging duty only upon the quantity of merchandise actually arriving at the port of destination.

NOTES.—1. If merchandise arrives, as it frequently does, without an invoice, it may be appraised and admitted; but when the value exceeds one hundred dollars, it is necessary to apply to the Secretary of the Treasury for permission to enter the same for appraisement.

2. Merchandise may be entered for consumption, warehousing, or for immediate transportation from one port to another within the limits of the United States, to be exported to a foreign country, or there entered for consumption or warehousing.

ESTIMATION OF DUTIES.

EXAMPLE 1. The following is a foreign Invoice of Merchandise, together with the form of a Cash Entry for Consumption. What is the amount of the duties, payable in gold?

INVOICE OF MERCHANDISE

Shipped by Ridgway, Acton & Co., on board American Steamer Asia, *Wilson, Master, for Account and Risk of whom it may concern, and consigned to Messrs. Hamilton, Phipps & Co.*

		£	s.	d.	£	s.	d.
R A C # 6	1 Case containing 366 Brch. Bordered Shawls, ... @ 7/4, In 15 Cartons.	134	4	0			
	Trade Discount, ...	2	17	0	121	7	0
	Net Weight 255 lbs.						
" # 7	1 Case containing 250 Brch. Bordered Shawls, ... @ 8/3 In 10 Cartons.	103	2	6			
	Net Weight 200 lbs. Trade Discount, ...	2	6	10	100	15	8
" # 8	1 Case containing 210 Pcs. 30 in. White Check. Muslins, 20 yds. each, @ 9/9,	102	7	6			
" # 9	1 Case containing 200 Pcs. 31 in. " " " " " @ 11/3,	112	10	0			
" # 10	1 Case containing 205 Pcs. 31 in. " " " " " @ 13/-,	133	5	0	348	2	6
					570	5	2
	Charges						
	Cases, Tickets, Patterns, 25 Cartons, Consul's Fees, $^{11}/_{4}$, ...				7	14	7
					577	19	9
	Commission, 1½°/o, ...				7	4	6
					585	4	3
	Manchester, April 15, 1871. RIDGWAY, ACTON & CO. E. & O. E.						

[*Cash Entry.*] [No. 22.—Form 80.]

Entry of Merchandise Imported by *Hamilton, Phipps & Co.*, in the *Steamer Asia*, whereof *Wilson* is Master, from *Liverpool*, NEW YORK, May 2, 1871.

PACKAGES AND CONTENTS.	INVOICE. WHERE DATED.	QUANTITY.	50 *cts. pr. lb.* & 40°/₀	5¼ *cts. per sq. yd.*	TOTAL.
R. A. O. Nos. 6 & 7 2 Cas. Worst. Sh.	Manchester, April 15, 1871.	455 lbs.	£222. 2. 8		
Nos. 8 & 10 3 Cas. Wht. Cot.		10,475 sq. yds.		£348. 2. 6	
Charges as per Invoice			£222. 2. 8 3. 0. 2	£348. 2. 6 4.14. 5	£570. 5. 2 7.14. 7
Commission 1½°/₀, as per Invoice.......			£225. 2.10 2.16. 2	£352.16.11 4. 8. 4	£577.19. 9 7. 4. 6
Less consul's certificate, 11/4............			£227.19. 0 5.	£357. 5. 3 6. 4	£585. 4. 3 11. 4
Shipping charges to Liverpool, 5/ ea.....			£227.14. 0 10.	£356.18.11 15.	£585.12.11 1. 5.
Commission to equal 2½°/₀...............			£228. 4. 0 2.16. 2	£357.13.11 4. 8. 5	£585.17.11 7. 4. 7
@ $4.8665....			£231. 0. 2 $1124.20	£363. 2. 4 $1767.10	£593. 2. 6 $2886.44

Duty. Inv. $1124 @ 40°/₀ $449.60
455 lbs. @ 50c. 227.50
10475 sq. yds. @ 5½c. 576.13
$1253.23

2. IMPORTATION FROM GERMANY.—Entered by C. A. Zoebish & Sons, an invoice from Christian Gottfr. Kessler, in Markneukirchen, per steamship *Hansa*, from Bremen:

Z. & S., Nos. 161-166.	30%	45%	35%
Six cases musical instruments.			
Harmonicas, violins, flutes, and guitars.	*Thal.* 2129.15.5		
Cornets of brass..................		88.0.0	
Metal strings.....................			137.20.0
	2129.15.5	88.0.0	137.20.0
Cases, shipping, etc., to Bremen........	30. 0.0	1.0.0	2. 0.0
	2159.15.5	89.0.0	139.20.0
Commission 2½%.........	53.29.7	2.6.9	3.14.9
	Thal. 2213.15.0	91.6.9	143.4.9
At 69c.	$1527.31	$62 95	$98.78

Duty, $1527, at 30% = $458.10
63, " 45% = 28.35
99, " 35 = 34.65
Total.................................$521.10

3. IMPORTATION FROM FRANCE.—Entered at the port of New York, by Henry Barbey & Co., an invoice from Henri Bergounioux & V. Richard, 4 Rue St. Joseph, Paris, by steamship *Pereire*, via Havre:

H. B. & C. No. 143.	50c 35%	60%
One case silk and worsted barèges Weight 175 lbs.	*Fr.* 3227.10	
Silks in piece		*Fr.* 2900.85
	3237.10	2900.85
Disc't, 7%	225.90	203.06
	3001.20	2697.79
Shipping charges to Havre, fr. 20	11.00	9.00
	3012.20	2706.79
Commissions, 3%	90.37	81.20
	Fr. 3102.57	*Fr.* 2787.99
At $19\frac{3}{10}$c	$598.79	$538.08

Duty, $599 at 35% = $209.65
175 lbs., " 50c = 87.50
$538, " 60% = 322.80
$619.95

4. IMPORTATION FROM SWITZERLAND.—Entered by Taylor, Olmsted & Taylor, an invoice from Breitling & Laederich, in Chaux de Fonds, per steamship *Main*, from Bremen:

T. O. One case watches, fr. 6750, duty at 25%. Charges, cases, shipping, etc., to Bremen, fr. 11.50; commissions, 2½%. Required the amount of the duty. *Ans.* $333.84.

NOTE.—In reducing foreign currencies, if the cents are fifty or over, they are taken as $1; if less than fifty, they are rejected.

5. Imported by M. Good, New York, July 20, 1871, from Rosario, *viâ* Buenos Ayres, S. A., the following mdse., *duty free.*

D. & C. Ten bales, 8000 goat skins, $70,062.50 paper dollars, at 25 to a Patacon dollar = $2802.50 (P.). Charges, inland freight, and export duties, $239.96 (P.); com., 2½% on $3042.46 (P.). Charges, drawing com., and 1 month's interest, at 1%, on $3118.52 (P.), from May 15 to June 14, $77.95. Total, $3196.47, at 16 patacons to a patriot doubloon = oz. 199.12.47, at $15.58, U. S. currency = $3112.89.

NOTE.—Calculations, to convert the currencies, for impost purposes, are made on free goods, the same as if they were subject to duty.

RATIO.

406. **Ratio** is the relation which one quantity or number bears to another of the same kind, with respect to magnitude.

407. Two numbers may be compared in two ways—

1st. By their *difference,* which is called their *arithmetical ratio.* The ratio shows how much greater, or less, one is than the other.

2d. By the *quotient* of the one divided by the other, which is called their *geometrical ratio.* The ratio shows how many times one is contained in the other.

408. The word *ratio,* when used alone, signifies geometrical ratio.

409. It is evident, from the above definition, that either number may be taken as the divisor, or dividend, for the ratio is the measure of their comparative value. Thus, the ratio of 2 to 8 is $\frac{8}{2} = 4$, or $\frac{2}{8} = \frac{1}{4}$. Whichever is taken as the *standard,* is compared with the other; 2 is one fourth of 8, or 8 is four times 2.

But it is more in accordance with the general use of the terms, to consider the first the divisor, and the second the dividend. Ratios are chiefly used in forming proportions, in which the first term is almost invariably taken as the divisor. This rule is therefore here adopted.

410. Since a ratio is the quotient resulting from the division of one number by another, it follows that both numbers must be of the same kind; that is, if concrete, they must either be of the same denomination, or susceptible of reduction to a common unit. For example, there can be no comparison, in the form of a ratio, between 5 dollars and 4 pounds. But we can compare 5 dollars and 4 dollars, or 5 dollars and 4 cents, because dollars may be reduced to cents.

411. The **Terms** of a ratio are the two numbers compared. The first is called the *antecedent,* the second the *consequent.* The two terms taken together constitute a *couplet.*

412. A ratio is indicated in two ways: 1st. By placing two dots, or a colon, between the terms. 2d. By writing the antecedent under the consequent in the form of a fraction. Thus, the ratio of 9 to 3 is written 9 : 3, or $\frac{3}{9}$.

413. Simple Ratio is a ratio of two numbers; as, 6 : 12: and is read, as 6 is to 12.

414. A **Compound Ratio** is the indicated product of two or more simple ratios; as, $(7:2)\times(5:10)=\frac{2}{7}\times\frac{10}{5}=\frac{20}{35}=35:20$.

NOTE.—When the multiplication is performed, the product is a simple ratio.

415. A compound ratio is usually expressed by writing the simple ratios composing it, the one under the other; thus,

$$\left.\begin{array}{r} 7:\ \ 2\\ 5:10\\ 3:\ \ 4 \end{array}\right\} = \frac{2\times10\times4}{7\times\ 5\times3} = \frac{80}{105}.$$

$$\overline{105 : 80}, \text{ Product, a simple ratio.}$$

416. Since a ratio indicates division, and may be expressed in fractional form, we derive from the laws of division and their application to fractions the following

GENERAL PRINCIPLES OF RATIO.

1. *Multiplying the antecedent, or dividing the consequent, divides the ratio.*

2. *Dividing the antecedent, or multiplying the consequent, multiplies the ratio.*

3. *Multiplying, or dividing, both terms by the same number, does not alter the ratio.*

417. And since the ratio of one number to another is equal to the consequent divided by the antecedent, it follows that—

1. *The antecedent is equal to the consequent divided by the ratio.*

2. *The consequent is equal to the antecedent multiplied by the ratio.*

EXAMPLES.

What is the value of the following ratios?

1. 7 : 28. *Ans.* 4.
2. 14 : 7. *Ans.* $\frac{1}{2}$.
3. 13 : 39.
4. $15 : $45.
5. 3 lbs. : 18 lbs. *Ans.* 6.
6. 4 oz. avoir. : 3 lbs. avoir.
7. (6 : 4) × (16 : 8).
8. $\frac{1}{4} : \frac{1}{6}$.

9. If the antecedent be 6, and the ratio 4, what is the consequent?

10. If the consequent be 36, and the ratio 3, what is the antecedent?

11. The ratio of two numbers is 8, and the consequent $\frac{3}{8}$: what is the antecedent?

12. What is the reciprocal of the ratio 5 : 15? *Ans.* $\frac{1}{3}$.

NOTE.—The *reciprocal of a ratio* is 1 divided by the ratio, or the ratio of the terms inverted. This is sometimes, though improperly, termed *Inverse Ratio.* There is, in fact, no such thing as inverse ratio. In stating a proportion, it is frequently necessary to consider that one quantity *varies inversely* as another, when one increases in the same ratio as the other decreases. For example, if 10 men can perform a certain piece of work in 6 days, how long will it take 30 men to perform the same work? Now, no one would suppose that the employment of a greater force would *increase the time.* On the contrary, the time *varies inversely* as the number of men employed. But no ratio can be established between *time* and *men;* and the ratio which determines the time is that of 30 men to 10 men, which is a *direct ratio*, and cannot be construed into the absurdity of an inverse ratio.

The expression is perhaps admissible in such a connection as this: the given time is to the required time in the inverse ratio of 10 to 30, meaning in the ratio of these terms inverted; or, the *men* and the *time* are *inversely proportional.*

13. Reduce 18 : 9 to its lowest terms.

14. Reduce to a simple ratio

7 : 14
2 : 6
9 : 18

15. What is the ratio of $\frac{4}{5}$ to $\frac{3}{5}$? *Ans.* $\frac{3}{4}$

NOTE.—Since the fractions have a common denominator, they are to each other as their numerators, and the denominators may be omitted.

PROPORTION.

418. Proportion is an equality of ratios. A Proportion may be either *Simple* or *Compound.*

419. A **Simple Proportion** expresses the equality of two simple ratios. Thus,

$$5 : 10 : : 6 : 12$$

is a simple proportion, and indicates that the ratio of 5 to 10 is equal to the ratio of 6 to 12. It is read, 5 *is to* 10 *as* 6 *is to* 12.

420. The double colon between the ratios signifies their equality.

421. A **Compound Proportion** expresses the equality of two compound ratios, or of a compound and a simple ratio The following are examples:

$$\left.\begin{matrix}3:4\\3:2\end{matrix}\right\}::\left\{\begin{matrix}9:16.\\6:\ \ 3.\end{matrix}\right.$$

And,

$$\left.\begin{matrix}5:15\\3:\ \ 2\end{matrix}\right\}::6:12.$$

The first indicates that the ratio of 3 × 3 to 2 × 4, is equal to the ratio of 6 × 9 to 3 × 16; the second, that the ratio of 3 × 5 to 2 × 15, equals the ratio of 6 to 12. By performing the multiplications indicated, they may be reduced to simple proportions.

Instead of the four dots, or double colon, between the ratios, the sign of equality is sometimes used; as, $5 : 10 = 6 : 12$. A proportion may also be written in fractional form; as, $\frac{10}{5} = \frac{12}{6}$.

422. The *first* and *third* terms of a proportion are called the *Antecedents;* the second and fourth, the *Consequents.* The first and second together are the *first couplet;* the third and fourth, the *second couplet.*

423. The **Extremes** are the first and fourth terms.

424. The **Means** are the second and third terms.

425. All the terms are called *Proportionals,* and the last term is called a fourth proportional to the other three in order.

426. A **Mean Proportional** is the middle term of three in continual proportion, or the square root of the product of the first and third. Thus, 6 is a mean proportional between 4 and 9; $6^2 = 4 \times 9 = 36$.

427. The two antecedents, or the two consequents, are called *Homologous Terms*, because they have the same ratio or proportion.

428. By Geometry, it is demonstrated that a proportion admits of the following changes. Take the proportion,

$3 : 2 :: 9 : 6$

Then, by *Inversion*, $2 : 3 :: 6 : 9$

By *Alternation*, $3 : 9 :: 2 : 6$

By *Composition*, $3 + 2 : 2 :: 9 + 6 : 6$, or
$3 + 2 : 3 :: 9 + 6 : 9$

By *Division*, $3 - 2 : 2 :: 9 - 6 : 6$, or
$3 - 2 : 3 :: 9 - 6 : 9$

By *Conversion*, $3 : 3 - 2 :: 9 : 9 - 6$

By *Sum and Difference*, $3 + 2 : 3 - 2 :: 9 + 6 : 9 - 6$.

429. *In every proportion, the product of the extremes is equal to the product of the means.*

To demonstrate this proposition, let the proportion, $3 : 9 :: 4 : 12$, be written in fractional form; thus, $\frac{9}{3} = \frac{12}{4}$. Then, reducing the fractions to the same denomination, we have,

$$\frac{9 \times 4}{3 \times 4} = \frac{3 \times 12}{3 \times 4}$$

Now, since the fractions are equal, and have the same denominator, their numerators must be equal, or $9 \times 4 = 3 \times 12$. But, 9 and 4 are the *means*, and 3 and 12 are the *extremes*. Therefore, the product of the extremes is equal to the product of the means. Hence, any three terms of a proportion being given, the other may be found by the following

RULE.—I. To find either extreme, *divide the product of the means by the other extreme.*

II. To find either mean, *divide the product of the extremes by the other mean.*

EXAMPLES.

Find the term that is wanting in each of the following examples:

1. 12 : 9 :: 8 : —,
2. 3 : 4 :: — : 8,
3. — : 7 :: 13 : $6\frac{1}{3}$,
4. $\frac{3}{4}$: — :: $\frac{2}{5}$: 8,
5. 2 : $3\frac{1}{3}$ :: 7 : —,
6. 1.5 : .6 :: — : 4.5,
7. $6 : $7 :: 4 : —,
8. 2 : 3 :: 6 lbs. : —.

9. Find a mean proportional between 4 and 16.

10. Reduce $\left.\begin{matrix} 3 : 6 \\ 7 : 5 \\ 4 : 10 \end{matrix}\right\} : 28 : 100$ to a simple proportion.

11. The first three terms of a proportion are 6, 8, and 7: what is the fourth term?

SIMPLE PROPORTION.

430. A problem in simple proportion always gives three terms of a proportion, and requires the fourth. Hence, the rule is sometimes called the *Rule of Three.*

431. The arrangement of the given terms in the form of a proportion is called a *statement.*

432. In stating a question, it is customary to take for the third term that number which is of the same kind, or denomination, as the answer sought. Then, the other two numbers, which will be of the same kind, will constitute the first and second terms, and must be arranged according to the conditions of the question. If these conditions imply that the answer will be greater than the third term, the ratio of the first couplet must be greater than 1, and the greater of the two remaining numbers must be the second term, and the other the first; if the answer must be less, the ratio of the first couplet must be less than 1, and this order must be reversed.

Example 1. If 6 yards of cloth cost \$24, what will 9 yards cost at the same rate?

STATEMENT AND OPERATION.

$$\begin{array}{r} 6 : 9 :: \$24 \\ 9 \\ \hline 6)216 \\ \hline \$36, \textit{ Ans.} \end{array}$$

Analysis.—The answer sought is to be in dollars; therefore \$24 is taken as the third term. Now, 9 yds. will cost more than 6 yds., or $\frac{9}{6}$ as much. The cost varies *directly* as the quantity; hence the ratio of the given quantity, 6 yds., to the increased quantity, 9 yds., is the same as the ratio of the given cost to the increased cost. In other words, the cost of 9 yds. will be *greater* than the cost of 6 yds.; therefore, 6 is the first term, and 9 the second. In the three given terms, 9 and 24 are the *means*, and 6 is one of the *extremes*. The product of the means is found to be 216, and this product, divided by 6, gives 36, the other extreme.

Cancellation.—As already explained (Art. 67), equal factors may be cancelled from the first and second, or first and third terms. Thus, in the above example, cancelling 6 from the first and third terms, we have,

$$\begin{array}{c} \cancel{6} : 9 :: \cancel{\$24} : \text{—}, \\ 4 \\ \hline \$36 \end{array}$$

Or, cancelling 3 from the first and second terms, we have,

$$\begin{array}{r} \overset{2}{\cancel{6}} : \overset{3}{\cancel{9}} :: \$24 \\ 3 \\ \hline 2)72 \\ \hline \$36 \end{array}$$

Example 2. If 3 men can do a certain piece of work in 21 days, how long will it take 7 men to do the same work?

STATEMENT AND OPERATION.

$$\begin{array}{c} \cancel{7} : 3 :: \cancel{21} \\ 3 \\ \hline 9 \text{ days, } \textit{Ans.} \end{array}$$

Analysis.—The answer is required in days; hence 21 is the third term. Now, the *time* will vary *inversely* as the number of *men* employed; or, the time will decrease in the same ratio as the force increases. Therefore, the time will be less than 21 days, in the ratio of 7 to 3; and 7 is the first term, and 3 the second.

From these examples and explanations we derive the following

RULE.—I. *Take for the third term that number which is of the same kind as the answer sought.*

II. *If the question implies that the answer will be greater than the third term, make the larger of two remaining numbers the second term, and the other the first; but if the answer must be less than the third term, make the smaller of these numbers the second term, and the larger the first.*

III. *Divide the product of the second and third terms by the first term; the quotient will be the answer in the same denomination as the third term.*

NOTES.—1. To abridge the operation of multiplying and dividing, first cancel equal factors from the first and second, or first and third terms.

2. The first and second terms must be of the same denomination, or capable of being reduced to the same unit value. When necessary, reduce them to the same unit.

3. If the third term be a compound number, reduce it to its lowest unit, or to a decimal, or fraction of the highest.

EXAMPLES FOR PRACTICE.

1. If a man travel 900 miles in 8 days, how far will he travel in 3 days, at the same rate?

2. If 3 tons of coal cost $19.50, how much will 17 tons cost?

3. What must be paid for a certain piece of cloth, if ¾ of it cost $24?

4. Two numbers are to each other as 4 to 7, and the greater is 195; what is the less?

5. A garrison of 1000 men has provisions for 60 days. How many men must be discharged that the remainder may be supported 100 days?

6. If 40 men be required to build a wall 620 rods long, in 7 days, how many men will build 868 rods in the same time?

7. Suppose a railroad train to run at the rate of 20 miles in 50 minutes, in what time will it run 275 miles?

8. If a stake 5 feet 4 inches in length, standing perpendicularly, cast a shadow 7 feet long, what is the height of a tree which casts a shadow 45 feet 6 inches?

9. If a man can perform a certain piece of work in 14 days, by working 12 hours a day, in how many days can he perform the same work, by working 8½ hours a day?

10. A certain cistern has 3 pipes; the first will empty it in 20 minutes, the second in 40 minutes, and the third in 75 minutes: in what time will they all empty it? *Ans.* 11 m. $19\frac{13}{53}$ sec.

11. If a watch, showing correct time at noon on Monday, loses 2 minutes 15 seconds per day, what time will it show on Wednesday at 6 o'clock P.M.?

12. What quantity of water must be added to a cask of wine containing 120 gallons, which cost $210, to reduce the first cost to $1.50 per gallon?

13. A contractor, having engaged to do a job of work in 15 days, employed 9 men for that purpose; but, after they had worked 12 days, he found that it would take 8 days more, with that force, to complete the job: how many additional men must he employ to finish it within the stipulated time?

14. If a pile of wood, 36 feet long, 4 feet wide, and 5 feet high, costs $29.25, what will a pile 120 feet long, 4 feet wide, and 6 feet high, cost, at the same price?

15. How many days will it take 18 men to do a piece of work which 50 men can do in 5½ days?

16. A fox is 75 rods before a greyhound, and while the fox is running 20 rods the greyhound runs 30 rods: how far must the dog run to overtake the fox?

17. It is required to divide $1542 among three persons in the proportion of $\frac{1}{7}$, $\frac{2}{5}$, and $\frac{3}{8}$: how much will each receive?

18. How many yards of carpeting, ¾ of a yard wide, are equal to 40 yards 1¼ yards wide?

19. Two men purchase a lot of standing grass for $36.50, and, on cutting it, one takes 3½ tons of the hay, and the other tons: what ought each to pay?

COMPOUND PROPORTION.

433. A **Compound Proportion,** as applied to the solution of problems, involves two or more simple proportions. Hence, the rule is sometimes called the *Double Rule of Three.*

434. Every problem contains two or more conditions, upon each of which the answer depends. Each condition implies a separate ratio, and will give a simple proportion; but a combined statement is usually made, in the form of a compound proportion, consisting of a compound and a simple ratio.

435. The first and second terms of each couplet of the compound ratio, must be numbers of the same kind or denomination, and the third term of the proportion a number of the same kind as the answer sought.

EXAMPLE. If 8 men can earn $160 in 9 days, by working 12 hours a day, how much can 20 men earn, in 7 days, by working 10 hours a day?

436. We will first give a solution by Simple Proportion.

SOLUTION.—The answer to this question depends on three conditions; namely, the number of *men* employed, the number of *days* they work, and the number of *hours* they work each day.

Now, 20 men will earn more money, *in the same time,* than 8 men; and this leads to the following proportion:—

8 men : 20 men : : $160 : $400.

But, the 20 men only work 7 days. Therefore, assuming the hours to be the same, they will earn less than $400, in the ratio of 9 to 7. This gives

9 days : 7 days : : $400 : $$311\frac{1}{9}$.

But, again, they only work 10 hours a day, and will, therefore, earn less than $$311\frac{1}{9}$, in the ratio of 12 to 10.

Hence we have, finally,

12 hours : 10 hours : : $$311\frac{1}{9}$: $$259\frac{7}{27}$, *Ans.*

437. By Compound Proportion, these separate statements are combined in one, as follows:—

STATEMENT AND OPERATION.

$$\left.\begin{array}{rcr} & & 5 \\ \cancel{8} & : & \cancel{20} \\ 9 & : & 7 \\ 3\ \cancel{12} & & 10 \end{array}\right\} :: \overset{20}{\cancel{160}}$$

$$\frac{5 \times 7 \times 10 \times 20}{9 \times 3} = 259\tfrac{7}{27},\ Ans.$$

Or,

$$\begin{array}{r|l} & \cancel{160}\ \ 20 \\ \cancel{8} & \cancel{20}\ \ \ 5 \\ 9 & \ \ 7 \\ 3\ \cancel{12} & 10 \\ \hline 27 & 7000 \qquad (259\tfrac{7}{27},\ Ans. \\ & 54 \\ \cline{2-2} & 160 \\ & 135 \\ \cline{2-2} & \ \ 250 \\ & \ \ 243 \\ \cline{2-2} & \ \ \ \ \ 7 \end{array}$$

Analysis.—Making $160 the third term, we form the ratio required by each condition precisely as if the answer depended upon that condition alone. Thus, *other things being equal,* 20 men will earn *more* than 8 men; therefore 20 is the second term of one couplet, and 8 the first. Again, a certain number of men will earn *less* in 7 days than in 9 days; hence, for this condition, we have the ratio 9 : 7. Lastly, they will earn *less*, by working 10 hours a day, than by working 12, which gives the ratio 12 : 10. The continued product of the second terms multiplied by the third, and divided by the continued product of the first terms, gives the fourth term, or answer.

Note.—This, and every similar problem, may be solved by *Analysis*. Thus, if 8 men, in 9 days of 12 hours each, earn $160, 20 men will earn $\frac{20}{8}$ of $160 = $400. If, in 9 days, 20 men earn $400, in 7 days they will earn $\frac{7}{9}$ of $400 = 311\frac{1}{9}$. If, in 7 days, 20 men earn 311\frac{1}{9}$, by working 12 hours a day, they will earn by working 10 hours a day $\frac{10}{12}$ of 311\frac{1}{9}$ = 259\frac{7}{27}$.

438. From the foregoing illustrations and explanations, we derive the following

Rule.—I. *Make that number which is of the same kind as the answer required the third term.*

II. *Of the remaining numbers, take any two of the same kind, to form a ratio, and arrange them as in Simple Proportion, as if the result depended on them alone; then any other two, and so on, till all are used.*

III. *Multiply the continued product of the second terms by the third, and divide by the continued product of the first; the quotient will be the required answer.*

PROBLEMS.

1. If 120 men can mow 56 acres of grass in 9 days, how long will it take 45 men to mow 112 acres?

2. If 8 persons consume 3 barrels of flour in 3 months, how long will 18 barrels last 100 persons?

3. If 16 horses consume 84 bushels of grain in 24 days, how many bushels will 32 horses consume in 48 days?

4. If 12 masons can erect a wall 400 feet long, 12 feet high, and 18 inches thick, in 13¾ days of 10⅓ hours each, how long will it take 5 masons to build a wall 250 feet long, 18 feet high, and 22½ inches thick, working 12½ hours a day?

5. A ship's crew of 300 men were so supplied with provisions for 12 months, that each man was allowed 30 ounces per day; but after having been 6 months on their voyage, they found it would take 9 months longer to finish it, and fifty of their number had been lost. It is required to find the daily allowance of each man during the last 9 months.

6. How many men in 36 days of 16 hours, will do three times as much work as 24 men can perform in 28 days of 12 hours? *Ans.* 42.

7. If 9 men, in 12 days, can build a wall 36 yards in length, how many additional men must be employed to continue the wall 74 yards further, in 15 days? *Ans.* 13.

8. If 15 compositors, in 7 days of 8 hours 40 minutes each, can set 420 pages of type, each page containing 40 lines, and each line 39 letters, how many hours a day must 8 compositors work, in order to set, in 10 days, 240 pages of 52 lines each, and 44 letters in each line?

9. If a family of 4 persons consume a barrel of flour in 8 weeks, how many barrels, at the same rate, will 20 persons consume in a year of 52 weeks?

10. If 3 bushels of barley, weighing 45 lbs. per bushel, can be bought for 2 bushels of corn, weighing 58 lbs. per bushel, how much barley, weighing 48 lbs. per bushel, would be equivalent to

100 bushels of corn, weighing 56 lbs. per bushel, considering the price of each to vary with the weight?

11. Three pipes of equal size will fill a certain cistern with water in 1 hour and 40 minutes: in what time will 5 such pipes fill a cistern whose capacity is 2½ times as great?

Ans. 2½ hours.

12. If a man travel 120 miles in 3 days, travelling 8 hours a day, how far would he go in 7½ days, travelling 7½ hours a day?

13. How many pounds of cotton will it take to make 20 pieces of sheeting, each containing 28 yards, 1¼ yards wide, if it takes 22 pounds of cotton to make 2 pieces of sheeting, each containing 32 yards, 1⅛ yards wide?

14. If 3 men, by laboring 10 hours a day, can mow a field of 18 acres in 5 days, how long will it require 7 men and 4 boys to mow a field containing 30 acres, provided each boy can do ½ as much as a man?

ANALYSIS.

439. Analysis, in arithmetic, is the process of solving problems by applying general principles, according to the conditions of each case, independently of special rules.

EXAMPLES.

1. If one man can do a certain piece of work in 12 days, by working 8 hours a day, how many men will it require to do the same work in 4 days, if they work only 6 hours a day?

SOLUTION.

$$\begin{array}{r} 12 \\ 8 \\ \hline 4 \times 6 = 24)96 \\ \hline 4, \textit{ Ans.} \end{array}$$

ANALYSIS.—Since one man can do the work in 12 times 8 hours = 96 hours, and one man, in 4 days of 6 hours each, can do 4 × 6 = 24 hours' work, it follows that it will take as many men to do the whole in 24 hours, or 4 days of 6 hours each, as 24 is contained times in 96 = 4.

2. A merchant, owning $\frac{7}{8}$ of a ship, sold $\frac{3}{4}$ of his share for \$9450: how much is the whole ship worth, at the same rate?

SOLUTION. $\frac{7}{8} \times \frac{3}{4} = \frac{21}{32}$, Share sold.
And, \$9450 $\div \frac{21}{32}$ = \$14400, Value of the ship.

ANALYSIS.—Since $\frac{21}{32}$ of the ship is worth \$9450, $\frac{1}{32}$ is worth $\frac{1}{21}$ of \$9450 = \$450, and $\frac{32}{32}$, or the whole, is worth 32 times \$450 = \$14400, *Ans.*

3. A garrison of 1500 men has a supply of provisions for 9 months, at the rate of 14 ounces per day: how long will the supply last, if the garrison be reinforced by 500 men?

4. A person, having an income of \$1000 per annum, spent one half of it in 9 months: how much will he have remaining at the end of a year, if he reduce this rate of expenditure one fourth?

5. If 9 tons of coal cost \$67.50, what will .375 of a ton cost?

6. The hour and minute hands of a watch are together at 12 o'clock: when will they next be together?

7. A and B can saw a cord of four-feet wood, each length into two pieces, in 2 hours; but, with the assistance of C, they can saw each length into 3 pieces, in 3 hours: how long will it take C to saw each length into 3 pieces? *Ans.* 12 hours.

8. B's age is $1\frac{3}{4}$ times the age of A, C's is $1\frac{4}{11}$ times the sum of their ages, and the sum of all their ages is 78: what is the age of each? *Ans.* A's 12, B's 21, C's 45.

9. A can do a certain piece of work in 18 days of 10 hours each; B can do $\frac{4}{9}$ as much as A, in the same number of days, by working 8 hours a day: how long will it take both, working 10 hours a day, to accomplish the work? *Ans.* $11\frac{4}{7}$ days.

10. If the wages of 20 men for 11 days amount to \$550, for how many days can 15 men be employed for \$375?

11. A man who owned $\frac{3}{8}$ of a stone quarry, sold 25 per cent. of his share for \$15000: what part of the whole quarry did he sell, and what part did he retain?

12. A and B had the same income. A saved $\frac{1}{4}$ of his; but B,

by spending $150 a year more than A, found himself, at the end of 10 years, $250 in debt: what was the income of each?

13. A has a hog weighing 300 pounds, and B has another weighing 500 pounds. C buys both hogs, weighed together, for 5 cents per pound. The three men agree that A's hog is worth $\frac{1}{2}$ cent more per pound than B's, and shall be paid for at that rate: how much per pound will A and B each receive for his hog? *Ans.* A, $5\frac{5}{16}$ cents.
B, $4\frac{13}{16}$ cents.

14. If 12 oxen eat $3\frac{1}{3}$ acres of grass in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, how many oxen will eat 24 acres in 18 weeks, the grass being at first equal on every acre, and growing uniformly? *Ans.* 36 oxen.

15. A shepherd, being asked how many sheep he had, replied: "If I had as many more, and half as many more, and two sheep and a half, I would have a hundred:" how many had he?
Ans. 39.

16. A and B have each the same sum of money. If A had $20 more, and B $10 less, A would have $2\frac{1}{3}$ times as much as B: what has each? *Ans.* 32.50

STOCKS AND BONDS.

440. Stock, or **Capital Stock,** is the general name of the capital of incorporated companies.

441. Shares are portions of the stock owned by individuals, who are termed *Stockholders.*

442. Certificates of Stock are the written instruments issued to the stockholders, signed by the proper officers, certifying the number of shares to which they are respectively entitled. These are transferable, and are bought and sold like any other property.

443. The **Par Value** of a share of stock is its original, or nominal value,—the amount specified in the certificate.

444. Stock is *above*, or *below par*, when it sells for more, or less, than its nominal or par value. When above par, it is said to be at a *premium;* when below, at a *discount.*

445. A **Dividend** is a sum divided among the stockholders from the profits of the business of a company.

446. A **Bond** is a formal written obligation under seal to pay a certain amount of money, at a specified time, usually with interest. Bonds of railroad companies and other corporations are generally issued for loans, and are secured by mortgages upon their property.

447. Bonds of the General Government, States, counties, and cities, are also issued for loans, but are secured only by public faith.

448. **Coupon Bonds** are bonds with interest certificates printed at the bottom, to be cut off and presented for payment, when due. These bonds, being made payable to the holder, are negotiable by delivery.

449. **Registered Bonds** are bonds payable to the order of the holder or owner, and can be transferred only by assignment duly acknowledged.

450. The term *Stocks* is frequently applied to Government bonds, as well as to the shares of incorporated companies.

451. The market value of stocks is generally quoted by the per cent. of their par value which they bring. For example, if shares of the par value of $100 sell at 87 per cent., they are quoted "87," and if at 112 per cent., they are quoted "112." So, if the par value is $50 per share, and the market value is $49, the quotation will be "98." There are exceptions to this rule. In some cities, the quotations signify the actual prices of the shares.

452. **Consols** is a term abbreviated from the words *Consolidated Annuities,* in the Act of Parliament consolidating the Government bonds of Great Britain, which had been issued at various times for money borrowed. They bear interest at the rate of 3 per cent. per annum, payable semi-annually. These bonds are payable only at the option of the Government, and are, therefore, practically, perpetual annuities.

GENERAL INFORMATION

REGARDING U. S. GOVERNMENT SECURITIES.

453. The following are descriptions of the Government bonds issued under various acts of Congress:—

454. 6's of '81, dated in 1861, and redeemable 20 years from January 1st and July 1st of that year. They are called 1881's, because they are due in the year 1881. The interest on them is 6 per cent. in gold, payable January 1st and July 1st. There are three series of these bonds; to wit:—

1*st Series*, issued under Act of February 8th, 1861.
2*d Series*, issued under Acts of July 17th and August 5th, 1861.
3*d Series*, issued under Act of March 3d, 1863.

455. 5-20's of 1862, sometimes called *old*, because they were the first of these popular bonds issued, and designated 5-20's, from the time they have to run. They bear date May 1st, 1862, are redeemable in *five* years, or May 1st, 1867, and payable in *twenty* years, or May 1st, 1882. The interest is 6 per cent., in gold, payable May 1st and Nov. 1st.

The coupon bonds of this issue sell higher than the other 5-20's, because, having been the first to be placed upon the foreign market, they were largely sold abroad, and are always in demand for remittance and investment there.

These bonds were issued under Act of February 25th, 1862.

456. 5-20's of 1864, dated November 1st, 1864; redeemable after November 1st, 1869; payable November 1st, 1884. They bear interest at 6 per cent., in gold, due May 1st and November 1st.

These bonds were issued under Acts of March 3d and June 30th, 1864.

457. 5-20's of 1865, November issue. These bonds bear date November 1st, 1865; are redeemable after November 1st, 1870, and payable November 1st, 1885. The interest is 6 per cent., in gold, due May 1st and November 1st. The coupon bonds of this issue command, generally, a higher premium than the bonds of 1864, in consequence of foreign demand.

They were issued under Act of March 3d, 1865.

458. 5-20's of 1865, July issue. These bonds were issued in exchange for 7-30's, and are dated July 1st, 1865. They are redeemable after July

.st, 1870, and payable July 1st, 1885. They bear 6 per cent. interest, in gold, payable January 1st and July 1st.

They were authorized by Act of March 3d, 1865.

459. **5-20's of 1867,** issued also in exchange for 7-30's, dated July 1st, 1867; redeemable after July 1st, 1872, and payable July 1st, 1887. The interest on them is 6 per cent., in gold, due January 1st and July 1st.

These were also authorized by Act of March 3d, 1865.

460. **5-20's of 1868.** These bonds were also issued by authority of the Act of March 3d, 1865, in exchange for 7-30's. They are dated July 1st, 1868; redeemable after July 1st, 1873, and payable July 1st, 1888. The interest is 6 per cent. in gold, due January 1st and July 1st.

461. **10-40 Bonds.** These bonds are so named from the time they have to run, being redeemable after *ten* years, and payable in *forty* years from their date, March 1st, 1864. The interest is 5 per cent., in gold, payable on the $500 and $1000 coupon bonds, and on all the registered bonds, March 1st and September 1st, and on the $100 and $50 coupons, yearly, on March 1st. They were authorized by the Act of March 3d, 1864.

462. **U. S. Pacific Railroad Currency Sixes.** These were issued by the Government to the companies chartered by Congress to construct railroads to and from the Pacific Coast, and on the completion of twenty miles of track, at the rate of $16,000, $32,000, and $48,000 per mile, according to the difficulty of construction. They bear 6 per cent. interest, in currency, payable January 1st, and July 1st and 16th. They are all registered bonds of $1000, $5000, and $10,000, and are redeemable in thirty years from the date of their issue.

463. The **7-30 Treasury Notes,** issued in 1864 and 1865, were payable in three years, or then convertible, at the option of the holders, into bonds, and have nearly or quite all been redeemed, or converted, and are no longer on the market. They were called *Seven-Thirties*, from the rate of interest they bore; namely, 7$\frac{3}{10}$ per cent. per annum. The interest was paid semi-annually.

BONDS OF THE FUNDED LOAN.

464. The Funded Loan of the United States of 1880, 1885, and 1900, under an "Act to authorize the refunding of the national debt," approved July 14th, 1870, comprises three classes of bonds, namely—

1st. Bonds to the amount of $200,000,000, payable at the pleasure of the United States after ten years from the date of their issue, and bearing interest, payable semi-annually in coin, at the rate of 5 per cent. per annum.

2d. Bonds to the amount of $300,000,000, payable in like manner after fifteen years, with interest semi-annually, in coin, at 4½ per cent. per annum.

3d. Bonds to the amount of $1,000,000,000, payable in like manner after thirty years, with interest semi-annually, in coin, at 4 per cent. per annum.

An amendment to this Act, approved January 20, 1871, authorized the increase of the 5 per cent. bonds to $500,000,000, and provided that the interest on all these bonds may, at the discretion of the Secretary of the Treasury, be made payable quarter-yearly.

These bonds are issued to subscribers to the National Loan, in accordance with the terms of a Circular issued by the Treasury Department, October 8, 1870.

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465. All gold-bearing bonds are either coupon or registered.

466. Coupon bonds are issued in denominations of $50, $100, $500, and $1000; registered the same, with the additional denominations of $5000 and $10000.

467. Registered bonds are sold and transferred by indorsement, which may be in blank, with an acknowledgment before a United States officer, according to directions on the back of the bonds.

468. Coupon bonds can be converted into registered bonds of the same loan, by sending the coupon bonds to the Secretary of the Treasury, with a letter detailing the amounts, numbers, date of the Act of Congress authorizing the loan, and stating in whose name the bonds are to be issued, and where interest is to be paid. Always inclose bonds of different loans in separate letters.

469. Letters relating to the transfer of registered bonds, or payment of interest on the same, should be addressed to the Register of the Treasury. Powers of attorney to draw interest should be addressed to the First Auditor of the Treasury.

STOCK INVESTMENTS AND DIVIDENDS.

470. The problems presented under this head relate to the finding of the cost of stocks at given quotations; the percentage of gain or loss on purchases and sales; the per cent. of gain derived from dividends; and the comparative productiveness of different investments.

471. The market value of stocks and bonds depends mainly upon their character as securities, and the income they produce, as compared with other investments. For instance, if a certain stock yields an annual income of 12 per cent., while the average rate of interest obtained by loans upon other securities is 6 per cent., the stock will, other things being equal, rise above par. On the contrary, if the stock is liable to considerable fluctuations, is uncertain as a permanent security, and produces an income less than the average rate of interest, it will fall below par. Independent of these considerations, however, stocks, as well as bonds bearing a fixed rate of interest, are subject to fluctuations, arising from speculation, the state of finances, and other causes.

CASE I.

472. To find the value of Stock when at a premium or discount.

RULE.—*Multiply the number of shares by the price per share;* or,

Multiply the nominal value by the price per unit.

EXAMPLES.

1. Bought 100 shares of Michigan Central R. R. stock, par value $100, at 8 per cent. premium: what did the stock cost me?

SOLUTION. 100 × 108 = 10800
Or, $10000 × 1.08 = $10800. *Ans.* $10800.

2. A broker purchased for me 50 shares of Erie R. R. stock, par $100, at 27½ per cent., and charged me ¼ per cent. of the par value for brokerage: what was the cost of the stock?

Ans. $1387.50.

NOTE.—As the brokerage and cost are both to be computed by percentage on the par value, the rate of brokerage may be added to the price. Thus, 27½% + ¼% = 27¾%; or, the cost per share is $27¾.

3. How much money must I place in the hands of my broker to purchase ten $500 U. S. 5–20 bonds, at 106, allowing him ⅛% brokerage on the par value?

4. What will be the cost of 100 shares Reading R. R. stock, at 49⅛ per share, the par being $50, and brokerage ¼%?

CASE II.

473. **To find the rate per cent. of the income on any investment.**

RULE.—*Annex two ciphers to the amount of the income, and divide the result by the amount of the investment.*

EXAMPLES.

1. Received a dividend of 10% per annum upon a certain stock, for which I paid $106 per share, par value $100: what was my income per cent. upon the investment?

SOLUTION. $1000 \div 106 = 9\frac{23}{53}$. *Ans.*, $9\frac{23}{53}$% per annum.

2. Bought $10000 U. S. 5–20 bonds, at $107\frac{1}{2}$, interest payable in gold, at 6% per annum: what rate per cent. in currency will my annual income be upon this investment, if I sell the gold for currency, at 12% premium? *Ans.* $6\frac{54}{215}$%.

3. If an annual dividend of $12 per share be received on Camden and Amboy R. R. stock, purchased at $115\frac{1}{2}$, par $100, what is the rate per cent.?

CASE III.

474. **To find the price at which Stock must be purchased to yield a given rate of income on the investment.**

RULE.—*Annex two ciphers to the rate which the stock produces, and divide the result by the required rate.*

EXAMPLES.

1. The U. S. 10–40 bonds bear 5% interest in gold: at what price per hundred dollars must they be purchased to yield 6% on the investment in gold?

SOLUTION. $500 \div 6 = 83\frac{1}{3}$. *Ans.* $83\frac{1}{3}$.

2. If a dividend of 18% per annum is paid upon a certain stock, at how much above par may it be purchased so that the person investing may realize 6% on his investment?

Ans. 200%.

3. The U. S. 5-20's bear 6% interest, in gold: at what price must they be bought, to yield 7% in currency, when gold commands a premium of 12 per cent.?

CASE IV.

475. To find the income that any given investment will produce.

RULE.—*Find the amount of stock which the investment will purchase, and then compute the income at the given rate upon the par value.*

EXAMPLES.

1. What income per annum, in gold, will be obtained by investing $11487.50 in U. S. 5-20's, at 114⅞?

SOLUTION. 11487.50 ÷ 1.14⅞ = 10,000,
6% of $10,000 = $600, *Ans.*

2. Which will produce the greater income upon the same investment, Pennsylvania 6 per cent. bonds, interest payable in currency, purchased at 101½, or U. S. 10-40's, interest at 5%, payable in gold, purchased at 111½, provided the gold commands 10% premium?

CASE V.

476. To find what sum must be invested to secure a given income.

RULE.—I. *Divide the given income by the rate of income produced by the stock; the quotient will be the amount of stock required, at its par value.*

II. *Multiply this amount by the cost of one dollar of the stock; the product will be the required investment.*

EXAMPLES.

1. I wish to invest in N. Y. Central R. R. bonds, bearing 6% interest, a sufficient sum of money to secure an income of $1500.

If the bonds can be purchased at 95% of their par value, how much money must I invest?

SOLUTION. $1500 \div .06 = 25000.$
$25000 \times .95 = 23750.$ *Ans.* $23750.

2. How much must I invest in Camden and Amboy R. R. bonds, bearing 6% interest, at 93¾, brokerage ¼%, to secure an income of $2400? *Ans.* $37600.

THE STOCK EXCHANGE.

477. A **Stock Exchange**, or a **Board of Brokers**, is an association of brokers organized for the purpose of transacting the business of buying and selling stocks and bonds.

478. Exchanges are established in the leading cities, and, through the members, acting as agents for their customers, and on their own account, sales and purchases of stocks are mostly made.

479. Every association prescribes its own rules and regulations, which are stringent, and rigidly enforced. Members are elected by ballot. In New York, ten black balls exclude an applicant; in Philadelphia, five.

480. The qualifications for membership are, in general, good character and standing, and solvency. A certain term of residence in the place, and a steady prosecution of the business of a stock and exchange broker, for a specified period, are also requisite.

481. The fee for admission in New York is $10,000; in Philadelphia, $5000; in Boston, $2000. A membership is transferable, and passes by descent, in case of the death of the holder.

482. The word of a member with regard to any contract, made during the sessions, must be as good as his bond.

483. A member failing to comply with his contracts, loses the privileges of the Board, and, in case of insolvency, he must make satisfactory arrangements with his creditors, in accordance with the rules, before he can be reinstated.

484. The Board holds daily sessions, at which the stocks in the market are called by the president, or clerk. "Bids" and "offers" are made, and the word "sold," or "take," binds the bargain, which is recorded by the secretary.

485. The rates of commission that may be charged for buying and selling are fixed by the Boards. ¼%, or 25 cents on every $100, is the usual charge on Stocks and Bonds, except United States Government Securities, which may be dealt in at ⅛% on their par value; and except stocks selling at $5, or under, per share, on which a charge of from 2½ to 6¼ cents per share may be made.

STOCK EXCHANGE TERMS AND PHRASES.

486. To "sell short," is to contract to deliver a certain amount of stock, at a fixed price, within a specified time, when the seller has not the stock on hand. The "Shorts" are nicknamed "Bears," from the habit of these animals to tear, or pull down with their claws. They sell at the ruling prices when they believe there will be a decline in the market, and then operate to depress prices as much as possible, so as to buy at a lower figure, deliver the stock, and pocket the "difference."

To "buy long," is to contract to purchase stock, deliverable within a stipulated time, in anticipation of a rise in the market. The "Longs" are nicknamed "Bulls," from the propensity of these animals to toss with their horns. Their interest is to raise prices as much as possible, in order to sell and make a profit.

A "corner" is the result of certain operations between these opposing parties. When it becomes known that there is a large number of "short contracts" out in a certain stock, advantage is taken of the fact by the buyers, who purchase all the floating shares they can get hold of, so that when the time arrives for the fulfillment of their contracts, the holders have the "Shorts" at their mercy. The latter are compelled to purchase at greatly advanced prices, and are "cornered," unless by sharp practice they can "break down the corner," when their opponents are beaten, and lose by the operation.

A "time sale," or "purchase," made simply with a view to speculate in the "margin," or rise or fall in prices, without any design to actually deliver or take the stock, requires no investment, except to cover the margin, and even this is not always demanded of known and responsible parties. A contract is made to buy, for instance, a certain number of shares of stock at a fixed price, within so many days. The broker acts under instructions, and if a favorable opportunity occur before the expiration of the time, he sells and settles with his customer, allowing interest on his deposit, if he has made any, and charging him interest on the cost of the stock for the time he has "carried" it, and his brokerage. If the stock has advanced beyond these charges, the speculator makes the difference. If it has declined, he loses, and the deposit is taken to make good the loss; or, if no deposit has been made, the purchaser is called upon to pay the difference, which is transferred to the pocket of the seller. This is what is sometimes opprobriously termed "Stock Gambling."

A "put," or a "call," is a contract whereby, for the payment of a small sum of money, one dealer may require another to take or deliver, within a limited time, say one day, a certain amount of stock at a stated price.

EXPLANATION OF QUOTATIONS.

487. Stock quotations are reports of sales at the Stock Exchange. The following are selections from the New York list for Oct. 31st, 1870:—

SALES AT THE NEW YORK STOCK EXCHANGE.

WALL ST., Monday, Oct. 31, 1870.

$1000	U. S. 5-20's, c., '65	111⅞	700	Chicago & N. West. pf., c.	88⅜
5000	U. S. 5-20's, r., '65	110⅛	500	Reading R. R	100¼
25000	U. S. 6's, '81, c.,	113¾	200	do. b. 5	100⅜
11000	Cent. Pac. R. R., g. bds.	91	100	Chicago & R. I. R. R., s. 3.	110⅛
5000	do. b. c.	90⅞	100	Milw'kee & St. P., pref., s.30.	80⅜
1000	Erie 4th mtge	80¾	10	Fourth Nat. Bank	106¼
3000	Gr't West'n 1st m., '88.	89	200	N. Y. Cent. & H. R. R.	91⅜
11000	U. S. 5's, 10-40, r.,	106½	50	Michigan Cent	120⅛
5000	U. S. 5-20's, r., ex. coup.	107⅞	62	Pitts. F. W. & C., gtd.	93⅛

488. EXPLANATIONS.—"1000 U. S. 5–20's, c., '65, 111⅞," means, $1000 United States 5–20 coupon bonds, of the issue of 1865, at $111⅞ per $100, or 11⅞ per cent. premium.

"700 Chicago and N. West., pf., c., 88⅜," means, 700 shares (or $70000 worth at par) Chicago and Northwestern Railroad Preferred Stock, for cash, at 88⅜ per share, or 88⅜ per cent.

"r" means *registered bonds;* also *regular way,* in connection with stocks; that is, the stocks are sold for cash, deliverable the next day.

"c," before the prices, means *cash.*

"s," *seller's option;* thus, "s. 30" signifies that the stock was sold with the privilege, on the part of the seller, to deliver it at any time within thirty days; but he *must* deliver it on the last day.

"b," *buyer's option;* thus, "b. 60" means that the buyer has the privilege of calling for the stock at any time within sixty days; but he *must* take it at the end of that time.

"b. c.," *between calls,* or during an intermission of the Board.

"1st mtge.," or "1st m. bds.," *first mortgage bonds;* "g. bds.," *gold bonus,* "gtd.," *guaranteed,* as, by some State, city, etc.

"ex. coup.," *without coupons;* that is, the coupons for the interest due about the time of the sale are not included.

"pref.," or "pf.," *preferred stock,* or stock which has been issued to bondholders, and is entitled to dividends in preference to the common stock.

"5's," "6's," and "7's," mean bonds bearing 5, 6, and 7 per cent. interest.

489. Other abbreviations are used in stock quotations, but they will be readily understood from the above examples and explanations.

490. A notice of intention to take or deliver stock, on any day, under a time-contract, must be given previous to the meeting of the Board on that day, unless there is an agreement to the contrary. "b. 30 w. n." means *buyer* 30 *without notice.*

491. Contracts to be fulfilled within five days are considered as *without interest,* unless otherwise expressed; but when the time is more than five days, *with interest* is understood. "b. 5 & int." means *buyer* 5 *and interest;* "b. 10 flat," *buyer* 10 *without interest.*

492. In New York, stocks are always quoted by percentage; that is, the price per $100 is given, whatever the nominal value of the stock. In some other cities, the price per share is quoted.

493. In buying and selling U. S. bonds bearing interest payable in coin, the accumulated interest is included in the price named, except by special agreement.

On all "paper loans," that is, where the interest is payable in currency, as was the case with the Seven-Thirties, the interest is added to the price stated.

OPERATIONS
IN THE
STOCK EXCHANGE.

494. The following examples afford practical illustrations of the operations of buying and selling stocks, and other transactions at the Stock Exchange.

REMARKS.—In all transactions, buyer's or seller's option, over three days, it is customary with brokers to exchange contracts, in which it is stipulated that either party may call for deposits to the amount of 10 per cent. on the par value of all stocks bought or sold, to be kept good, to meet the fluctuations of the market. The deposits are usually made in a Trust Company, and receipts are issued to both parties for the amounts of their respective deposits, but are made payable only on their *joint order*.

Commission Stock Houses require deposits from their customers of the same amount, and that the same shall be kept good. If a purchase on time has been made, and the stock declines, or a sale, and it rises, a further deposit is demanded to cover the loss. Their safety depends entirely on a rigid enforcement of this rule.

The seller *gets* interest on stock sold seller's option.

The buyer *pays* interest on stock bought on time.

No interest is allowed on "short" sales for customers, as the broker usually "borrows" the stock to deliver on sale, or sells on time.

EXAMPLES.

1. Gave my broker 100 shares Delaware and Hudson Canal Co.'s stock, with orders to sell the same at the market price. He reported sale at 120. What amount did I receive, allowing him a commission of $\frac{1}{8}$%, and Government tax of 1 cent per $100?

SOLUTION.	100 shares, @..........	$120.	$12000
	Less com., $\frac{1}{8}$%..........	12.50	
	Gov. tax..............	1.20	13.70
		Ans.	$11986.30

2. Philadelphia, September 2d. Deposited with my broker $500, and instructed him to buy 200 shares Reading R. R. stock, at 48½, b. 30. He made the purchase the same day, and sold the stock March 20, for cash, at 49½. His charges were,—for brokerage ¼%, and interest at 6%; and he allowed me interest on my deposit. What was my profit?

SOLUTION.	200 shares, @ 48½	$9700.
	Interest on same, 18 days	29.10
	Brokerage, on purchase and sale	50.00
		$9779.10
	Less interest on deposit, $500, 18 days	1.50
	Net cost	$9777.60
	Proceeds of sale	9900.00
	Profit	$122.40

3. My broker purchased $10000 U. S. 5-20's of 1867, at 109. Paid him ⅛% commission. What was the amount of my bill?

Ans. $10912.50.

4. I had 100 shares of Erie R. R. stock in a distant city, which I desired to have sold at the market price, which was 72, fearing it would decline materially before I could receive it. My broker sold the same, April 2d, at 72, s. 30, and delivered it at the end of the option. What did the stock produce me?

SOLUTION.	March 2, 100 Erie, @ 72	$7200
	Interest on contract, 30 days, at 6%	36
		$7236
	Less com., $12.50; Gov. tax, 73c.	13.23
	Net	$7222.77, *Ans.*

5. The account of A. Burroughs, on the books of Jones & Co., stands as follows:—

Dr. A. BURROUGHS. **Cr.**

Bo't	100 N. Y. Cent.,	@ 115	$11500	By Cash, dep.	$6000.
	200 St. Paul,	" 100	20000		
	300 Lake Shore,	" 112	33600		
	200 Read. (par. 50)	" 110	11000		

N. Y. Central is selling at 90; Reading at 96; St. Paul at 95; and Lake Shore at 93. What must Jones & Co. call from A. Burroughs to cover the loss, and make the margin good?

SOLUTION.

100 N. Y. Central,	@ 90,	will bring	$9000
200 St. Paul,	" 95,	" "	19000
300 Lake Shore,	" 93,	" "	27900
200 Reading,	" 96, (par. 50),	" "	9600
			$55500
		Add Cash received......	6000
			$71500
		Amount of the debit side of the account.....	76100
		Loss, or decline	$4600
		10% on 700 full shares	7000
		Amount to be called for	$11600, *Ans.*

6. The account of James Smith appears on the books of Jones & Co. as "short."

Cr. 200 Erie, @ 60, $12000.
400 Lake Shore, @ 90, 36000.

Erie is now selling at 75, and Lake Shore at 98. What amount must Smith deposit to give his brokers a full 10% margin?

SOLUTION.

200 Erie,	@ 75 =	$15000
400 Lake Shore,	@ 98 =	39200
		$54200
Amount received on sales,		48000
Loss,		$6200
10% margin on 600 shares,		6000

Amount due from Smith, $12200, *Ans.*

7. My broker in New York purchased for me, March 2, 100 shares N. Y. Central and Hudson stock, at 92. Deposited with him 10% margin, $1000. He sold out March 20, at 97. Interest was charged at 7%, and commission $\frac{1}{8}$%: what did I make?

SOLUTION.

Cost of Purchase, 100 shares @ 92, and com. ⅛%,		$9212.50
Interest on same, 18 days, at 7%,		32.25
Total...............................		$9244.75
PROCEEDS OF SALE, 100 shares @ 97....	$9700.00	
Int. on deposit, $1000, 18 days,	3.50	
	$9703.50	
Less com., $12.50, and Gov. tax, .97...	13.47	$9690.03
Net profit......................		$445.28, *Ans.*

8. If, in the case presented by the last example, N. Y. C. & H. stock had, under a panic, declined on the 10th of March to 81, what amount of cash would my broker require of me, to protect himself, and hold 10% margin, exclusive of interest? *Ans.* $1112.50.

9. My broker in Philadelphia bought, April 14, 100 shares Penn. R. R. stock, at 57½, b. 30, and sold it the same day, s. 30. No deposits were made, and settlement was effected April 24. Required my gain or loss, interest at 6 per cent., commission ⅛%, and revenue tax. *Ans.* Loss, $25.58.

10. Suppose, in the above case, the stock had risen to 70 on the 20th of April, what amount of deposit must my broker require of the party of whom he purchased it, to render the transaction secure? *Ans.* $2250.

11. Bought, Nov. 1, 200 shares Harlem R. R. stock (par 50), at 134. Sold the same Nov. 10, b. 30, at 134½. Delivered the stock on the expiration of the contract. Reckoning interest at 6% on the cost of the stock and the commission, commission at ⅛%, and Government tax on the sale, what was my gain or loss? *Ans.* Gain, $3.63.

12. New York, Oct. 2. Sold short 200 Reading (par 50), at 98, seller 30 days. On the 1st of Nov., was obliged to buy the stock at 100⅜. How did I stand on settlement, paying ⅛% commission, and interest at 6 per cent.? *Ans.* I had lost $214.48.

13. Bought, reg., Jan. 2, 500 Wabash, at 49. Held the same for ten days, on a declining market. My broker informed me

that a further decline was probable, and advised me to "sell out," and "go short." I sold my 500 shares, at 39, and also 1000 additional shares, at 38, *short.* The 1000 shares I bought back in five days, at 32⅝. Without reckoning interest on my account, or revenue tax, paying my broker the usual commission of ⅛% on each lot, what did I make, or lose? *Ans.* No gain, or loss.

NOTE.—The sale of 1000 shares, at 38, and the re-purchase of the same, at 32⅝, covered the loss on the 500 shares, and all commissions.

14. Bought, May 2, 100 Erie, at 50. Sold, May 2, 100 N. Y. Cent. at 94, short. May 17, bought 100 N. Y. Cent., at 97, and sold 100 Erie, at 54: what was my gain, or loss, paying ⅛% commission, but no interest or revenue tax? *Ans.* Gain, $50.

15. My broker bought, April 2, 100 Lake Shore, at 96. I deposited with him, the same day, $1000. April 10, the company paid a dividend of 5 per cent. Sold my stock April 30, at par: what was my gain, interest at 6%, commission ⅛%, and no revenue tax? *Ans.* $836.48.

16. New York, Oct. 4. A broker purchased, on his own account, 100 Ft. Wayne, at 96, s. 30. The stock was delivered to him Oct. 19. Oct. 13, the company paid a dividend of 4 per cent. For what sum did he draw his check to pay for the same, Oct. 19, paying interest on the stock, and receiving interest on the dividend, at 6%, but having no commission to pay? *Ans.* $9223.60.

NOTE.—Stock purchased before the payment of a dividend, but not delivered till after, carries the dividend.

17. A broker sold, on his own account, June 1, 100 N. Y. Central, at 90, s. 60. The company paid a 5 per cent. dividend June 15. He expects to deliver the stock July 1. For what amount shall he make a bill, interest at 6 per cent.? *Ans.* $8543.67.

NOTE.—In this case the buyer is entitled to the dividend, it having been paid after the purchase, and before delivery.

18. A broker, to raise money, loaned 200 N. Y. Central, at 96. The stock advanced to 110. The borrower of the stock failed. How much did the broker lose? *Ans.* $2800.

19. A broker, having purchased for customers more stocks than he could carry, was forced, in a pressure, to "turn" them for one day, at 1%, to raise money, and save a failure. He sold 100 Erie, at 40, and 500 Wabash, at 50, cash. He bought them back, 100 Erie, at 41, and 500 Wabash, at 51, regular way: what did it cost him? *Ans.* \$600.

495. Mode of estimating the value of stocks upon which instalments are due, when quoted at a certain price.

20. Bought 100 shares Chicago and Alton scrip stock, at 108, par value \$100 (\$75 paid in): what did I pay?

SOLUTION. Quoted price, \$108 × 100 \$10800
Instalment due, \$25 × 100 2500
Cost, \$8300, *Ans.*

$$\text{Or, } \$\overline{108 - 25} \times 100 = \$8300.$$

RULE.—*Find the difference between the value at the quoted price, and the amount not paid, or instalments due.*

AMERICAN STOCKS AND BONDS IN EUROPE.

496. The three leading markets, in Europe, for United States Stocks and Bonds, are London, Paris, and Frankfort. At these and other places on the continent they find a ready sale, and the prices which they command are daily reported. It is therefore important to understand the quotations, and to be able to compare them with our own.

497. The quotations are the prices of the stocks or bonds per \$100 in gold, but are made with reference to an assumed or conventional par of exchange. The real prices, in our funds, are ascertained by comparisons and calculations.

498. At London, United States Bonds, and the majority of the Railroad Stocks and Bonds, are quoted at an exchange of 4*s.* 6*d.* for \$1, at the nominal par of \$4.44$\frac{4}{9}$ for £1 sterling. As this is below the actual par, about 9½% must be added to bring it to par, while the current rate of exchange may be more or less than 9½% premium, causing a corresponding variation in the estimate.

A few American Railroad stocks and bonds are quoted at 4*s.* 2*d.* for $1, which makes the pound $4.80, or 8% above the nominal par; and some at 4*s.* 4*d.*

497. At Paris, the quotations are made at the conventional par of 5 francs for $1, while exchange usually sells here below this rate, or more than 5 francs are given for $1.

498. At Frankfort, the basis is 2½ florins for $1, or a little below the true par. The average rate of exchange on Frankfort is between 40 and 41 cents per florin.

499. Sales are made at London and Paris, "flat;" that is, the accrued interest goes with the bonds at the quoted price. At Frankfort, the interest is not included, and must be added.

500. From these explanations it will be seen that, in comparing any foreign quotation of stocks and bonds with our own, it is necessary to take into consideration the basis of the quotation, the rate of exchange, and the difference in price between gold and our currency.

501. Bankers and Brokers, in order to determine the availability of bonds for remittances, estimate the quoted price at the *buying rate* of exchange.

502. To ascertain the proceeds of a sale, the cost of remittance, commission, insurance, etc., must be deducted; and if drafts are sold against the proceeds, the *selling rate* of exchange should be taken in the calculation.

FOREIGN QUOTATIONS, PER CABLE.

LONDON, JULY 17, 1871, 5 P.M. American Securities closed;—Five-twenty Bonds, 1862, 92½; 1865, 92¼; 1867, 91¼; Ten-forties, 91⅜.

FRANKFORT, JULY 17, EVENING. United States Five-twenty Bonds, 96⅛, for the issue of 1862.

EXAMPLES.

1. Required the equivalent price, in our currency, of U. S. 5-20 bonds, sold in London at 92½, reckoning gold at 110.

SOLUTION.

92½ × 1.09½ = 101.29, percentage, gold.
101.29 × 1.10 = 111.41+, percentage, currency. *Ans.*

2. What rate of exchange here would correspond with the London quotation of 91½ for U. S. Bonds?

SOLUTION.

$4.8665 × .91½ × 1.09½ = $4.875. *Ans.*

Note.—In solving all similar problems, the table on the following page will be found very convenient. Thus, the gold equivalent of 91½ is shown by the table to be 100.19; and the corresponding rate of exchange is $4.8665 × 1.0019 = $4.875+

3. What would be realized, in currency, per $100, by a sale of U. S. bonds at Frankfort, July 17, at 96⅛, gold at 112½, and exchange 41 cents per florin?

SOLUTION.

96.125.		243.46	
1.26	Int. 77 days.	41	Exchange value.
97.385		24346	
2½		97384	
194770		8)99.8186,	Gold.
48692		12.47	12½% for currency.
243.462,	Florins.	$112.29,	*Ans.*

4. Suppose the Paris quotation to be 104½, what would be the equivalent in U. S. currency, at 5 francs for $1, exchange at 5.12½, and gold at 112½?

SOLUTION.	104½		
	5		
	5.12)5522.5	($101.95,	Gold.
	5125	12.74,	12½% for currency.
	10000	$114.69,	*Ans.*
	5125		
	48750		
	46125		
	26250		

Table showing the true percentage, corresponding to the quotation in London, of United States Bonds in fictitious percentage.

PERCENTAGE	0.	$\frac{1}{8}$.	$\frac{1}{4}$.	$\frac{3}{8}$.	$\frac{1}{2}$.	$\frac{5}{8}$	$\frac{3}{4}$.	$\frac{7}{8}$
80	87.60	87.74	87.87	88.01	88.15	88.28	88.42	88.56
81	88.69	88.83	88.97	89.11	89.24	89.38	89.52	89.65
82	89.79	89.93	90.06	90.20	90.34	90.47	90.61	90.75
83	90.88	91.02	91.16	91.30	91.43	91.57	91.71	91.84
84	91.98	92.12	92.25	92.39	92.53	92.66	92.80	92.94
85	93.07	93.21	93.35	93.49	93.62	93.76	93.90	94.03
86	94.17	94.31	94.44	94.58	94.72	94.85	94.99	95.13
87	95.26	95.40	95.54	95.68	95.81	95.95	96.09	96.22
88	96.36	96.50	96.63	96.77	96.91	97.04	97.18	97.32
89	97.45	97.59	97.73	97.87	98.00	98.14	98.28	98.41
90	98.55	98.69	98.82	98.96	99.10	99.23	99.37	99.51
91	99.64	99.78	99.92	100.06	100.19	100.33	100.47	100.60
92	100.74	100.88	101.01	101.15	101.29	101.42	101.56	101.70
93	101.83	101.97	102.11	102.25	102.38	102.52	102.66	102.79
94	102.93	103.07	103.20	103.34	103.48	103.61	103.75	103.89
95	104.02	104.16	104.30	104.44	104.57	104.71	105.85	104.98
96	105.12	105.26	105.39	105.53	105.67	105.80	105.94	106.08
97	106.21	106.35	106.49	106.63	106.76	106.90	107.04	107.17
98	107.31	107.45	107.58	107.72	107.86	107.99	108.13	108.27
99	108.40	108.54	108.68	108.82	108.95	109.09	109.23	109.36
100	109.50	109.64	109.77	109.91	110.05	110.18	110.32	110.46

NOTE 1.—If a bond of the United States is sold at actual par in London, the *quotation* of the bond would be 91$\frac{5}{16}$ per cent. only. For the real par, say of a one-thousand-dollar bond (at \$4.8665 per £), being stated decimally £205.486, and the nominal par at 54 pence per dollar being £225, the London dealer by quoting 91$\frac{5}{16}$ per cent. on the latter sum arrives at the true, or nearly true, par value.

2. In the above Table, column 1 shows the fictitious percentage without fractions; column 2, the true percentage corresponding thereto based upon the real par as calculated on the units of coinage in Great Britain and the United States. The subsequent columns indicate the true percentage corresponding to successive increases of one-eighth per cent. in the fictitious rate. For example, if United States new 5's are quoted in London at 91$\frac{1}{4}$, opposite 91 and under $\frac{1}{4}$ will be found 99$\frac{92}{100}$ per cent., or eight-hundredths of one per cent. below the true par.

PARTNERSHIP AVERAGE.

503. A Partnership is the union or association of two or more persons, formed for the purpose of carrying on any kind of business for their mutual benefit.

504. The individuals forming the partnership, taken collectively, are styled the *Firm;* and they usually take a firm name; as, Jones & Smith; Brown, Williams & Co.

505. The money, merchandise, or other property, which they invest, is called their *Capital Stock,* or *Stock in Trade.*

506. Property of all kinds belonging to a firm, and debts due them, are also called their *Resources, Effects,* or *Assets.* The debts they owe are termed their *Liabilities.*

507. Net Capital is the excess of resources over liabilities; *Net Insolvency,* the excess of liabilities over resources.

508. Partnership Average is the process of finding the share of the whole gain or loss which is to be apportioned to each member of a firm, on a final settlement, or at the expiration of a year, or other period of time.

509. Partnership Average is of two kinds; namely, *Simple* and *Compound.*

510. Simple Average applies when the several shares of capital have been invested for the same length of time, and the gains or losses are to be divided proportionally to the investments.

511. Compound Average applies when the shares of capital have been invested for different periods of time, and the gains and losses are to be divided according to the average investments; that is, in proportion to the several *amounts* and the *times* for which they have been employed.

512. Gains and losses are divided according to agreement, upon the basis of the amounts of capital invested by the partners respectively, or in certain stipulated proportions. Frequently one partner furnishes all the capital, while another participates in the gains. So the gains and losses are often shared equally, though the partners may have contributed unequally to the capital stock. In some cases, each partner is allowed interest on his capital, and is charged with interest on all sums withdrawn.

SIMPLE PARTNERSHIP AVERAGE.

513. To divide gains and losses between copartners in proportion to their investments, when employed for the same period of time.

EXAMPLES.

1. H. W. Barton and William Woods engaged in business as partners, and agreed to share the gains and losses in proportion to their investments. Barton invested $9000; Woods, $7000. They gained $2245.62: what was each one's share?

SOLUTION

$$\begin{array}{rl} \$9000 & \\ 7000 & \\ \hline \$16000 & \text{whole capital.} \end{array}$$

$\frac{9000}{16000} = \frac{9}{16}$, B's fractional part of the capital.

$\frac{7000}{16000} = \frac{7}{16}$, W's fractional part of the capital.

$\frac{9}{16}$ of $2245.62 = $1263.16 +, B's share of gain.

$\frac{7}{16}$ of $2245.62 = $982.46 —, W's share of gain.

Or, *by Proportion:*

$16000 : $9000 : : $2245.62 : $1263.16 +, B's share.

$16000 : $7000 : : $2245.62 : $982.46 +, W's share.

ANALYSIS.—The whole capital being $16000, B's share is $\frac{9000}{16000} = \frac{9}{16}$; and W's share is $\frac{7000}{16000} = \frac{7}{16}$. And since the gain is to be divided in proportion to the investments, B is entitled to $\frac{9}{16}$ and W to $\frac{7}{16}$ of $2245.62, the whole gain.

By proportion, $16000 : $9000 expresses the ratio of the whole capital to B's investment; and $16000 : $7000 the ratio of the whole to W's share; and, making the whole gain $2245.62, the third term of a proportion in each case, and finding the fourth term, we have the same result as before. Hence the following

RULE.—*Multiply the whole net gain or loss by each partner's capital, and divide the product by the joint capital.* Or, by proportion, *The whole capital is to each partner's share, as the whole gain or loss is to his share of the gain or loss.*

NOTES.—1. We may first find the per cent. of gain or loss to be shared by all the partners, by dividing the whole gain by the whole capital. That per cent. of each one's capital will be his share of the gain, or loss. Dividends of incorporated companies are computed in this way.

2. The difference between the net capital on commencing, and the net capital on closing, is the *net gain*, when the capital has *increased*, and the *net loss*, when it has *decreased.*

3. The net capital of any partner, at the expiration of any term, is his investment plus his share of gain, or minus his share of loss.

4. If a firm, or any member, be insolvent at the commencement, a gain will reduce the insolvency, and must be subtracted to show the net insolvency at closing. A loss will increase it, and must be added. Whenever the gain exceeds such net insolvency, the excess represents net capital.

514. When each partner is to receive or sustain a specified *fractional* part of the profit or loss, the whole gain or loss multiplied by the fraction designating such part, will give his share, as in the following example :—

EXAMPLE 2. At the expiration of a year from the commencement of their business, Morton, Clark & Co., present the following statement of their resources, liabilities, and investments :—

RESOURCES.

Amount of inventory of stock............	$8475.80
" cash on hand................	4150.00
" debts due the firm..........	5472.50

LIABILITIES.

Amount of debts owed by the firm	3267.90

INVESTMENTS.

Samuel Morton invested.................	5000.00
David Clark invested....................	4000.00
Charles Wadsworth invested	2000,00

Morton is to receive $\frac{2}{5}$ of the gain; Clark, $\frac{1}{3}$, and Wardsworth, $\frac{4}{15}$: what is each partner's share?

SOLUTION.

Total capital at the end of the year.......	$18,098.30
Total liabilities.........................	3,267.90
Net capital	14,830.40
Total investment.............	11,000.00
Net gain.....................	3,830.40

Ans.	Morton's $\frac{2}{5}$ is	$1532.16
	Clark's $\frac{1}{3}$ is	1276.80
	Wadsworth's $\frac{4}{15}$ is	1021.44

Examples for Practice.

1. Find each partner's share of gain in the firm of Messrs. Jones, Johnson & Co., from the following statements:—

RESOURCES AND LIABILITIES.

Inventory of stock	$6325.74
Debts due the firm......................	8304.25
Debts owed by the firm.................	2473.40

INVESTMENTS.

Samuel Jones invested..................	2500.00
H. C. Johnson "	3000.00
T. H. Brown "	3500.00

The gains and losses are to be divided in proportion to the amounts invested.

Ans.	Samuel Jones' share is	$876.83
	H. C. Johnson's "	1052.20
	T. H. Brown's "	1227.56

NOTE.—In closing a set of books, the exact total amount of gain or loss must be entered on the Profit and Loss account, and a fraction of a cent is added to, or subtracted from any partner's share, as the case may require, to produce this result.

2. Required the gain or loss to be apportioned to each partner in the firm of Wheelock, Gould & Co., at the end of one year from the commencement of their business. The following is a memorandum of their effects, liabilities, and investments:—

Amount of stock, per inventory	$13,175.18
Cash on hand	3,500.00
Debts due to the firm	24,262.37
Debts due by the firm	8,275.45
Charles Wheelock invested	25,000.00
John Gould "	20,000.00
James Seely "	10,000.00

Each partner is to share in proportion to his capital invested.

3. S. P. Townsend, James Ross, and Hiram Bowers, engaged in trade under the firm name of Townsend, Ross & Co., and, at the expiration of their copartnership, exhibit the following statement of their affairs:—

RESOURCES.

Merchandise on hand, as per inventory	$7,138.50
Cash on hand	2,567.00
Bills receivable	4,265.00
Book accounts due the firm	3,122.62

LIABILITIES.

Bills payable outstanding	2,850.00
Book accounts against the firm	1,575.42

INVESTMENTS AND WITHDRAWALS.

J. P. Townsend invested $4,500; withdrew	$800
James Ross invested $4,000; withdrew	700
Hiram Bowers invested $2,500; withdrew	500

The gains and losses are to be divided in proportion to the original investments: what is each partner's share of the gain or loss, and his net capital at closing?

4. Find each partner's net insolvency, in the firm of George Walker & Sons, from the following statement:—

Inventory of stock	$3,456.62
Debts due to the firm	2,331.75
Debts due by the firm	17,834.32
George Walker invested	4,000.00
Henry Walker “	3,000.00
Chas. S. Walker “	2,000.00

George Walker is to have or sustain one half of the gains or losses, and Henry Walker and Chas. S. Walker are to share the remainder, in proportion to their investments.

5. A partnership for conducting a general banking business is closed with the following effects: what is each partner's share of the profits?

Bills receivable, on hand	$125,346.75
Stocks on hand, valued at	42,875.00
Cash	18,350.18
Banking House, estimated at	22,500.00
Debts due to the firm	62,450.35
Debts owed by the firm	24,368.98

INVESTMENTS.

J. R. Thompson invested	50,000
Walter Bennett “	30,000
Charles Hargrave “	25,000

Charles Hargrave is to receive ⅖ of the profits of the interest account, as his whole share of gain. The profits of the interest account amount to $87,468.32. J. R. Thompson and Walter Bennett are to share the balance of the profits, in proportion to the capital invested by each.

6. Three mechanics, S. Mason, J. Carpenter, and S. Painter, perform a certain piece of work, and furnish materials, for $1500, in partnership, with an agreement that they shall share the profits equally; but that each shall be charged $2.50 per day for lost time. They advanced for purchasing materials as follows:—Mason and Carpenter each $100, and Painter $50. On the completion of the work, Mason had lost four days' time;

Carpenter, three; and Painter, five: how much should each receive?

Ans.	Mason,	$516.66
	Carpenter,	519.17
	Painter,	464.17

7. Henry, James, and William Brown, own a pasture as tenants in common, their shares being as 4, 3, and 2. The pasture was leased for one year, at a rent of $100, and the taxes for that year were $18.75: what was each one's share of the profits?

8. Huntingdon, Parkinson & Co. close their business in insolvency. The following is a statement of their affairs, and a list of their creditors, with the amounts due them: what is each partner's share of insolvency, the gains or losses being shared in proportion to their investments; and what will each creditor receive, if the effects be distributed among them *pro rata?*

RESOURCES.

Bills receivable, available, on hand	$65,431.62
Cash	4,321.18
Debts due on account	21,343.20

LIABILITIES.

Bills payable	30,000.00
Debts due on account	95,182.30

INVESTMENTS.

Edward Huntingdon invested	20,000.00
John Parkinson "	14,000.00
Charles Walters "	12,000.00

CREDITORS.

Mulford, Reeves & Co	12,365.81
Harrison & Jameson	18,872.28
Harlan, Davidson & Co	31,711.29
Samuel Wright	41,118.80
Southworth & Co	21,114.12

9. A is engaged in business, and is insolvent. His indebtedness exceeds his resources $3500. B forms a copartnership with him, under an agreement that A shall share ½ of the gains or

losses, and B ¾, and invests $5000. At the expiration of a year, the following is an exhibit of the condition of the business:—

Total resources	$15,160.16
Total liabilities	13,275.86

What is each partner's share of gain or loss, and his net capital, or net insolvency?

Ans. to last. A's net insolvency, $3403.92.
B's net capital, 5288.22.

10. Three partners, A, B, and C, commenced business January 1, 1869, and invested $1000 each, agreeing to share the gains and losses equally. On closing their books, January 1, 1870, they found the concern insolvent, the partners' accounts standing as follows:—A's debit balance, or net insolvency, was $835.40; B's debit balance, $724.85; and C's debit balance, $945.15. They continued the business, and January 1, 1871, their balance-sheet showed the following result:—

A's account Dr.	$1835.40
B's account Dr.	1624.85
C's account Dr.	1745.15
Amount of stock, per inventory	3721.18
Cash on hand	2560.50
Debts due the firm	4216.90
Debts owed by the firm	2375.25

What was each partner's share of gain, and his capital, January 1, 1871?

Ans. A's capital, $2607.51.
B's " 2818.06.
C's " 2697.76.
A's gain, 4442.91.
B's " 4442.91.
C's " 4442.91.

NOTE.—Persons not familiar with Book-keeping are informed that the increase of the debit balances of the partners' personal accounts is caused by the withdrawal of certain sums for private use during the year. A has taken $1000; B, $900; and C, $700.

COMPOUND PARTNERSHIP AVERAGE.

515. To divide gains and losses between copartners, in proportion to their average investments, when their shares of capital have been employed for different periods of time.

EXAMPLE 1. Henry W. Parker, Samuel Warren, and Milton W. Davis, commenced business July 1, 1870, under an agreement to share the accruing gains and losses in proportion to their average investments. Parker invested, July 1, $20,000; July 19, $5,000; and July 25, he withdrew $230. Warren invested, July 1, $10,000. Davis invested, July 1, $10,000. Their net gain, August 31, is found to be $1441.94. What is each partner's share, reckoning the exact time by days?

SOLUTION.

The time from July 1 to August 31, is 61 days.

Parker's investment, July 1, $20,000, for 61 days =	$1,220,000	for 1 day.
" " " 20, 5,000, " 42 " =	210,000	"
Total......	1,430,000	"
Amount withdrawn July 25, $230, for 37 days =	8,510	"
His average investment.......	1,421,490	"
Warren's investment, July 1, $10,000, for 61 days =	610,000	"
Davis' " " 10,000, " " =	610,000	"
Total average investment........	2,641,490	"

And, by Proportion,

$2641490 : $1421490 : : $1441.94 : $775.96, Parker's share.
$2641490 : $610000 : : $1441.94 : $332.99, Warren's "
$2641490 : $610000 : : $1441.94 : $332.99, Davis' "

516. The *average investment* of a partner is a sum which, if employed for a certain period of time, would be equivalent to his actual investments for the specified periods. Hence the

RULE.—*Multiply each investment by the time it has been employed. Then, as the sum of all the products is to each product, so is the whole net gain, or loss, to each partner's share.*

In the following example the time is reckoned by months, as is usual in business, and the gain is computed by percentage.

Example 2. John Jackson, Hiram Dickerson, and William Wilson, entered into partnership, and invested, and drew out as follows:—

John Jackson invested,	July……1, 1869 ………	$1,000
" " "	November 1, 1869 ………	1,500
" " drew out,	April…..1, 1870 ………	300
Hiram Dickerson invested,	July …..1, 1869 ………	2,500
" " "	January ..1, 1870 ………	600
" " "	" ..1, 1870 ………	1,000
William Wilson invested,	July …..1, 1869 ………	3,000
" " drew out,	November 1, 1869 ………	2,000
" " " "	March ….1, 1870 ………	500

Their whole gain was $15140, and each partner shares in proportion to his amount of capital invested, and for the time it was employed. What does each partner withdraw July 1, 1870?

SOLUTION.

	Amt.	Time.			
John Jackson invested, July 1, 1869 …….	$1000 ×	12 =		$12000	
Nov. 1, " …….	1500 ×	8 =		12000	
				24000	
Drew out, April 1, 1870 ………….	300 ×	3 =		900	
Capital for 1 month ………..					$23100
Hiram Dickerson invested, July 1, 1869 ….	2500 ×	12 =		30000	
Jan. 1, 1870 ….	600 ×	6 =		3600	
June 1, " ….	1000 ×	1 =		1000	
Capital for 1 month …. ……					$34600
William Wilson invested, July 1, 1869……	3000 ×	12 =		36000	
Drew out, Nov. 1, 1869…..	$2000 ×	8 =	16000		
Mar., 1870 ……	500 ×	4 =	2000	18000	
Capital for 1 month ………..					$18000
Whole average capital for 1 month…..					$75700

$15140 ÷ 75700 = .20 per cent. of gain on the capital for 1 month.

		Gain.
Jackson's average investment	$23,100 × .20 =	$4,620
Dickerson's " "	34,600 × .20 =	6,920
Wilson's " "	18,000 × .20 =	3,600
Whole gain (proof)		$15,140
Jackson's balance of capital is		2,200
" share of profit		4,620
" net capital		6,820
Dickerson's balance of capital is		4,100
" share of profit		6,920
" net capital		11,020
Wilson's balance of capital is		500
" share of profit		3,600
" net capital		$4,100

EXAMPLES FOR PRACTICE.

1. Evans & Lincoln closed their partnership with the following effects and liabilities:—

Inventory of stock in store	$3,504.20
" lumber and coal	9,216.80
Individual accounts due the firm	2,834.61
" " owed by the firm	3,620.18
Bills payable outstanding	675.00
R. Evans invested, Jan. 1, 1866	3,500
" drew out, July 15, 1868	700
D. J. Lincoln invested, Jan. 1, 1866	3,500
" drew out, Jan. 1, 1869	1,000

What is each partner's net capital at the dissolution of the partnership, Jan. 1, 1871, the gains and losses being divided in proportion to the average investments?

2. Bigelow & Smith present the following statement of their business, January 1, 1871:—

RESOURCES.

Amount of stock, per inventory	$8,345.65
Due to the firm on personal accounts	14,641.18
Bills receivable on hand	12,340.20
Farm in Goshen, valued at	4,350.00
Mortgage on John Holmes' farm	1,800.00

LIABILITIES.

Due on personal accounts	$5,346.22
Bills payable	8,264.18

INVESTMENTS.

J. Bigelow invested, Jan. 1, 1868	$10,000
" drew out, Oct. 31, 1868	2,500
C. H. Smith invested, Jan. 1, 1868	8,000
" drew out, Jan. 1, 1869	3,500

What was each partner's share of the gain or loss, and net capital, Jan. 1, 1871, the gain or loss being divided in proportion to time and amount?

3. The copartnership of Field, Maxwell & Co. expired by limitation January 1, 1871. The following is a statement of their accounts, and the gains and losses were divided according to their average investments. What did each partner draw out?

Inventory of stock	$4.565.25
Debts due to the firm	1,562.40
Debts due by the firm	$675.00

The accounts of the partners stood upon the ledger at closing as follows:—

DR. (*Withdrawn.*)	CHESTER W. FIELD.	(*Invested.*)	CR.
1869, Oct. 1, To cash	$150	1869, Jan. 1, By cash	$500
1870, Mar. 1, "	200	" July 1, "	200
		1870, Sept. 1, "	100

DR.	SAMUEL W. MAXWELL.		CR.
1869, Oct. 1, To cash	$100	1869, Jan. 1, By cash	$600
1870, May 1, "	100	1870, " 1, "	400

Dr.	John B.	Loring.	Cr.
1870, May 1, To cash	$100	1869, Jan. 1, By cash	$400
" Oct. 1, "	50	" Oct. 1, "	200
		1870, Apr. 1, "	200
	——		——

4. On the 1st of January, the firm of Bowers & Murphy was insolvent; and on closing their books, the debit balance against Bowers was $1437.35, and the debit balance against Murphy, $927.45. They admit two partners, Thomas Wilson, who invests $4000, and Henry Brown, who invests $5000; and continue the business, agreeing that the gains and losses shall be divided as follows:—Bowers is to receive or sustain $\frac{1}{6}$; Murphy $\frac{1}{6}$; and Wilson and Brown are to share the remainder in proportion to their average investments. At the end of a year their total resources were $22,543.75, and their total liabilities $7,946.45. Bowers and Murphy had each drawn out, as per agreement, $1200, for personal expenses. Thomas Wilson drew out as follows:—April 1, $300; July 1, $500; Oct. 1, $700. Henry Brown drew out, April 1, $250; July 1, $430; Oct. 1. $600: what was the debit balance against, or credit balance in favor of, each partner at the end of the year?

5. A, B, and C, hire 10 acres of pasture for $140, in which A grazes 60 head of cattle for 20 days; B, 20 head for 40 days; and C, 30 head for 30 days. What proportion of the rent should each pay?

6. A and B enter into partnership in business, Jan. 1, 1870, under an agreement that the gains and losses shall be shared equally; but that each shall be allowed interest, at 6%, on his capital invested, and be charged with interest, at the same rate, on all sums withdrawn. A invested, January 1, $10000, and March 1, withdrew $3000. B invested, January 1, $5000, and April 1, $5000 more; and withdrew, July 1, $4000. On taking an inventory of stock, and closing their books, January 1, 1871, their Total Resources were found to be $29,148.75, and thei Total Liabilities, $12,246.25. What did each one gain by interest, and what was his whole gain and net capital at closing?

7. In the firm of Dean, Hobart & Co., Silas Dean is a special partner, and invested $25,000. Richard Hobart and John Simmons are general partners. Hobart invested, at first, $10,000, and four months afterward, $5000 more. Simmons invested, at first, $5000, and at the end of six months added $3000. Each partner is to be allowed interest on his capital out of the profits, at 7 per cent. per annum; and Hobart, being the chief manager of the business, is to receive $100 per month extra for his services, besides his interest and share of the profits. The balance of profits, or net loss, is to be divided in proportion to the average investments. Hobart was paid the $100 per month regularly for twelve months, which was charged to his account; and, besides, he drew out, at the end of six months, $1000. Simmons withdrew, at the end of eight months, $1500. At the end of twelve months, their net profits amount to $8456: what is each partner's share?

Ans.	Dean's share,		$4138.40
	Hobart's	"	3324.38
	Simmons'	"	993.22

STORAGE.

517. When goods are taken on storage by commission and forwarding merchants, they are governed in their charges by the rates established by the Board of Trade or Chamber of Commerce of the city in which the storage is made, unless a special agreement is made to the contrary. These rates are fixed at a certain price per month per barrel, box, bale, etc.

518. In most cities, a full month's storage is chargeable for any part of a month that the goods remain in store; in others, fifteen days or less are reckoned half a month, and any part of a month over fifteen days, a whole month.

519. When the charges are made according to the actual time the goods are stored, the amount of storage is computed by average. Hence, we have the following

RULES FOR COMPUTING STORAGE.

520. When the goods have been received at different dates, but no part of them *delivered* during the time for which storage is to be charged:

Multiply the number of bushels, barrels, or packages received at each date by the number of days they have been in store, and divide the sum of the products by 30. *The quotient will be the quantity for which to charge storage for one month.*

When the goods have been received at different dates, and a part of them delivered during the time:

I. *Multiply each quantity by the number of days between the date of its receipt and the date of the first delivery, or of the second receipt.*

II. *Multiply the balance, or increased quantity, by the number of days to the date of the next delivery, or receipt, and so on.*

III. *Divide the sum of the products by* 30, *and the quotient will be the average quantity in store for one month.*

EXAMPLES.

1. Received the following quantities of wheat on storage, at 1 cent a bushel per month. What is the average quantity per month, and what is the amount of the storage due April 30th?

March 6th	300	bushels.
" 24th	200	"
April 7th	200	"
" 25th	150	"

SOLUTION.

Dates.	No. bush.		Days.		Products.			
March 6th	300	×	55	=	16500	bushels	in store	for one day.
" 24th	200	×	37	=	7400	"	"	"
April 7th	200	×	23	=	4600	"	"	"
" 25th	150	×	5	=	750	"	"	"

30)29250

975, Average quantity for one month.
.01

$9.75, Amount of storage.

2. Received on storage, and delivered, the following merchandise. Settlement was made May 31st: what was the amount of the storage at 3 cents per month per barrel or bale?

Received.	*Delivered.*
March 14th, 200 bbls. flour.	April 10th, 300 bbls. flour.
" 31st, 400 " "	" 20th, 50 " pork.
April 16th, 100 " pork.	" 28th, 50 " "
" 25th, 200 bales hemp.	" 28th, 300 " flour.
May 20th, 300 bbls. flour.	May 1st, 200 bales hemp.
	" 25th, 300 bbls. flour

SOLUTION.

	Packages.		Days.		Products.
Mar. 14th, Received.........	200	×	27	=	5400, in store for one day.
" 31st, "	400	×	10	=	4000, " "
Amount...	600				
Apr. 10th, Delivered.........	300				
Balance...........	300	×	6	=	1800, " "
Apr. 16th, Received..........	100				
Amount..........	400	×	4	=	1600, " "
Apr. 20th, Delivered	50				
Balance...........	350	×	5	=	1750, " "
Apr. 25th, Received..	200				
Amount	550	×	3	=	1650, " "
Apr. 28th, Delivered.........	350				
Balance...........	200	×	3	=	600, " "
May 1st, Delivered.........	200				
Balance...........	000				
May 20th, Received..........	300	×	5	=	1500, " "
" 25th, Delivered.........	300				
	000				

30)18300,

610, Average quantity in store for 1 month.

.03

$18.30, Amount of storage.

521. The following is another method of solving the same question:—

SECOND METHOD OF SOLUTION.

Received.	*Delivered.*
Mar. 14th, 200 × 78 = 15600	April 10th, 300 × 51 = 15300
" 31st, 400 × 61 = 24400	" 20th, 50 × 41 = 2050
April 16th, 100 × 45 = 4500	" 28th, 350 × 33 = 11550
" 25th, 200 × 36 = 7200	May 1st, 200 × 30 = 6000
May 20th, 300 × 11 = 3300	" 25th, 300 × 6 = 1800
55000	36700
36700	
30)18300	

610 × .03 = $18.30, as before.

Explanation.—Here the calculation is made with reference to the goods received, as if they had all remained in store up to the date of settlement. Then each quantity delivered is multiplied by the number of days it was *out of the store.* The difference of the products is the average number of packages *in store* for one day; and the quotient found by dividing this by 30, is the same as before.

3. Received on storage and delivered, as follows:—

Received.	*Delivered.*
Oct. 4th, 1600 bush. wheat.	Oct. 8th, 750 bush. wheat.
" 17th, 900 " rye.	" 16th, 350 " "
" 26th, 490 " corn.	" 20th, 500 " rye.
Nov. 1st, 350 " oats.	Nov. 4th, 200 " "
	" 8th, 350 " oats.

What is due for storage Nov. 15th, at ½ cent a bushel per month?

4. Received on storage, and delivered, the following merchandise:—

Received.	*Delivered.*
Jan. 1st, 100 bbls. apples.	Jan. 15th, 100 bbls. apples.
" 15th, 200 " "	" 30th, 100 " "
Feb. 10th, 300 " corn meal.	Feb. 20th, the balance.

What is the amount of storage due Feb. 20th, at 2½ cents a barrel per month?

Note.—When different kinds of goods are received at different rates of storage, a separate calculation is necessary for each kind.

AVERAGE OF ACCOUNTS.

522. **Average of Accounts,** or **Equation of Payments,** is the process of finding the mean or average time for the payment, in one amount, of several debts due at different dates, without allowing interest to either debtor or creditor.

523. The time when the amount falls due by average is called the *Equated Time.*

524. The methods of averaging accounts are based upon the principle that the interest or use of any sum of money *paid before it is due,* is equivalent to the interest or use of an equal sum *retained for the same length of time after it becomes due.* The process, therefore, consists merely in finding a point of time when the *interest* on sums then overdue equals the *discount* on the sums not due, reckoning what is here termed discount as simple interest, like bank discount.

ILLUSTRATION.—Suppose A owes B $100 due in one year, and $100 due two years, without interest, the average maturity of both sums is eighteen months from the date of the first; for A will then have had the use of $100 for six months after it became due, which is equivalent to the use of $100 paid six months before it is due.

SIMPLE AVERAGE.

525. **Simple Average** is the process of averaging an account containing debit or credit items only.

EXAMPLES.

1. A owes B, Jan. 1, 1871, $1800; of which $700 is payable in 6 months; $300 in 4 months, and $800 in 18 months: when can the whole be paid without gain or loss of interest to either party?

ANALYSIS.—It is evident that, if nothing should be paid till the maturity of the debt last due, A would owe B $1800, besides the interest of $700 for

12 months, which, at 6%, is $42, and the interest of $300 for 14 months, which is $21. But, the question is, when may the whole be paid without interest? Clearly, it should be paid as long prior to the expiration of 18 months as it will take $1800 to gain the interest then due, namely, $42 + $21 = $63. The interest of $1800 for one month, at 6%, is $9; and 63 ÷ 9 = 7. Therefore, it will take $1800 7 months to gain $63 interest, and the equated time is 7 months prior to July 1, 1872, or 11 (18 — 7) months from Jan. 1, 1871, which is Dec. 1, 1871.

PRACTICAL SOLUTION.

Product Method.

$700 × 6 = $4200
300 × 4 = 1200
800 × 18 = 14400
1800)19800(11
1800
1800

Interest Method.

Int. of $700 for 6 mos. = $21.00
" " 300 " 4 " = 6.00
" " 800 " 18 " = 72.00
2)18.00 $99.00
9)99(11
99

EXPLANATION.

The *use* of $700 for 6 months = the use of $1 for 4200 months.
" " $300 for 4 months = the use of $1 for 1200 months.
" " $800 for 18 months = the use of $1 for 14400 months.
" " $1800 for 11 months = the use of $1 for 19800 months.

The use of $1 for 19800 months is equal to the use of $1800 for as many months as 1800 is contained times in 19800 = 11.

By the *Interest Method*, the total interest of the different sums for their respective terms of credit is $99, and the interest of $1800 for one month is $9. Therefore, it will take $1800 as many months to gain $99 as 9 is contained times in 99 = 11. Hence, the equated time is 11 months from Jan. 1, 1871, or Dec. 1, 1871.

The work may be shortened by subtracting 4 from all the multipliers, and also by rejecting the ciphers in the amounts. Thus,

700 × 2 = 1400 Or, 7 × 2 = 14
300 × 0 = 0000 3 × 0 = 0
800 × 14 = 11200 8 × 14 = 112
1800)12600(7 18)126(7
12600 126

This makes the first *maturity* the starting point in reckoning the time, and gives the same result. The equated time is 7 months from April 1, or Dec. 1, as before.

PROOF.

The interest of $700 from July 1 to Dec. 1, 5 months =	$17.50
The interest of $300 from May 1 to Dec. 1, 7 months =	10.50
Interest against A,	$28.50
The interest of $800 from Dec. 1, 1871, to July 1, 1872, 7 months, or, discount in favor of A,	$28.50

526. The **Focal Date** is any date assumed from which to compute the interest, or count the time. The *earliest* or *latest date* in an account, or the earliest or latest *maturity,* is usually selected.

527. The last day of the month previous to that in which the earliest date occurs, is, as will be shown, the most convenient date for this purpose.

528. In reckoning the time by days, the 1st of January (inclusive), prior to the date of any entry in the account, may be taken as the focal date.

The TIME TABLE on the following page may then be used to great advantage.

DIRECTIONS FOR USING THE TIME TABLE.

Find the number of the day of the year corresponding with the date of each entry, and place the same opposite the item, first adding the term of credit, if there be any. Then, to abridge the work of multiplying, subtract from all the multipliers so obtained the smallest one, which will be the number of days from the first day of January (inclusive) to the earliest maturity, which then becomes the *focal date.* Compute the average time as usual, and count it forward from the earliest maturity, to find the equated time.

To find the number of days from any given date to any other, *find the difference between the numbers opposite those dates.*

EXAMPLE 1. Required the number of days from March 4 to July 10.

TIME TABLE,

Showing the number of days from January 1st, inclusive, to any day in the year; also (in italics) to any day in the second year.
See directions on the preceding page.

January.			Feb'ary.			March.			April.			May.			June.			July.			August.			Sept'ber.			October.			Nov'ber.			Dec'ber.		
1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d	1st Yr.		2d
1	1	*366*	**1**	32	*397*	**1**	60	*425*	**1**	91	*456*	**1**	121	*486*	**1**	152	*517*	**1**	182	*547*	**1**	213	*578*	**1**	244	*609*	**1**	274	*639*	**1**	305	*670*	**1**	335	*700*
2	2	*367*	**2**	33	*398*	**2**	61	*426*	**2**	92	*457*	**2**	122	*487*	**2**	153	*518*	**2**	183	*548*	**2**	214	*579*	**2**	245	*610*	**2**	275	*640*	**2**	306	*671*	**2**	336	*701*
3	3	*368*	**3**	34	*399*	**3**	62	*427*	**3**	93	*458*	**3**	123	*488*	**3**	154	*519*	**3**	184	*549*	**3**	215	*580*	**3**	246	*611*	**3**	276	*641*	**3**	307	*672*	**3**	337	*702*
4	4	*369*	**4**	35	*400*	**4**	63	*428*	**4**	94	*459*	**4**	124	*489*	**4**	155	*520*	**4**	185	*550*	**4**	216	*581*	**4**	247	*612*	**4**	277	*642*	**4**	308	*673*	**4**	338	*703*
5	5	*370*	**5**	36	*401*	**5**	64	*429*	**5**	95	*460*	**5**	125	*490*	**5**	156	*521*	**5**	186	*551*	**5**	217	*582*	**5**	248	*613*	**5**	278	*643*	**5**	309	*674*	**5**	339	*704*
6	6	*371*	**6**	37	*402*	**6**	65	*430*	**6**	96	*461*	**6**	126	*491*	**6**	157	*522*	**6**	187	*552*	**6**	218	*583*	**6**	249	*614*	**6**	279	*644*	**6**	310	*675*	**6**	340	*705*
7	7	*372*	**7**	38	*403*	**7**	66	*431*	**7**	97	*462*	**7**	127	*492*	**7**	158	*523*	**7**	188	*553*	**7**	219	*584*	**8**	250	*615*	**7**	280	*645*	**7**	311	*676*	**7**	341	*706*
8	8	*373*	**8**	39	*404*	**8**	67	*432*	**8**	98	*463*	**8**	128	*493*	**8**	159	*524*	**8**	189	*554*	**8**	220	*585*	**7**	251	*616*	**8**	281	*646*	**8**	312	*677*	**8**	342	*707*
9	9	*374*	**9**	40	*405*	**9**	68	*433*	**9**	99	*464*	**9**	129	*494*	**9**	160	*525*	**9**	190	*555*	**9**	221	*586*	**9**	252	*617*	**9**	282	*647*	**9**	313	*678*	**9**	343	*708*
10	10	*375*	**10**	41	*406*	**10**	69	*434*	**10**	100	*465*	**10**	130	*495*	**10**	161	*526*	**10**	191	*556*	**10**	222	*587*	**10**	253	*618*	**10**	283	*648*	**10**	314	*679*	**10**	344	*709*
11	11	*376*	**11**	42	*407*	**11**	70	*435*	**11**	101	*466*	**11**	131	*496*	**11**	162	*527*	**11**	192	*557*	**11**	223	*588*	**11**	254	*619*	**11**	284	*649*	**11**	315	*680*	**11**	345	*710*
12	12	*377*	**12**	43	*408*	**12**	71	*436*	**12**	102	*467*	**12**	132	*497*	**12**	163	*528*	**12**	193	*558*	**12**	224	*589*	**12**	255	*620*	**12**	285	*650*	**12**	316	*681*	**12**	346	*711*
13	13	*378*	**13**	44	*409*	**13**	72	*437*	**13**	103	*468*	**13**	133	*498*	**13**	164	*529*	**13**	194	*559*	**13**	225	*590*	**13**	256	*621*	**13**	286	*651*	**13**	317	*682*	**13**	347	*712*
14	14	*379*	**14**	45	*410*	**14**	73	*438*	**14**	104	*469*	**14**	134	*499*	**14**	165	*530*	**14**	195	*560*	**14**	226	*591*	**14**	257	*622*	**14**	287	*652*	**14**	318	*683*	**14**	348	*713*
15	15	*380*	**15**	46	*411*	**15**	74	*439*	**15**	105	*470*	**15**	135	*500*	**15**	166	*531*	**15**	196	*561*	**15**	227	*592*	**15**	258	*623*	**15**	288	*653*	**15**	319	*684*	**15**	349	*714*
16	16	*381*	**16**	47	*412*	**16**	75	*440*	**16**	106	*471*	**16**	136	*501*	**16**	167	*532*	**16**	197	*562*	**16**	228	*593*	**16**	259	*624*	**16**	289	*654*	**16**	320	*685*	**16**	350	*715*
17	17	*382*	**17**	48	*413*	**17**	76	*441*	**17**	107	*472*	**17**	137	*502*	**17**	168	*533*	**17**	198	*563*	**17**	229	*594*	**17**	260	*625*	**17**	290	*655*	**17**	321	*686*	**17**	351	*716*
18	18	*383*	**18**	49	*414*	**18**	77	*442*	**18**	108	*473*	**18**	138	*503*	**18**	169	*534*	**18**	199	*564*	**18**	230	*595*	**18**	261	*626*	**18**	291	*656*	**18**	322	*687*	**18**	352	*717*
19	19	*384*	**19**	50	*415*	**19**	78	*443*	**19**	109	*474*	**19**	139	*504*	**19**	170	*535*	**19**	200	*565*	**19**	231	*596*	**19**	262	*627*	**19**	292	*657*	**19**	323	*688*	**19**	353	*718*
20	20	*385*	**20**	51	*416*	**20**	79	*444*	**20**	110	*475*	**20**	140	*505*	**20**	171	*536*	**20**	201	*566*	**20**	232	*597*	**20**	263	*628*	**20**	293	*658*	**20**	324	*689*	**20**	354	*719*
21	21	*386*	**21**	52	*417*	**21**	80	*445*	**21**	111	*476*	**21**	141	*506*	**21**	172	*537*	**21**	202	*567*	**21**	233	*598*	**21**	264	*629*	**21**	294	*659*	**21**	325	*690*	**21**	355	*720*
22	22	*387*	**22**	53	*418*	**22**	81	*446*	**22**	112	*477*	**22**	142	*507*	**22**	173	*538*	**22**	203	*568*	**22**	234	*599*	**22**	265	*630*	**22**	295	*660*	**22**	326	*691*	**22**	356	*721*
23	23	*388*	**23**	54	*419*	**23**	82	*447*	**23**	113	*478*	**23**	143	*508*	**23**	174	*539*	**23**	204	*569*	**23**	235	*600*	**23**	266	*631*	**23**	296	*661*	**23**	327	*692*	**23**	357	*722*
24	24	*389*	**24**	55	*420*	**24**	83	*448*	**24**	114	*479*	**24**	144	*509*	**24**	175	*540*	**24**	205	*570*	**24**	236	*601*	**24**	267	*632*	**24**	297	*662*	**24**	328	*693*	**24**	358	*723*
25	25	*390*	**25**	56	*421*	**25**	84	*449*	**25**	115	*480*	**25**	145	*510*	**25**	176	*541*	**25**	206	*571*	**25**	237	*602*	**25**	268	*633*	**25**	298	*663*	**25**	329	*694*	**25**	359	*724*
26	26	*391*	**26**	57	*422*	**26**	85	*450*	**26**	116	*481*	**26**	146	*511*	**26**	177	*542*	**26**	207	*572*	**26**	238	*603*	**26**	269	*634*	**26**	299	*664*	**26**	330	*695*	**26**	360	*725*
27	27	*392*	**27**	58	*423*	**27**	86	*451*	**27**	117	*482*	**27**	147	*512*	**27**	178	*543*	**27**	208	*573*	**27**	239	*604*	**27**	270	*635*	**27**	300	*665*	**27**	331	*696*	**27**	361	*726*
28	28	*393*	**28**	59	*424*	**28**	87	*452*	**28**	118	*483*	**28**	148	*513*	**28**	179	*544*	**28**	309	*574*	**28**	240	*605*	**28**	271	*336*	**28**	301	*666*	**28**	332	*697*	**28**	362	*727*
29	29	*394*				**29**	88	*453*	**29**	119	*484*	**29**	149	*514*	**29**	180	*545*	**29**	210	*575*	**29**	241	*606*	**29**	272	*637*	**29**	302	*667*	**29**	333	*698*	**29**	363	*728*
30	30	*395*				**30**	89	*454*	**30**	120	*485*	**30**	150	*515*	**30**	181	*546*	**30**	211	*576*	**30**	242	*607*	**30**	273	*638*	**30**	303	*668*	**30**	334	*699*	**30**	364	*729*
31	31	*396*				**31**	90	*455*				**31**	151	*516*				**31**	212	*577*	**31**	243	*608*				**31**	304	*669*				**31**	365	*730*

Opposite July 10, we find 191, and opposite March 4, 63; and 191 − 63 = 128, *Ans.*

2. When will a note dated Aug. 5, payable 90 days after date, mature? Adding 3 days' grace we must count forward 93 days. Opposite Aug. 5, we find 217; and 217 + 93 = 310, which number stands opposite Nov. 6, *Ans.*

3. What date will be reached by counting forward 230 days from July 20 of this year? July 20 is the 201st day of the year. 201 + 230 = 431. 431 is found, *in italics*, opposite March 7. *Ans.* March 7, next year.

In *Leap-Years* add or subtract one, as the case may require, for the 29th day of February.

2. When will the following bill become due per average?

NEW YORK, March 1, 1871.

MR. E. C. HENRY,

To H. B. CLAFLIN & CO., DR.

1871.				
Jan.	1	To Mdse., @ 3 mos.	$460	00
Feb.	12	" " " 30 days	300	00
Mar.	1	" " " 60 "	585	00
			$1345	00

It will be observed that in the preceding case the terms of credit begin at the same date, while in this example they begin at different dates. The mode of averaging is, however, the same in principle in both cases; for, by assuming a focal date, the terms of credit are always taken as beginning at the same time. The two rules given by most authors are, therefore, unnecessary.

SOLUTION.

1st. *The earliest maturity assumed as the Focal Date.*

DUE.	ITEMS.	DAYS.	PROD.
April 1,	$460	× 18 =	8280
March 14,	300	× 0	
April 30,	585	× 47 =	27495
	$1345		)35775(27−

Due 27 days after March 14, or April 10.

2d. *The latest maturity assumed as the Focal Date.*

DUE.	ITEMS.	DAYS.	PROD.
April 1,	$460	× 29 =	13340
March 14,	300	× 47 =	14100
April 30,	585	× 0	
	$1345		)27440(20+

Due 20 days before April 30, or April 10.

EXPLANATION.—We first find the time when each item becomes due, and then from the earliest or latest maturity, as a focal date, reckon the time to the maturity of each of the other items. Thus, in the first solution, the item of $300 is first due, March 14. This is taken as the focal date. From March 14 to April 1 is 18 days, and from March 14 to April 30 is 47 days. The terms of credit so found form the multipliers, as in the preceding case, though in days instead of months. Performing the multiplication, and dividing the sum of the products by the sum of the items, we find the average time to be a fraction less than 27 days; but as it is nearer 27 than 26, we call it 27. Now, is this time to be counted *forward* or *backward* from March 14? Evidently forward, for only one item is due March 14, and the average maturity of all must be later. Counting forward 27 days, we reach April 10, the equated time.

In the second solution, the latest maturity, April 30, is taken as the focal date, and the average time, or *term of discount*, is found to be 20 days. Now, the sum of $585 is due April 30, and all the other sums are then *overdue*. Consequently, interest has commenced to accumulate *against the debtor*, and the time must be counted backward. Hence, the equated time is 20 days prior to April 30, or April 10, as before.

THE SAME EXAMPLE BY THE INTEREST METHOD.

Due......	April 1,	$460.	—Interest, 18 days, =	$1.38
Focal Date,	March 14,	300.	—No interest.	
Due......	April 30,	585.	—Interest, 47 days,	4.58
		$1345		$5.96

The Interest of $1345 for 1 day, is $0.224, and 5.96 ÷ .224 = 27 +

529. The Interest and Product Methods are mathematically the same. Which is the shorter depends on circumstances. Persons not ready at computing interest, will doubtless find it better to use the Product Method; but, in general, especially when the time is in months and days, the Interest Method is to be preferred.

530. The *rate* of interest forms no element of the calculation; and, therefore, any rate may be adopted. When the time is expressed in months, 12% is a convenient rate, from the fact that the interest at this rate may be found by multiplying the principal by the number of months, and pointing off two figures for decimals. But in the great majority of the cases which occur in ordinary practice, it is better to compute the interest at 6 per cent.

531. By the Product Method, the months, with the days expressed as fractions of a month, may be taken as multipliers. The work will then amount to the same thing, substantially, as the operation of calculating the interest at 12 per cent.

3. Find the equated time for the settlement of the following account:—

MESSRS. WALKER & Co.,
In account with H. J. FIELDS & Co.

1871.				
May	10	To Mdse., @ 60 days	$450	00
July	15	" " " 30 "	850	00
Sept.	1	" " " 90 "	500	00
"	5	" " " 60 "	862	00
Oct.	20	" " " 60 "	325	00
			$2987	00

SOLUTION. 1st.—BY THE INTEREST METHOD,

At 6%, assuming April 30 as the Focal Date.

DATE.	AMOUNTS.	CREDIT.	TIME.	INTEREST.
May 10,	$450.	60	2 mos. 10 ds.	$5.25
2 July 15,	850	30	3 mos. 15 ds.	14.87
4 Sept. 1,	500	90	7 mos. 1 d.	17.58
4 " 5,	862	60	6 mos. 5 ds.	26.58
5 Oct. 20,	325	60	7 mos. 20 ds.	12.46
	$2987			$76.74

2)29.87

14.935)76.740(5 mos. 4 days +, average term of credit.
74675

Equated time, Sept. 4.

2065
30

61950
59740

2210

EXPLANATION.—We here compute the interest at 6%, reckoning the time in months and days, and then divide the total amount of interest by the interest on the sum of the debts for one month. The quotient is the number of months and fraction of a month in the average term of credit, or 5 months and 4 days, to be counted forward from April 30. The equated time is, therefore, Sept. 4.

The figures 2, 4, 4, 5, at the left, are written in pencil, to show the number of months from the focal date to the month opposite. Then, reckoning 30 days to the month in the terms of credit, the time is found with rapidity and ease. Thus, in the first line, we read directly, 2 months, 10 days; in the second, 3 months, 15 days, and so on.

2d.—BY THE PRODUCT METHOD,

Reckoning the time in months and fractions of months.

DATE.	AMOUNTS.	CREDIT.	TIME.	PRODUCTS.
May 10,	$450	60	$2\frac{1}{3}$ mos.	1050
2 July 15,	850	30	$3\frac{1}{2}$ "	2975
4 Sept. 1,	500	90	$7\frac{1}{30}$ "	3516
4 " 5,	862	60	$6\frac{1}{6}$ "	5316
5 Oct. 20,	325	60	$7\frac{2}{3}$ "	2492
	$2987			15349

2987)15349(5 mos. 4 days +, the same as before.
14935
414
30
12420
11948
472

If two figures be pointed off in each of these products as decimals, the results will represent the interest at 12 per cent.; and one half of the same, the interest at 6 per cent., as found in the preceding solution. So, in all cases, the products really represent interest.

3d. *The same with the use of the* TIME TABLE.

DATE.	AMOUNTS.	TIME IN DAYS.	PRODUCTS.
May 10,	$450	× 190 − 190 = 0	
July 15,	850	× 226 − 190 = 36 =	30600
Sept. 1,	500	× 334 − 190 = 144 =	72000
" 5,	862	× 308 − 190 = 118 =	101716
Oct. 20,	325	× 353 − 190 = 163 =	52975
	$2987		257291

2987)257291(86 + days.
23896
18331
17922
409

Explanation.—By the Time Table we find May 10, the first date, to be the 130th day of the year, and 130 + 60, the term of credit = 190, which we write opposite $450 as a multiplier. July 15 is the 196th day, and 196 + 30 = 226; and in like manner the time is found for each of the other items. Next, we subtract 190 from all the multipliers (which does not affect the result, while it lessens the labor of multiplying), and multiply by the remainders. The sum of the products divided by the amount of the debts, is the average time, in days, to be counted forward from the maturity of the item first due, or from the 190th day of the year; 190 + 86 = 276, opposite which, in the Table, stands Sept. 3, the equated time.

The variation of one day from the result obtained by the preceding methods of solution, is accounted for by the fact that there are intervening months which contain more than thirty days each.

From the foregoing examples and elucidations, we derive the following rules for Simple Average:—

1st. Product Method.

I. *Assume as a Focal Date the last day of the month previous to that in which the earliest date in the account appears, or the first date, or the earliest date upon which any item falls due.*

II. *Multiply each sum by the time between this assumed date and the day of its maturity, and divide the sum of the products by the amount of the debts. The quotient will be the time, in the same denomination as the multipliers, to be reckoned forward from the focal date to find the equated time.*

2d. Interest Method.

I. *Assume a Focal Date as directed above.*

II. *Find the interest on each sum for the time intervening between the focal date and the day of its maturity, and divide total amount of interest by the interest of the amount of the debts for any unit of time, as one day, month, or year. The quotient will be the time, in the same denomination as this unit, to be reckoned forward from the focal date to find the equated time.*

Notes.—1. The latest date upon which any item falls due may be taken as the focal date. In that case the time must be counted *backward.*

2. The cents are usually rejected in the calculation, if less than fifty, and counted one dollar, if fifty or more than fifty. The work is also sometimes abbreviated by rejecting the units of dollars.

EXAMPLES FOR PRACTICE.

1. Sold goods to A, as follows:—

1871,	June 10,	a bill	@	60	days	$2,500.00
	Aug. 18,	“	“	30	“	475.32
	Sept. 20,	“	“	90	“	500.00
	Oct. 1,	“	“	60	“	219.75
	Nov. 1,	“	“	30	“	150.25
						$3,845.32

When will the above bill become due by average?

Ans. Sept. 12.

2. Bought of Hendricks & Co., mdse., as follows:—

1871,	Jan. 1,	mdse.,	@	30	days	$192.30
	Feb. 13,	“	“	60	“	137.50
	Mar. 15,	“	“	90	“	742.25
	Apr. 11,	“	“	30	“	350.00
						$1,422.05

When will this account average due?

3. Sold Smith & Co.,

1871,	July 25,	mdse.,	@	3	mos.	$1,350.00
	Aug. 20,	“	“	2	“	463.75
	Sept. 30,	“	“	3	“	189.25
	Oct. 26,	“	“	2	“	300.00
						$2,303.00

The purchasers propose to settle the account October 30, 1871, by giving a promissory note for the entire amount: what time should be expressed in the note, to make it fall due at the average maturity of the account?

4. A holds three notes against B; one dated January 1, 1871,

at 30 days, for $250; one dated January 13, at 60 days, for $300; and the other dated January 25, at 90 days, for $463. B asks for an extension of 30 days, and proposes to give a new note for the whole amount with interest added, payable 30 days after the average maturity of the three notes. For what amount should the note be drawn, and for what time, if dated January 28?

5. Average the following account:—

MESSRS. HARRISON & Co.,
Bought of PHILIP BROWN & Co.

1870.				
Dec.	1	Mdse., @ 90 days..............................	$3,756	37
1871.				
Jan.	20	" " 60	253	87
Feb.	26	" " 30 "	1,892	25
Mar.	21	" " 90 "	122	67
April	2	" " 60 "	368	50
			$6,393	66

6. S. Jones bought of Stewart & Co. several bills of goods, as follows:—

1871, May 15, a bill of $375.50 on 3 months' credit.
June 5, " 632.40 " " "
July 18, " 725.00 " " "
Aug. 10, " 137.80 " " "

When must a note, dated August 10, for the whole amount, be made payable?

NOTE.—As the sales have a common term of credit, we may first find the average date of *purchase*, and to this add 3 months.

7. Sold Marvin & Co. goods, as follows:—

1871, April 3, a bill of $356.25, on 3 months' credit.
May 15, " 1,250.60, on 4 " "
June 11, " 186.00, on 2 " "
July 1, " 275.00, on 5 " "

Required the average maturity of these bills.

8. Average the following account:—

MESSRS. CHAMBERS & Co.,

To A. B. HORNER & Co., DR.

1871.				
Jan.	15	To Mdse., @ 60 days	$375	00
"	30	" " (cash)	150	00
Feb.	1	" " @ 10 days	187	00
"	16	" " " 30 "	300	00
Mar.	8	" " (cash)	210	00
			$1,222	00

NOTE.—The items marked "Cash" are sales on which no time was allowed, but as payment was neglected, they are treated as due at their respective dates.

532. When partial payments have been made on a debt before it is due, the debtor is entitled to interest on the payments, or, in equity, to an extension of time on the balance.

EXAMPLE 1. Suppose a person owes $1800, payable without interest in 8 months. If, at the end of 4 months, he should pay $1000, what extension should be granted for the payment of the balance?

SOLUTION.

$1000 \times 4 = 4000.$

$4000 \div 800 = 5.$

Ans. 5 months.

ANALYSIS.—According to the principles of average, the debtor will be entitled to retain the balance, $800, as long after the maturity of the debt as it will take $800 to gain the interest of $1000 for 4 months. The use of $1000 for 4 months is equal to the use of $1 for 4000 months, or of $800 for $\frac{1}{800}$ of 4000 months = 5 months. Hence the

RULE.—*Multiply each payment by the time it was made before maturity, and divide the sum of the products by the balance of the debt.*

2. A owes B $1600, due in 90 days. At the end of 30 days he pays $400, and at the end of 60 days $500: required the equitable time for the payment of the balance.

3. Jones & Co. sold goods to Henry Brown, as follows:—

Sept. 4, a bill of $1340 on 3 months' credit.
Oct. 15, " " 922 " 4 " "
Nov. 10, " " 462 " 2 " "

On the 4th of Oct. Brown paid on account $340, and on the 15th of Oct., $500: when should the balance of the account be paid?

Note.—First find the average maturity of the sales, and then, by the last rule, what extension of time should be granted on account of the partial payments.

COMPOUND AVERAGE.

533. Compound Average is the process of finding the mean or equitable time for the payment of the balance of an account containing both debits and credits.

534. The **Equated Time** is the date when there is an equilibrium of interest between the two sides, so that the balance of the account is then due without interest.

535. The object of averaging an account current may be either to fix a date for settlement *in the future*, or to determine the time from which to compute interest on the balance, when the equated time is *past*.

Example 1. Find the equated time for the payment of the balance of the following account:—

New York, Nov. 5, 1871.

Mr. John W. Davis,

In account with S. H. Thomas.

1871.					1871.				
Aug.	18	To Mdse., 90 days,	$2500	00	Sept.	20	By Cash,	$1000	00
Sept.	20	" " 60 "	500	00	Oct.	10	" "	500	00
Oct.	1	" " 30 "	475	00	Nov.	5	" "	275	00
Nov.	5	" " 30 "	335	00			*Balance,*	2035	00
			$3810	00				$3810	00

536. To explain the principles involved, we will first equate the debit and credit sides of the account separately, by simple average. The result of the work is as follows:—

Dr.	Cr.
Due Nov. 16, $3810	Due Oct. 3, $1775.

The question now is, How shall we determine, from this statement, when the balance of the account becomes due?

If a settlement should be made Nov. 16, Davis would then owe Thomas $2035, but Thomas would owe him the interest of $1775, the amount of his payments, from Oct. 3. In other words, the balance of the account would be against Davis, but the balance of interest in his favor. Therefore, Davis is entitled to retain the $2035 long enough after Nov. 16 for it to gain the interest of $1775 from Oct. 3 to Nov. 16, which, at 6%, is $13.02. The interest of $2035 for one day is $0.34, and 13.02 ÷ .34 = 38 +. Hence, the equated time for the payment of the balance, is 38 days from Nov. 16, or Dec. 24.

537. But the better method is to take the same Focal Date for both sides, as shown in the following

SOLUTION.

The latest maturity, *Dec.* 5, *assumed as the Focal Date for both sides of the account.*

DEBIT SIDE.			CREDIT SIDE.		
Amounts.	*Time.*	*Int. 6%.*	*Amounts.*	*Time.*	*Int. 6%.*
$2500	19 days,	$7.92	$1000	76 days,	$12.67
500	16 "	1.33	500	56 "	4.67
475	35 "	2.77	275	30 "	1.38
335	0 "	0	*Bal. ac.* 2035		
	Bal. Int.	6.70			
$3810		$18.72	$3810		$18.72

6)2.035, Bal. Int.

Int. of $2035 for 1 day, .339)6.700(19 + days, to be reckoned forward
339 from Dec. 5, making the average
date Dec. 24, as before
3310
3051

Explanation.—At the assumed date in this solution, all the items of the account are *overdue*, and are supposed to draw interest from their maturity. The interest is, therefore, reckoned on each item from the time it falls due to the focal date. The amount of interest on the debit side is $12.02, and the amount on the credit side, $18.72. This shows an excess of interest, *in favor of Davis*, of $6.70, while the balance of the account is against him. Hence, according to principles already explained, he may retain the balance of the account till it shall earn the $6.70 interest due him, or 19 days, which must consequently be counted *forward*.

SECOND SOLUTION.

July 31, *assumed as the Focal Date.*

DEBIT SIDE.			CREDIT SIDE		
Amounts.	*Time.*	*Discount.*	*Amounts.*	*Time.*	*Discount*
$2500	108 days,	$45.00	$1000	51 days,	$8.50
500	111 "	9.26	500	71 "	5.92
475	92 "	7.28	275	97 "	4.45
335	127 "	7.09	*Bal. ac.* 2035		*Bal. disct.* 49.76
$3810		$68.63	$3810		$68.63

6)2.035, Bal. Disct.

Int. on $2035 for 1 day, .339)49.760(146 + days, to be counted forward from July 31, which gives Dec. 24, as before.

339

1586

1356

2300

2034

Explanation.—At the focal date adopted in this solution, nothing is due on either side of the account. Therefore, all the items are supposed to be subject to *discount* for the time intervening between this date and their respective dates of maturity. Computing the discount as simple interest, we find that the debit amount is $68.63, and the credit amount $18.87, showing a balance, *in favor of Davis*, of $49.76.

Now, if a settlement could be made July 31, Davis would be entitled to a deduction of $49.76 from the balance which he owes. Hence, upon the principles of average, he may retain the balance long enough after this date for it to earn the excess of discount in his favor, or 146 days.

2. Required the average date of the following account:—

Philadelphia, Oct. 15, 1871.

Messrs. Hall & Kennedy, In account with Stewart & Co.

1871.					1871.				
March	18	To Mdse.,	$314	68	May	28	By Cash,	$600	00
April	24	" "	834	20	June	30	" "	400	00
"	26	" "	237	67	July	21	" "	300	00
May	14	" "	516	31	Aug.	16	" "	450	00
June	20	" "	614	38	Sept.	24	" "	100	00
					Oct.	12	" "	250	00
					Oct.	15	" *Balance*	417	24
			$2517	24				$2517	24

SOLUTION, PRODUCT METHOD.

Feb. 28 assumed as the Focal Date.

DEBITS.	$315 × 18 =	5670	**CREDITS.**	$600 × 89 =	53400
	834 × 55 =	45870		400 × 122 =	48800
	238 × 57 =	13566		300 × 143 =	42900
	516 × 75 =	38700		450 × 169 =	76050
	614 × 112 =	68768		100 × 208 =	20800
				250 × 226 =	56500
	2517	172574			
	2100			2100	298450
	417				172574
					417)125876(302
					1251
					776

The average time is found to be 302 days. The balance of the account, and the balance of *discount*, represented by the products, are both against Hall & Kennedy. Therefore, the balance of the account is *overdue*, and the time must be reckoned *backward*. 302 days, counted backward from Feb. 28, 1871, gives May 2, 1870, as the *average date* of the account. And, if settlement be made Oct. 15, 1871, interest must be added to the balance, for 531 days.

SECOND SOLUTION.

The latest date, Oct. 12, taken as the Focal Date.

DEBITS.	$315 × 208 =	65520	**CREDITS.**	$600 × 137 =	82200
	834 × 171 =	142614		400 × 104 =	41600
	238 × 169 =	40222		300 × 83 =	24900
	516 × 151 =	77916		450 × 57 =	25650
	614 × 114 =	69996		100 × 18 =	1800
				250 × 0 =	0
	2517	396268		2100	1761550
	2100	176150			
	417	)220118(528			
		2085			
		1161			
		834			
		2278			
		3336			

Average time 528 days, which, counted backward from Oct. 12, 1871, gives May 2, 1870, as before.

Here the products represent *accumulated interest*, and since, at the focal date, both the balance of interest and the balance of the account are against the debtors, it follows that the equated time is in the *past*, or 528 days before Oct. 12.

538. The solutions of these examples prove that—

1st. When the latest maturity (or any later date) is assumed as the Focal Date, if the balance of interest and the balance of the account are on *opposite sides*, the average time must be reckoned *forward*, to find the equated time; but if they are on the *same side*, it must be reckoned *backward*.

2d. When the earliest maturity (or any earlier date) is chosen as the Focal Date, the time must be reckoned *backward*, if these balances fall on *opposite sides*, but *forward*, if they fall on the *same side*.

These principles may be stated in other words, thus:—

If, at any assumed date, the balance of interest (or discount) is in favor of the person who owes the balance of the account, the time must be reckoned FORWARD; *if the reverse*, BACKWARD.

539. From these examples and elucidations we derive the following rules:—

1st. PRODUCT METHOD.

I. *Assume as a Focal Date for both sides of the account the last day of the month previous to that in which the earliest date appears, or the first date, or the earliest date upon which any item falls due.*

II. *Multiply each sum by the time between this assumed date and the day of its maturity, and divide the balance of the products by the balance of the account. The quotient will be the time, in the same denomination as the multipliers, to be reckoned from the focal date,*—FORWARD, *if the balance of the products and the balance of the account are on the same side, but* BACKWARD, *if they are on opposite sides of the account.*

2d. INTEREST METHOD.

I. *Assume a Focal Date as directed above.*

II. *Compute the discount on each sum for the interval of time* *between this focal date and its maturity, and divide the balance of interest by the interest of the balance of the account for any unit of time, as one day, month, or year. The quotient will be the time, in the same denomination as the unit, to be reckoned from the focal date,*—FORWARD, *if the balance of discount is in favor of the person owing the balance of the account, but* BACKWARD, *if it is against him.*

NOTE.—When the latest maturity is assumed as the focal date, for "discount" read "interest," and reverse the rule for reckoning the time.

EXAMPLES FOR PRACTICE.

1. Average the following account:—

DR. A. P. MANNING & CO. **CR.**

1871.					1871.				
June	15	To Mdse., 30 days.	$1447	25	June	1	By Mdse., 30 days.	$856	25
July	1	" " 60 "	128	75	July	5	" Cash,	400	00
"	20	" " 60 "	875	50	Aug.	1	" "	300	00
Aug.	20	" " 30 "	264	37	"	21	" "	100	00
							Balance,	1059	62
			$2715	87				$2715	87

SOLUTION.

May 31, *assumed as the Focal Date.*

Amounts.	Time.	Products.	Amounts.	Time.	Products.
$1447	$1\frac{1}{2}$	2171	$856	$1\frac{1}{30}$	884
129	$3\frac{1}{30}$	391	400	$1\frac{1}{6}$	467
876	$3\frac{2}{3}$	3212	300	$2\frac{1}{30}$	610
264	$3\frac{2}{3}$	968	100	$2\frac{7}{10}$	270
$2716		6742	$1656		2231
1656		2231			
$1060		)4511(4 — 8			
		4240			
		271			
		30			
		8130			

Average time, 4 months, 8 days. Hence, the balance of the account is due October 8.

NOTE.—When this method is employed, the exact number of days may be obtained, if required, by adding one for every month *passed* which contains 31 days, and subtracting the number wanting in February. That is not done here, as the work is regarded as sufficiently accurate for all practical purposes, without this correction.

2. When is the balance of the following account due per average?

DR. A. B. COLLINS. **CR.**

1871.					1871.				
Sept.	12	To Mdse., @ 30 days,	$927	30	Oct.	10	By Cash,	$500	00
Oct.	15	“ “ “ 30 “	342	75	Nov.	20	“ “	300	00
Nov.	18	“ “ “ 60 “	212	13	“	30	“ “	250	00
Dec.	1	“ “ “ 30 “	175	50					

Ans. Nov. 20, 1871.

3. Average the following account:—

DR. GEO. HOMER in account with JOHNSON & COLEMAN. **CR.**

1871.					1871.				
Jan.	1	To Mdse., 30 days,	$586	40	Feb.	1	By Cash,	$460	00
Feb.	15	“ “ “ “	322	10	Mar.	15	“ “	280	00
Mar.	3	“ “ “ “	863	14	Apr.	1	“ Draft at 10 days,	400	00

4. Find the equated time for the settlement of the following account:—

DR. JAMES WALTON & CO. **CR.**

1871.					1871.				
March	1	To Balance,	$325	00	March	16	By Cash,	$250	00
April	16	“ Mdse., @ 2 mos.,	623	47	May	20	“ “	300	00
May	11	“ “ “ 2 “	1722	30	July	30	“ Note, 30 days,	1000	00
June	16	“ “ “ 30 days,	975	12	Aug.	31	“ Cash,	560	00
July	30	“ “ “ 60 “	146	35					
Aug.	17	“ “ “ 30 “	1650	25					

5. When is the following account due per average?

DR. H. C. LYMAN. **CR.**

1870.					1870.				
Oct.	13	To Mdse., @ 3 mos.	$275	16	Dec.	1	By Cash,	$150	00
Nov.	18	“ “ “ 4 “	186	37	“	30	“ “	300	00
Dec.	1	“ “ “ 2 “	917	16	1871.				
“	5	“ “ “ 3 “	875	00	Feb.	18	“ “	560	00
1871.					March	1	“ “	262	00
Jan.	10	“ “ “ 2 “	327	16					
Feb.	16	“ “ “ 3 “	186	70					

6. Sold goods to A as follows:—April 10, a bill of $275.60; May 15, a bill of $300; June 16, a bill of $862.25; Aug. 11, a bill of $972.75. Received cash, May 1, $150; July 15, $200; July 20, $340. When is the balance of the account due per average?

7. Average the following account:—

DR. DAVID W. MANSFIELD. **CR.**

1871.					1871.				
Jan.	1	To Mdse., Cash,	$500	00	Jan.	10	By Cash,	$500	00
Feb.	12	" " @ 30 days,	227	50	March	12	" "	200	00
March	25	" " " 30 "	275	00	April	30	" "	300	00
April	18	" " " Cash,	1246	00	May	1	" "	1000	00
June	18	" " " 90 days,	862	00	June	1	" "	200	00
Aug.	5	" " " 60 "	347	92					

CASH BALANCE.

540. The **Cash Balance** of an account is the sum that will be required to settle the balance at any given date.

541. The averaging of an account determines the date when the balance may be paid without interest. If it be proposed to settle at any other time, with an adjustment of interest between debtor and creditor, the account may first be averaged, and the interest or discount computed on the balance; or it may be reckoned on each item for the interval of time between its maturity and the date of settlement. The latter method is generally adopted.

8. Required the average date of the following account, and the *Cash Balance* for settlement, Jan. 1, 1872:—

DR. WALKER & JONES. **CR.**

1871.					1871.				
Aug.	4	To Mdse., 6 mos.	$450	00	Sept.	20	By Cash,	$1500	00
Sept.	18	" " Cash,	1826	37	"	28	" Mdse., 4 mos.	283	62
"	25	" " 4 mos.	217	67	Oct.	14	" Cash,	312	81
Oct.	15	" " 8 "	912	33	Nov.	9	" Mdse., 3 mos.	187	00
Nov.	12	" " 6 "	1200	00	Dec.	31	" Cash,	700	00
Dec.	17	" " Cash,	714	34					

9. Average the following account, and find its *cash value*, Jan. 1, 1872:—

New York, Oct. 11, 1870.

Mr. E. R. Jameson,
Dr. (6 months). In account with E. S. Jaffray & Co. Cr.

1870.					1870.				
Jan.	15	To Mdse.,	$1450	00	July	1	By Cash,	$1200	00
Feb.	25	" "	197	35	Aug.	1	" "	300	00
March	16	" "	2516	38	Sept.	16	" "	2000	00
April	10	" "	200	00	Oct.	11	" "	100	00
Aug.	5	" "	316	75					
Sept.	20	" "	1620	37					

AVERAGE OF ACCOUNTS OF SALES.

542. An **Account of Sales** is an account rendered by an agent or consignee, of goods sold by him for his principal, or consignor, containing a statement of the sales, expenses, and charges, and showing the net proceeds due the consignor.

543. The sales constitute the credits; the charges, the debits, and the balance in the net proceeds.

544. When sales are made on time, the consignee is entitled by custom to the same term of credit for the payment of the proceeds.

545. Expenses incurred in receiving the goods, and all charges paid in cash, are considered due the consignee when paid. Commissions and after-charges are, upon strict principle, due at the average maturity of the sales; yet this rule is not always observed.

546. The object of averaging an account of sales is to ascertain when the net proceeds become due. Except as regards the adjustment of the date of the commissions and charges made on closing a consignment, the process is precisely the same as that of equating an account current.

547. In joint consignments, the consignee's share of the invoice may be considered due at the date of the receipt of the goods, or the date of shipment, or not until sales are effected; but this depends altogether upon the agreement between the parties. The consignee's share of the gain or loss takes date with the commissions and charges, being due at the average maturity of the sales.

EXAMPLE 1. Required the equated time for the payment of the net proceeds of Cooper's consignment, the account of sales being as follows:—

ACCOUNT-SALES *of Mdse. received from* J. B. COOPER, *Lancaster, to be sold on his account and risk.*

1871.						
July	14	Sold at 10 days, 500 bush. corn, @ $1.10............	$550	00		
"	16	Sold 500 bush. corn, @ $1.10 .....$550.00				
		25 " clover seed, @ $8.50 212.50				
		Received cash	500	00		
		Balance on ac., 10 days	262	50		
"	16	Sold for cash, 500 bush. oats, @ 60 c......... ....$3,000				
		25 " clover seed, @ $8........ 200	3,200	00		
					$4,512	50
		Charges.				
July	11	Paid freight in cash $300.00				
		Drayage 10.50	310	50		
"	16	Commission on sales, $4,512.50, @ 2½%....	112	81		
					423	31
		Net proceeds due per average ——.			$4,089	19

STATEMENT.

DR.	COOPER'S	CONSIGNMENT.	CR.
Due July 11..............	$310.50	Due July 24..............	$550 00
Commission..............	112.81	" " 16..............	500.00
Balance	4,089.19	" " 26..............	262.50
		" " 16..............	3,200 00
	$4,512.50		$4,512.50

SOLUTION.

Focal Date, July 26; Interest at 6%.

$310.50, int. 15 days	$0.78	$550.00, int. 2 days	$0.18
112.81, " 8 "	0.15	500.00, " 10 "	0.84
Balance of interest	5.42	262.50, " 0 "	
		3,200.00, " 10 "	5.33
	$6.35		$6.35

The interest of $4512.50 for one day, is $0.75, and 6.35 ÷ .75 = 8 +. Therefore, the average maturity of the sales is 8 days prior to July 26, or July 18. Affixing this date to the commission item, we proceed with the calculation, and find the balance of interest, July 26, in favor of the credit side, $5.42. The interest of the balance of the account, $4089.81, for one day, is $0.68, and 5.42 ÷ .68 = 8 —. The small amount of interest upon the charges in this case varies the result from the average maturity of the sales only by the fraction of a day. *Ans.* July 18.

2. Average the following account-sales:—

ACCOUNT-SALES *of Mdse. received from* E. C. HOWARD & CO., *for sale on joint account of themselves,* C. S. WALKER, *and ourselves, each ⅓.*

1871.						
July	11	Sold on ac., at 3 days, 100 bbls. petroleum, 4,000 gals., @ 48½ c.	$1,940	00		
"	11	Sold for cash, 250 bbls. do., 1,000 gals., @ 48 c.	4,800	00		
"	14	Sold on a note, at 30 days, 50 bbls. do., 2,000 gals., @ 49 c.	980	00	$7,720	00
-	15	100 bbls. on storage, destroyed by fire; no insurance.				
		Charges.				
"	10	Paid cash for freight and drayage, $750.00				
"	17	Storage and cooperage 25.50				
		Commission, 2½% on $7720 193.00	968	50		
		Less our ⅓ net loss	482	84	485	66
		Total net proceeds			$7,234	34
		Amount of invoice $8,200.00				
		E. C. H. & Co.'s, and our ⅔ 5,466.67				
		Less their ⅓ net loss 482.83				
		$4,983.84				
		Due by equation ——.				

Philadelphia, July 17, 1871. PARKER, WARREN & CO.

SOLUTION.

Focal Date, Aug. 16, *the latest maturity.*

Sales.	Due July 14	$1940	Int., 33 days,	$10.67
	" " 11	4800	" 36 "	28.80
	" Aug. 16	980	" 0	
Loss.	" July 16	1448.50	Int., 31 days,	7.48
		$9168.50		$46.95

6)9.168

1.53 +)46.95(31 — Average maturity of sales, 31 days
459 prior to Aug. 16, or July 16.
105

Charges.	Due July 10	$750	Int., 37 days,	$4.63	
	" " 16	218.50	" 31 "	1.13	
Balance of ac.		8200.		——	5.76
		$9168.50			*Bal. Int.*, $41.19

6)8.200

1.37 +)41.19(30 + Equated time for the payment of the Net
41.1 Proceeds, 30 days prior to Aug. 16, or
July 17.
9

In the above solution, the amount of the invoice is considered the balance of the account, while the total loss appears as a credit item. By subtracting our partners' shares of the loss from the total loss and the invoice, the result of the operation will be the same. The account, after the average date of the sales has been found, will then stand as follows:—

Dr.		Cr.	
July 10, To Charges	$750.00	July 16, By Sales	$7720.00
16, " "	218.50	" 16, Profit & Loss	482.84
Net Proceeds	7234.34		
	$8202.84		$8202.84

Assuming the latest date as the focal date, we have only to compute the interest on $750 for 5 days, which is $0.625. The interest of $7234.34 for one day is $1.205. The division gives a little more than half a day, which being reckoned a day, we have for the equated time,

July 17, as before.

It follows, therefore, that, having found the average maturity of the sales, it is only necessary to consider the cash charges and the net proceeds. This reduces the equation of an account-sales to a very simple formula. Thus, in this case, we have—

Dr.	Cr.
July 10, To Charges....$750	July 16, By Net Proceeds....$7234.34

This leaves but one simple question to be answered:—How long after July 16 may $7234.34 be retained for the use of $750 for 5 days?

EXCHANGE.

548. **Exchange** is the mode of remitting money, or canceling debts between distant places, by means of Drafts, or Bills of Exchange. The term is also applied, by common usage, to the bills themselves.

549. **Exchange** is of two kinds; *Domestic*, or *Inland*, and *Foreign*.

550. **Domestic**, or **Inland Exchange**, is exchange between different places in the same country.

551. **Foreign Exchange** is exchange between different countries.

DRAFTS AND BILLS OF EXCHANGE.

552. To comprehend the subject of exchange in all its bearings, it is necessary to consider—

First, the nature and forms of Drafts, or Bills of Exchange, which are the *medium* through which all exchanges are effected; and, in brief, some of the laws and usages which govern their operation.

Secondly, the methods of determining the comparative values of the moneys of different nations, or of converting the denominations of any given currency into those of another.

553. A **Draft,** or **Bill of Exchange,** is a written order, or request, from one party to another, directing the latter to pay to some person designated a specified sum of money.

554. Bills of Exchange are classified as *Foreign* and *Inland.* A *Foreign Bill* is a bill drawn in one country upon a person residing in another. An *Inland Bill* is one drawn and made payable in the same country.

555. A **Set of Exchange** is a bill drawn in duplicate, or triplicate, each of the copies having all the validity of the original bill, but so worded that when either the original or one of the copies is paid, the others become void. These are designed for transmission by different mails or conveyances, to guard against loss, or delay, by accident or miscarriage.

556. There are primarily three parties to a draft; namely, the *Drawer, Drawee,* and *Payee.* There may also be *Indorsers,* and *Indorsees.*

557. The **Drawer** is the party who makes or signs the draft,—the person who orders the money to be paid.

558. The **Drawee** is the party on whom the draft is drawn, or who is directed to pay the money.

559. The **Payee** is the party in whose favor it is drawn, or to whom the money is directed to be paid.

560. After the drawee has accepted the draft, he is also called the *acceptor;* and the draft itself, an *acceptance.*

561. The drawer may make himself the payee, in which case there are but two original parties to the draft.

562. The payee may transfer the draft to another party, by writing his name upon the back. This is termed an *indorsement,* and he is then called the *indorser.* The party to whom the bill is transferred is the *indorsee.*

563. The **Holder** of a bill is the legal owner or possessor of it, and the money must be paid to him.

564. Drafts are drawn payable at sight, or on demand, or a specified time after sight or after date, or at usance.

FORM OF A SIGHT DRAFT.

$1000. New York, *September 1, 1871.*

At sight, pay to the

2-cent Stamp.

order of ______CHARLES MASON,______

__________One Thousand Dollars,__________

value received, and charge to the account of

To H. C. ORAM & CO.,
Philadelphia, Pa.

James Richmond.

565. A draft in this form is payable on presentation, in most of the States; but in Massachusetts, New Hampshire, Maine, New Jersey (except on banks), North Carolina, South Carolina, Alabama, Texas, Kentucky, Iowa, Indiana, Michigan, and Minnesota, the drawee is entitled to three days' grace, and the draft must first be presented for acceptance. In several of these States, the statutes especially provide that drafts payable "on demand" shall not be entitled to grace.

In the above draft, James Richmond is the *drawer*, Charles Mason the *payee*, and H. C. Oram & Co. the *drawees.*

The theory of the draft is, that H. C. Oram & Co. owe James Richmond One Thousand Dollars, and that Richmond owes Mason, or has some dealings with him, and wishes the money paid to him, or his order. Two debts are, therefore, canceled by means of the draft, without the expense or trouble of transmitting the money, both ways, between New York and Philadelphia.

Drafts are, however, not always used for the double purpose of collecting and paying debts at the same time. They are often employed simply for collecting debts, and, as bought and sold extensively by bankers and merchants, they become a great medium of exchange between individuals and nations,—the legitimate money of commerce.

DRAFT PAYABLE AFTER SIGHT.

$475 $\frac{25}{100}$. Philadelphia, *July 1, 1871.*

Thirty days after sight,

25-cent Stamp.

Pay to JOHN WILSON, *or order,*

Four Hundred and Seventy-five $\frac{25}{100}$ Dollars,

value received, and charge the same to the account of

S. C. Davis.

To WILLIAM PORTER,
Boston, Mass.

566. This draft must first be presented to the drawee for his acceptance. An acceptance is a promise to pay according to the terms of the draft, and is usually written across its face, thus:—

"*Accepted July* 3, 1871.
WILLIAM PORTER."

The time, thirty days, with three days' grace added, commences to run from the date of the acceptance; that is, if the draft is accepted July 3, it will be due and payable thirty-three days after that date, which is Aug. 5.

In New York, no grace is allowed on checks, bills, or drafts, drawn upon any bank or banker, though payable after sight or after date.

A draft may be accepted so as to make it payable at a particular place; as at a bank, by adding the words "payable at —— Bank;" in which case it must be presented at the bank, and, according to custom, will be paid out of the funds of the acceptor, without further attention on his part.

Sight drafts are sometimes so accepted *pro forma*, the acceptance being equivalent to a check for the amount. The drawee writes upon the face of the draft, "Accepted, payable at —— Bank," with his signature, and returns it to the holder, who then presents it at the bank for payment.

A bill drawn payable after sight must be presented for acceptance within a reasonable time. Inexcusable delay will deprive the holder of all remedies against the drawer or indorsers. It must be presented during the usual hours of business, and at the drawee's place of business, or domicil. After acceptance, it must be again presented, for payment, on the day of maturity.

DRAFT PAYABLE AFTER DATE.

$1450. Cincinnati, O., *April 10, 1871.*

75-cent Stamp.

Ten days after date, pay to the order of GEORGE HARDY, *Fourteen Hundred and Fifty Dollars, value received, and charge the same to the account of*

To HIRAM A. CARLTON,
St. Louis, Mo.

J. C. Coleman & Co.

567. The holder of a draft of the above form is not bound to present it for acceptance. It being payable a certain time after date, it is only necessary to present it at maturity for payment. It is customary, however, as a matter of courtesy, to present such drafts at the earliest opportunity for acceptance.

The acceptance, in this case, does not require a date. Thus, the following form is sufficient:—"*Accepted, Hiram A. Carlton.*" The reason of this is, that the time is counted from the date of the draft, and it is immaterial when it is accepted.

No particular form of words is necessary to constitute a valid acceptance. Anything which amounts to promise to pay is sufficient. Even a verbal acceptance is good; but there must be a witness to prove it, in case of litigation.

PROTEST AND NOTICE.

568. In case of the refusal or failure of the drawee of a draft to accept or pay, the holder must cause the requisite steps to be taken to fix the liability of the drawer and indorsers,—which before this event is only contingent,—if he intends to hold them responsible. This must be done through a notary public. This officer presents the paper in his official capacity, and demands acceptance, or payment, and, if refused, *he notes the facts*, and afterward draws up a formal declaration called a *Protest*, giving the particulars of the presentment and refusal, and signs and seals the same. This must be done on the day the paper matures. As early as the subsequent day, he must notify all the parties whose liability is contingent of the protest.

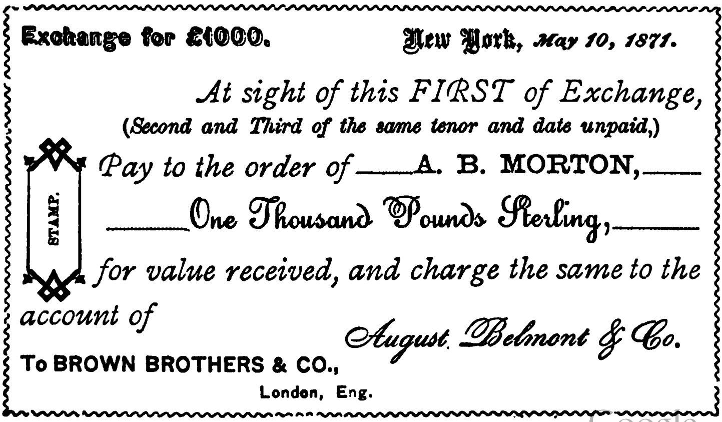
INDORSEMENTS.

569. When a draft is delivered to the payee, he is called the *Holder*. In his hands, it is negotiable paper, if made payable to his order, and may be transferred to any other party by indorsement and delivery. The indorsement may be either in blank,—that is, his name only written on the back of the draft,—or he may restrict its payment to his indorsee, or order, by writing over his name the words, "*Pay to the order of* (*indorsee*)." The indorsee must then, in turn, indorse in order to transfer the same to another. If the indorsement be not so restricted, the instrument becomes virtually payable to the bearer, and may pass through any number of hands before payment, by simple delivery. Any holder may write over the indorser's name the words, "*Pay to the order of* (*himself or other person*)," and thus put an end to the negotiability of the paper, without further indorsement by the person named. He may, if he choose, entirely stop the currency of the bill, by giving a bare authority to receive the money; such ~~as~~, "*Pay to* A for my use," or to "A, or order, for my use," or "to A only," or "for my account," or "for collection."

The payee may relieve himself from liability as indorser by adding to his signature the words, "without recourse."

570. Foreign Drafts are often drawn in Sets, for the protection of purchasers, as explained in the definition (Art. 555). The following is the form of a

SET OF FOREIGN EXCHANGE.



Exchange for £1000. New York, *May 10, 1871.*

At sight of this FIRST of Exchange,

(*Second and Third of the same tenor and date unpaid,*)

STAMP.

Pay to the order of——A. B. MORTON,——

——One Thousand Pounds Sterling,——

for value received, and charge the same to the account of

August Belmont & Co.

To BROWN BROTHERS & CO.,
London, Eng.

Exchange for £1000. New York, *May 10, 1871.*

At sight of this SECOND of Exchange,
(First and Third of the same tenor and date unpaid,)

STAMP.

Pay to the order of A. B. MORTON,
One Thousand Pounds Sterling,
for value received, and charge the same to the account of

August Belmont & Co.

To BROWN BROTHERS & CO.,
London, Eng.

Exchange for £1000. New York, *May 10, 1871.*

At sight of this THIRD of Exchange,
(First and Second of the same tenor and date unpaid,)

STAMP.

Pay to the order of A. B. MORTON,
One Thousand Pounds Sterling,
for value received, and charge the same to the account of

August Belmont & Co.

To BROWN BROTHERS & CO.,
London, Eng.

Any one of these Bills may be presented for payment, and, if paid, the others become useless, and void. In case of time drafts, if one has been accepted, and, before payment, is lost, the drawee should be notified of its loss, and the second presented for acceptance.

USANCE.

571. **Usance** is a term which signifies the customary or usual time given to Bills of Exchange, at the place of payment, when drawn from given points or places. The Bills read, "At Usance, pay to," etc. *Double usance* means double the time allowed for usance. The practice of drawing Bills at usance is fast wearing away, the time being now most commonly specified upon their face. It does not prevail in the United States.

The following is the time allowed in different countries:—

In ENGLAND	Bills drawn from *France*, *Holland*, and *Germany*, at usance	30 days' date.
	From *Spain* and *Portugal*	60 " "
	From *Italy*	3 months' "
In the NETHERLANDS.	Bills drawn from *England*, at usance,	1 " "
In HAMBURG	Bills drawn from *England*, *France*, and *Holland*, at usance	1 " "
In FRANKFORT-ON-THE-MAIN	Bills drawn from any place, and *not payable at the fairs of this place*	14 days' sight.
In AUSTRIA	Bills drawn at usance from any place	14 " "
In TUSCANY	Bills drawn from *England*, at usance	3 months' date.
In SPAIN	Bills drawn at usance from *England* on *Cadiz*	2 " "
	On *Madrid*	60 days' "
In GENEVA	Bills drawn from *England*, at usance,	30 days' sight.

DAYS OF GRACE.

572. DAYS OF GRACE on a Bill of Exchange, are a certain number of days granted by custom to the acceptor for paying it, after the term specified on its face has expired. These days in different countries are as follows:—

In the United States, Great Britain, and Austria, 3; in the Netherlands, none; in Hamburg and Altona, 12; in Frankfort-on-the-Main, 4; in Paris, Bordeaux, Dantzic, and Russia, 10; in Geneva, 5; in Bremen, 8; in Antwerp, Amsterdam, Cadiz, Lisbon, Oporto, Rio Janeiro, Rotterdam, and Venice, 6. In Russia, 3 days are allowed on all bills payable after sight.

When a bill is drawn "on demand," or if no time of payment be specified upon its face, it must be paid on presentation, no grace being allowed. If the last day of grace fall on Sunday, or a legal holiday, the bill is payable the day previous.

The legal holidays in most of the States are New-year's day, the Fourth of July, and Thanksgiving day, appointed by the Governor, or President of the United States. In New York, and some other States, Washington's Birthday, the 22d of February, is added. In England, they are Christmas and Good Friday.

DAMAGES ON PROTESTED BILLS.

573. In most of the United States, laws are in force which subject the drawers and indorsers of Bills of Exchange to certain rates of damages, for their non-acceptance or non-payment; that is, when a bill is returned protested, the holder can recover not only the amount of the bill, but a certain per cent. of its face, as damages, besides interest, the cost of protesting, and in some cases the cost of re-exchanges. These rates vary from 1 to 15 per cent., depending on the place on which the bills are drawn.

Want of space forbids a recapitulation of these laws in this connection, but we will give an abstract of the law of New York on this subject.

The damages on bills of exchange, negotiated in New York, and payable in other States, and returned under protest for non-acceptance, or non-payment, are as follows:—

1. If payable in Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New Jersey, Pennsylvania, Delaware, Maryland, Virginia, District of Columbia, and Ohio, 3 per cent.

2. If payable in North Carolina, South Carolina, Georgia, Kentucky, or Tennessee, 5 per cent.

3. If drawn upon parties in any other State or Territory of the Union, or in the West India Islands, or any place north of the Equator on this continent, or in Europe, 10 per cent.

PAR OF EXCHANGE.

574. The **Par of Exchange** is the established value of the coin, or standard of value, of one country, when expressed in the coin or standard of another; as the value of the pound sterling in the currency of the United States.

575. The **Intrinsic Par of Exchange** is the actual value of the coins of one country as compared with those of another by an assay of their relative weight and purity.

576. The **Commercial Par of Exchange** is the comparative value of the currency or coin of different countries, according to commercial usage.

577. The **Rate of Exchange** is the market-price of Bills of Exchange where they are drawn or negotiated.

578. The market value or cost of exchange fluctuates according to the conditions of trade, or the temporary scarcity or superabundance of funds to cover the demand for remittances between two countries, or places in the same country. As a general rule, it commands a premium in the country, or place, against which there is a balance of trade in favor of the place or country upon which the bills are drawn. For instance, if we import more goods from England than we export to pay for them, the balance of trade is against us, and specie must be exported to pay the difference. Under these circumstances, exchange on England will sell at a premium; and, this state of things being reversed, it will, unless affected by other contingencies, fall below par. If between New York and New Orleans, the balance of trade is in favor of New Orleans, in New York exchange on New Orleans will command a premium, and in New Orleans exchange on New York will be at a discount. Hence, upon general principles, the current rate of exchange between two countries or places, depends mainly upon the cost of transporting specie between them to settle the balance of trade.

The issue of government notes, and the establishment of our national banks, with a circulation current in all parts of the country, have reduced the premium or discount on domestic exchange, to a merely nominal rate, except between currency and gold. Formerly, when the notes of State banks were to a great extent irredeemable out of the States where they were issued, and were current only at home, the price of exchange corresponded very nearly with the market value of the money paid for it at any given place. Thus, in the Western States, the notes of Eastern banks commanded a premium of from ½ to 5 per cent., for western currency, and in the East, western notes, or "uncurrent funds," were at a discount. Then, at the West, eastern exchange was at a premium, and at the East, western exchange was at a discount. The principle may be still more forcibly illustrated, by comparing our present national paper currency with gold, or the gold standard of any foreign country. For example, if exchange on England is selling at $4.86 for gold, it will cost in currency, when gold commands a premium of 10%, as it is called, 10% more than $4.86, or $5.346 for £1, while in England all drafts on this country payable in currency will be sold at a discount from the current gold rates.

It thus appears that exchange has, commercially, no fixed value. It varies with the circumstances of trade, with the demand and supply, with the value of the money paid for it, and numerous other causes. The main object, therefore, of all calculations respecting exchange, is the determination of the comparative values of the moneys of different countries, according to some established standards.

DOMESTIC EXCHANGE.

579. Domestic, or **Inland Exchange,** is exchange between different places within the limits of our own country.

NOTE.—In the laws of some of the States, for special purposes, all bills drawn and payable within those States, are termed Inland bills, and all so drawn and payable elsewhere, Foreign Bills; but this distinction does not affect any calculations under this head.

CASE I.

580. To find the cost of a draft, the amount, or "face of the draft," and the rate of exchange being given.

RULE.—*Multiply the given amount by the cost of* $1, *at the given rate;* or, *compute the percentage of the premium, or discount, and add it to, or subtract it from, the amount of the draft.*

EXAMPLE 1. Howes & Macy, of New York, sell to Thomas Howard their sight-draft on Drexel & Co., of Philadelphia, for $785.60, at ¼% premium: what amount must be paid for the draft?

SOLUTION.

$785.60 × 1.00¼ = $787.564. Or, $785.60, face of draft.
1.964, ¼% added.
$787.564, *Ans.*

2. Required the cost of a sight-draft for $1500, purchased in Chicago, on Denver City, at 1¼% discount.

SOLUTION.

1.00 − .01¼ = .98¾.
$1500 × .98¾ = $1481.25.

Or, $1500.00, face of draft.
11.75, less ¼%.
$1481.25, *Ans.*

EXAMPLES FOR PRACTICE.

1. E. F. Davis & Co., of Cleveland, O., sell their sight-draft on the First National Bank of New York, for $2450.75, to Wm. Winters, at ½% premium. Write the draft, and calculate its cost?

2. What must be paid for a draft for $640, drawn by Jay Cook & Co., of New York, on the Third National Bank of Boston, at 1% premium?

3. James Harvey & Co., of Cincinnati, purchase of Manning & Brothers, their draft at sight on Lee & Pickering, Buffalo, for $1500, at 1% discount: what must be paid for the draft?

4. Suppose you are in Boston, and wish to remit $1000 to St. Louis, per draft, what will the draft cost you, if exchange is at 1% premium?

CASE II.

581. To find the face of draft, the cost and rate of exchange being given.

RULE.—*Divide the cost of the draft by the cost of* $1, *at the given rate.*

EXAMPLE 1. What is the face of a draft which can be purchased for $1500, at 1% premium?

SOLUTION. $1500 \div 1.01 = 1485.15$. *Ans.* $1485.15.

2. A merchant in Philadelphia purchased a draft on Chicago, for which he paid $4962.50, exchange being at ¾% discount: what was the face of the draft?

SOLUTION. $4962.50 \div .99\frac{1}{4} = 5000$. *Ans.* $5000.

EXAMPLES FOR PRACTICE.

1. What must be the amount of a draft which costs $227.50, at 1¼% discount?

2. A gentleman in New York has $1000 in his possession, belonging to James Wilson, of St. Louis, which he is requested

to forward per draft. The current rate of exchange is at ¾ per cent. discount. If he invests the whole sum in exchange, at this rate, and remits the same, how much will Wilson receive at St. Louis?

3. Messrs. Cook & Co. purchased a draft of Drexel, Read & Co., New York, on the First National Bank of Portland, Me., for which they paid $1362.75, exchange being at ¼% premium: what was the amount of the draft?

DISCOUNTING TIME DRAFTS.

582. Drafts payable after sight or date, upon being negotiated before maturity, are subject to discount the same as promissory notes.

Both the exchange and discount are computed on their face.

EXAMPLE.

1. Cooper & Conard, the drawees of the following draft, had the same discounted at the Farmers and Mechanics' National Bank, Philadelphia, on the day of its date. ½ per cent. was allowed for exchange, and discount was charged at the rate of 6% per annum: what was the amount of proceeds?

EXCHANGE FOR $2000. PHILADELPHIA, Dec. 1, 1870.

Sixty days after date, pay to the order of W. RUSHTON, JR., *Two Thousand Dollars*, value received, and charge the same to our account.

COOPER & CONARD.

TO MERION & CANFIELD,
Savannah, Ga.

SOLUTION.

$2000,	Face of draft.
10,	Premium of ½%.
2010,	Amount.
21.33,	Less discount, 64 days.
$1988.67,	Proceeds.

FOREIGN EXCHANGE.

583. Foreign Exchange is exchange between our own and foreign countries.

Foreign drafts are, with occasional exceptions, made payable in the money of account of the country on which they are drawn. Sometimes drafts are drawn on this country, payable in foreign money. In such cases, they must be paid at the current rate of exchange. When necessary, the kind of funds to be paid is specified upon their face; as, "Gold," "Currency," "Banco," etc.

584. Nearly all our exchanges with Europe are effected through certain prominent financial centres; as London, Paris, Antwerp, Hamburg, Amsterdam, Frankfort, Bremen, Vienna, Berlin, Leipsic, and Cologne.

Remittances to and from many other parts of the world are frequently made indirectly through some of these places, particularly London. Even between our own country and Central and South America, Sterling drafts are oftener employed than direct exchange. So it is with the East Indies, and to a greater or less extent with Asia and Africa.

585. We will therefore take as a basis of most of the problems to be given in this connection the following report:—

QUOTATIONS OF EXCHANGE.

WALL ST., NEW YORK, NOV. 5, 1874.

Sterling, 60 days, commercial	4.81	@	4.83
" Good to prime bankers'	4.84	"	4.85
" Short sight	4.87½	"	4.88½
Antwerp	5.20	"	5.15
Switzerland	5.18¾	"	5.13¾
Amsterdam	40⅝	"	41⅛
Hamburg	95¼	"	96¼
Frankfort	95¼	"	96¼
Bremen	95¼	"	96¼
Berlin	95¼	"	96¼
Paris, 60 days	5.16¼	"	5.15
" **Short sight**	5.12½	"	5.10

Explanation.—*Sterling* :—Commercial Bills are drafts of merchants, which usually sell for less than bankers' drafts, not being so reliable, nor in so great demand. "4.81 @ 4.83" means $4.81 to $4.83, for £1. "Short sight" signifies three days' sight.

France, *Belgium* (Antwerp), and *Switzerland* :—The quotations show the number of francs given for $1 ; as 5.16¼ ; that is, 5 francs 16¼ centimes.

Amsterdam :—"40⅝ @ 41⅛" signifies 40⅝ to 41⅛ cents per Florin.

Germany :—By reason of the new German coinage and regulations recently adopted in conformity therewith, exchange on Germany is now quoted and drawn in Reich-marks, and the quotations "95¼ to 96¼" mean so many cents for four Reich-marks.

The anomaly of quoting the price of four reich-marks instead of one, which ought to have been, and may yet be, avoided, sprang from the idea entertained before their exact value was known that four of them would be worth about $1 of our money. The par value, as estimated at our mint, is 23.82 cents; 23.82 cents × 4 = 95.28 cents, which will be the quoted rate when exchange is at par.

STERLING EXCHANGE,

Or, Exchange on Great Britain.

586. Drafts on England, Ireland, and Scotland, are called Sterling Bills, or Sterling Exchange, and were formerly negotiated upon the basis of the old par value of the pound sterling, $4.44⁴⁄₉, which was called the nominal par. The actual par, since the coinage of the new English sovereign, which is a pound, has been $4.866½, as determined by its standard weight and purity in comparison with our own gold coins, coined since 1837. Hence, the former method of reckoning sterling exchange, was to compute it at a certain per cent. above the nominal par, according to the market rates, the real par being about 9½ per cent. above the assumed base.

587. But this inconvenient mode of calculation has now been rendered obsolete, both in government transactions and in mercantile and banking business by an act of Congress, approved March 3d, 1873, entitled "*An Act to establish the Custom-house Value of the Sovereign or Pound Sterling of Great*

Britain, and to fix the par of Exchange." This act provides as follows:—

1. "That the value of foreign coin, as expressed in the money of account of the United States, shall be that of the pure metal of such coin of standard value; and the values of the standard coins in circulation of the various nations of the world, shall be estimated annually by the director of the mint, and proclaimed on the first day of January by the Secretary of the Treasury."

2. It then fixes the value of the pound sterling, for all governmental purposes, at $4.866½, and declares that "this valuation shall be the par of exchange between Great Britain and the United States;" and that "all contracts made after the first day of January, 1874, based on an assumed par of exchange with Great Britain, of fifty-four pence to the dollar, or four dollars forty-four and four-ninths cents to the sovereign or pound sterling, shall be null and void."

588. Since the passage of this act sterling exchange has been quoted in the money market and reckoned at so many dollars and cents to the pound, the rates fluctuating, according to circumstances, above and below $4.866½, the actual intrinsic par.

NOTE.—For a list of standard values of foreign currencies, as proclaimed by the Secretary of the Treasury, in accordance with this act, January 1st, 1875. see page 233.

EXAMPLES.

1. What is the value in United States money of £145 18*s.* 9*d.*, at the actual par?

SOLUTION. £145 18*s.* 9*d.* = £145.9375.
$4.866½ × 145.9375 = $710.204 +, *Ans.*

2. What is the value in English money of $4500, exchange at $4.86?

SOLUTION. Divide 4500 by 4.86, reducing the remainders as in the division of compound numbers. *Ans.* £925 18*s.* 6*d.* +

589. The current quotations of exchange denote the gold rates. In the present state of our currency, the premium on the gold must be added to find the cost in currency.

EXAMPLE 1. The rate of exchange being $4.80, and of gold for currency 110½, what will a bill for £45 sterling cost in currency?

SOLUTION. $4.80 × 1.10½ × 45 + $238.68, *Ans.*

2. What is the face of a bill of exchange on London that can be purchased for $500 in currency, when exchange is quoted at $4.85, and gold at 109?

SOLUTION. Divide 500 by 4.85 × 1.09.

590. EXAMPLES FOR PRACTICE.

1. Find the cost of the following Bill of Exchange, at $4.85.

EXCHANGE FOR £580. BOSTON, Jan. 1, 1875.

Sixty days after sight, pay to the order of JOHN HINCKLEY *Five Hundred and Eighty Pounds Sterling,* value received, and place the same to our account, with or without further advice.

KIDDER, PEABODY & CO.

TO BARING BROTHERS,
London.

Ans. $2813.

2. Required the cost in currency of a Bill on London for £300 10*s.*, exchange at $4.82, and gold at 110.

Ans. $1593.251.

3. For what amount can a Bill on Liverpool be purchased with $1350 gold, when exchange sells at $4.86?

Ans. £277 15*s.* 6*d.*+

4. A merchant in Boston owes £275 10*s.* 8*d.* in Manchester, England, and wishes to remit the same by draft. Exchange is $4.82, and gold 110½: what will the draft cost him in currency?

Ans. $1467.49+.

5. A in New York owes B in Edinburgh, Scotland, $3475 in gold, and desires to remit the same by a sterling Bill: required the face of the bill, the rate of exchange being $4.84½.

Ans. £717 4*s.* 8*d.*+

6. Find the exchange value of £150, at $4.87.

Ans. $730.50.

7. Find the exchange value of £148 18*s*., at $4.86½.
Ans. $724.398+.

8. What amount of sterling exchange can be purchased for $1450, in currency, the rate being $4.90 for gold, and gold at 109½?

9. Reduce $1375 currency to sterling money, exchange at par, and gold at 108.

10. Convert $2500 gold into sterling money, at $4.88.
Ans. £512 5*s*. 10*d*.+

11. What is the value in our currency of a draft for £1000 sterling, if exchange is reckoned at $4.86, and gold at 8½% premium? *Ans.* $5273.10.

12. What will a Bill of Exchange for £260 10*s*. cost in gold at par?

591. When brokerage is to be charged for drawing or remitting a bill, *add to the exchange value of £1 the brokerage on the same, and divide the sum to be invested by the amount.*

Examples for Practice.

1. To remit $1500 to London by a Sterling Bill, I am allowed for brokerage ¼% on the cost of the bill: what amount sterling shall I remit, reckoning exchange at $4.84?
Ans. £309 2*s*. 10*d*.+

2. I employed a broker to purchase and remit to Liverpool a Bill of Exchange for £400. He purchased the bill at 4.85, and charged me ¼% brokerage. What did it cost in gold?
Ans. $1944.85.

3. Required the proceeds in sterling exchange of $5000 in currency remitted through a broker who charges ¼ commission, gold at 109½, and exchange at 4.85½. *Ans.* £938 3*s*. 5*d*.+

4. Received from Peru, S. A., a sterling bill for £216 10*s*., and sold the same through a broker at 4.84. He charged me ¼% for brokerage, and paid me in currency at the rate of 110 for gold. What were the proceeds? *Ans.* $1149.76+.

EXCHANGE BETWEEN GREAT BRITAIN AND OTHER COUNTRIES.

592. The following Table shows the *Par of Exchange* between Great Britain and the countries named :—

Amsterdam	gives	12 Florins 3 Stivers for	£1	Sterling.
Belgium	"	25 Florins 20 Centimes for	1	"
Berlin *	"	6 Thalers 25 S. Groschen for.	1	"
Bremen *	"	6 Rixdalers 6 Groten for	1	"
Brunswick and Hanover	"	6 Thalers 16 Groschen for	1	"
Calcutta	"	1 Company's Rupee for	23d.	"
Copenhagen	"	9 Rigsdalers 16 Skillings for	£1	"
Dresden and Leipsic	"	6 Thalers 25 S. Groschen for .	1	"
Egypt	"	80 Piasters for	1	"
Frankfort *	"	12 Florins for	1	"
Gibraltar	"	1 Hard Dollar for	48½d	"
Genoa	"	25 Lire Centessimi for	£1	"
Greece	"	28 Drachmes 15 Leptas for	1	"
Hamburg *	"	14 Marcs Banco for	1	"
Leghorn	"	30 Lira Toscana 5 Centessimi for	1	"
Lisbon	"	1 Milrea for	53¼d.	"
Madrid	"	1 Peso of Exchange (dollar) for	47d.	"
Milan	"	30 Lira Austriache 30 Centessimi for	£1	"
Montevideo	"	1 Peso duro for	52d.	"
Naples	"	1 Ducat for	39⅝d.	"
Palermo	"	1 Onza for	119½d.	"
Paris	"	25 Francs 20 Centimes for	£1	"
Rio Janeiro	"	1 Milrea for	35d.	"
Smyrna and the Levant	"	105 Piastres for	£1	"
St. Petersburg	"	1 Silver Rouble for	38½d.	"
Sweden	"	12 Dalers banco for	£1	"
Switzerland	"	17 Francs 5 Rappes for	1	"
United States	"	1 Dollar for	49½d.	"
Venice	"	6 Lira Austriache for	47d.	"
Vienna	"	10 Florins 2 Kreutzers for	£1	"
Warsaw	"	32 Polish Gulden for	1	"

* The rates of exchange on all places in Germany will hereafter be given in Reich-marks.

The examples and solutions which follow, will sufficiently illustrate the method of making the calculations.

593. LONDON ON NEW YORK.

EXAMPLE 1.

EXCHANGE FOR $1000. LONDON, March 16, 1871.

At sight, pay to the order of *Henry Starkweather*, ONE THOUSAND DOLLARS, in Gold, value received, and place the same to the account of BROWN, SHIPLEY & CO.

To AUGUST BELMONT & CO.,
New York.

Required the cost of the above bill at 49½.

SOLUTION.

$$\begin{array}{r} 49\frac{1}{2} \\ 1000 \\ \hline 12)49500 \\ \hline 20)4125 \\ \hline £206\ 5s. \end{array}$$

NOTE.—The reason why England draws on the United States at a par less than 54*d.* for $1, is that the standard of fineness of British coins is higher than that of the United States coins, and the weight of our coins has been reduced since the old par was established.

For comparing the quotations with our own, the Table on the next page will be found very useful.

2. What must be paid in London for a draft on New York for $578.50, at 49?

594. LONDON ON PARIS.

EXAMPLE 1. Find the cost in London of the following bill, at 25.20 *fr.* for £1.

EXCHANGE FOR 4500 FRANCS. LONDON, May 1, 1871.

At three days' sight, pay the order of *Louis G. Dromel*, FOUR THOUSAND FIVE HUNDRED FRANCS, value received, and place the same to the account of

BROWN BROTHERS & CO.

To BOWLES BROTHERS & CO.,
Paris.

SOLUTION. $4500 \div 25.20 = 178\frac{144}{252}$.

And reducing the fraction, we have, *Ans.* £178 11*s.*

2. Required the cost in London of a Bill of Exchange on Paris for 1200 francs, at 25.25.

Comparative Table, showing the approximate equivalents, expressed by different methods, corresponding to exchange quoted in pence and eighths to the dollar.

Pence to the dollar.	Dollars and decimals to pound sterling.	Percentage of the true or mint par.	Percentage of the fictitious par.	Pence to the dollar.	Dollars and decimals to pound sterling.	Percentage of the true or mint par.	Percentage of the fictitious par.
47	5.10,6	104.9	114.9	50	4.80,0	98.6	108.0
47⅛	5.09,3	104.7	114.6	50⅛	4.78,8	98.4	107.7
47¼	5.08,0	104.4	114.3	50¼	4.77,6	98.1	107.5
47⅜	5.06,6	104.1	114.0	50⅜	4.76,5	97.9	107.2
47½	5.05,3	103.8	113.7	50½	4.75,3	97.7	106.9
47⅝	5.04,0	103.6	113.4	50⅝	4.74,1	97.4	106.7
47¾	5.02,6	103.3	113.1	50¾	4.72,9	97.2	106.4
47⅞	5.01,3	103.0	112.8	50⅞	4.71,8	96.9	106.1
48	5.00,0	102.7	112.5	51	4.70,6	96.7	105.9
48⅛	4.98,7	102.5	112.2	51⅛	4.69,5	96.5	105.6
48¼	4.97,4	102.2	111.9	51¼	4.68,3	96.2	105.4
48⅜	4.96,2	102.0	111.6	51⅜	4.67,2	96.0	105.1
48½	4.94,9	101.7	111.4	51½	4.66,1	95.8	104.9
48⅝	4.93,6	101.4	111.1	51⅝	4.64,9	95.5	104.6
48¾	4.92,3	101.2	110.8	51¾	4.63,8	95.3	104.3
48⅞	4.91,1	100.9	110.5	51⅞	4.62,7	95.1	104.1
49	4.89,8	100.6	110.2	52	4.61,5	94.8	103.8
49⅛	4.88,6	100.4	109.9	52⅛	4.60,4	94.6	103.6
49¼	4.87,3	100.1	109.7	52¼	4.59,4	94.4	103.4
49⅜	4.86,1	99.9	109.4	52⅜	4.58,3	94.2	103.1
49½	4.84,9	99.6	109.1	52½	4.57,2	93.9	102.9
49⅝	4.83,7	99.4	108.8	52⅝	4.56,1	93.7	102.6
49¾	4.82,4	99.1	108.5	52¾	4.55,0	93.5	102.4
49⅞	4.81,2	98.9	108.3	52⅞	4.53,9	93.3	102.1

NOTE.—The expressions on the same line are equivalent, or practically so. Thus, exchange quoted at 49¼ pence per dollar corresponds to \$4.87$\frac{3}{10}$ to the pound, or to 100.1 of the true par, (that is, one-tenth of one per cent. premium,) or to 109.7 of the old or fictitious par, (that is, falsely, 9$\frac{7}{10}$ per cent. premium.) The true par corresponds to \$4.86,65 to the pound, or 49$\frac{3}{10}$ pence to the dollar, equivalent to 109½ per cent. fictitious percentage, and therefore £15 are equal to \$73.

EXCHANGE ON FRANCE.

595. The denominations of the money of account of France are Francs and Centimes, forming a decimal currency. 100 Centimes equal 1 Franc. The intrinsic par value of the franc in United States money is 19.3 cents, as estimated at the mint, and proclaimed January 1st, 1874, by the Secretary of the Treasury. This gives $\$1 \div .193 = 5.18\frac{26}{193}$ francs as the par of exchange. If less than so many francs sell for $1, exchange on France is above par; if more, it is below.

596. EXAMPLES FOR PRACTICE.

1. Required the cost of a Bill of Exchange on Paris for 5000 francs, at 5.16.

SOLUTION. $5000 \div 5.16 = 968.99 +$. *Ans.* 968.99+.

2. Required the face of a draft which can be purchased for $1000, exchange being quoted at $5.18.

SOLUTION. $1000 \times 5.18 = 5180$. *Ans.* 5180 francs.

3. Convert 570 francs 50 centimes into United States money, at 5 francs 15 centimes per dollar.

4. Find the value of the following Bill of Exchange at 5.16½.

EXCHANGE FOR 6000 FR. NEW YORK, Nov. 5, 1874.

At three days' sight of this FIRST OF EXCHANGE (Second and Third of same tenor and date unpaid), pay to the order of JAMES CLAXTON, *Six Thousand Francs*, for value received, and place the same to the account of

READ, DREXEL & CO.

TO HARJES, DREXEL & CO.,
Paris.

Ans. $1161.66+.

5. What is the value of 1560 francs, exchange at 5.17?
Ans. $301.74+.

6. What amount in exchange on Paris can be purchased for $1750 currency, exchange at 5.14½ and gold at 110?
Ans. 8185.18+ francs.

7. What is the value in currency of 10000 francs, at par, when gold is worth 108½ in exchange for currency? *Ans.* $2094.05.

8. Change to U. S. money 3000 francs, at 5.18, at 5.17, at 5.16, at 5.15, at par.

Ans. $5.79.15 + ; $580.27 + ; $581.39 + ; $582.52 + ; $579.

EXCHANGE ON GERMANY.

597. By the new system, adopted in 1874, to conform to the new German coinage, drafts are now drawn on all parts of the German Empire payable in *Reich-marks,* instead of Florins and Thalers. According to our mint estimate, the par value of the mark is 23.82 cents, but the current quotations, as before stated (page 328), denote the prices for four marks. The 20-mark piece (gold) is worth $4.76+, or very nearly the value of the English sovereign.

TABLE.

$$\text{10 Groschen} = \text{1 Mark.}$$

$$\left.\begin{array}{r}\text{3 Marks, or}\\ \text{30 Groschen}\end{array}\right\} = \text{1 Thaler.}$$

NOTE.—Reich means royal; therefore, reich-mark means royal mark. The simple term mark is, however, generally used.

598. EXAMPLES FOR PRACTICE.

1. What is the value in United States money of 500 marks, at par?

SOLUTION. 23.82 cents × 500 = $119.10, *Ans.*

2. I wish to remit $3000 to Berlin: required the face of a bill which can be purchased for this sum, exchange being quoted at 95.

SOLUTION. $$\frac{3000 \times 4}{.95}\text{, or, } 3000 \div \frac{.95}{4} = 12631.57+.$$

Ans. 12631.57 + marks.

3. Required the cost of a Bill of Exchange on Frankfort for 1500 marks, exchange at 95½.

4. What is the value in currency of 2000 marks, reckoning gold 110 and exchange at 96? *Ans.* $528.

5. Purchased a Bill of Exchange on Hamburg for 1800 marks, at 95⅛: required the cost of the bill. *Ans.* $428.06¼.

6. What amount of exchange on Bremen can be bought for $5000 currency, gold at 109½, and exchange at 95?
Ans. 19226.14+ marks.

7. If in London exchange on Germany sells at 20.5 marks for £1 sterling, what is the cost in English money of 1000 marks?

Note.—Sterling bills are frequently sold in this country for remittance to Germany, instead of direct exchange.

8. If at Frankfort the exchange value of £1 sterling equals 20.4 marks, how much sterling must be remitted to Frankfort to pay a debt of 3600 marks? How much will it cost in our coin if exchange on London sells at $4.85?
Ans. £176 9*s.* 4*d.*+
$855.86+.

9. Suppose in the above example direct exchange sells here at par, what would a bill for 3600 marks cost?
Ans. $857.52.

10. Find the exchange value of 1200 marks, at par, at 95, at 95½, at 95¾, at 96.
Ans. $285.84; $285; $286.50; $287.25; $288.

11. Find the exchange value in marks of $1600, at par, at 95, at 95¼, at 95½, at 96.
Ans. 6717.04+ marks; 6736.84+ marks; 6724.40+ marks; 6701.57+ marks; 6666.66+ marks.

12. Find the value in marks of $5500 in currency, exchange 95¼, and gold 110.

13. Find the cost in currency of exchange for 4500 marks, exchange at 95, and gold at 109.

EXCHANGE ON AMSTERDAM.

599. Bills of Exchange on Amsterdam are drawn payable in Florins (or Guilders). The money of account consists of Florins and Centimes. 100 centimes equal 1 florin. The par value of the florin is 40.5 cents.

EXAMPLES FOR PRACTICE.

1. What will a draft on Amsterdam for 300 florins cost at 40⅛? *Ans.* $120.37+.

2. What is the value in exchange on Amsterdam of $800 at 41⅛? *Ans.* 1945.28+ florins.

3. Find the cost in currency of a Bill for 1000 florins, exchange at 40½, and gold at 108. *Ans.* $437.40.

4. What amount of exchange on Amsterdam can be purchased for $850 in currency, exchange at 41¼, and gold at 110?

5. Received $1000 in gold to remit to Amsterdam. If I charge ¼% brokerage, and the rate of exchange is 41⅛, how many florins shall I remit?

NOTE.—To ascertain the fractional parts of a unit in any of the foregoing currencies, consult the Table on page 233.

ARBITRATION OF EXCHANGE.

600. Arbitration of Exchange is the process of computing the cost of indirect exchange between two places, by means of remittances through one or more intermediate places.

The object of arbitration of exchange is to ascertain, by a comparison of the current rates between several places, the best mode of drawing and remitting bills. If, for example, owing to the contingencies of trade, or other causes, the rate of exchange between Paris and London is such that a bill on Paris, purchased in the United States, will, when collected and the proceeds invested in exchange of Paris on London, buy a larger amount of sterling exchange than the original sum will purchase in the United States, there may be an advantage in remitting to London through Paris. This is the philosophy of indirect exchange.

Arbitration is termed either *Simple* or *Compound.*

601. Simple Arbitration consists in finding the rate or price of exchange between two places, through a third.

602. Compound Arbitration consists in finding the rate or price of exchange between two places, through several others.

EXAMPLES.

1. A merchant in the United States, wishing to remit $4000 to London, desires to ascertain whether it will be more to his advantage to purchase direct exchange on London, or make his remittance through Paris. He finds on inquiry that exchange on London is quoted at 4.85, on Paris 5.16, and that the prevailing rate between Paris and London is 24.50 francs per pound sterling: how much will be gained by remitting to London, through Paris, at these rates, allowing ⅛% for brokerage in Paris?

$4000 will purchase direct exchange on London for £824 14*s.* 10*d.*
$4000 will purchase direct exchange on Paris for 20640 francs.
Deducting brokerage, ⅛%.............. 25.80

Net proceeds to be invested in Paris.... 20614.20 francs.
20614.20 francs will purchase exchange, Paris on London, for £841 7*s.* 11*d.*

Ans. Gain by the indirect exchange, £16 13*s.* 1*d.*

2. A man in Chicago has $1500 due to him in New Orleans, and wishes to negotiate a draft for the same. Direct exchange on New Orleans is 1% discount; exchange on New York is 1½% premium, and at New York exchange on New Orleans is ½% discount: how much will be gained by drawing on New York and paying ½% brokerage for re-exchange, and for what amount must he draw on New York?

Ans. By direct exchange he would receive $1485. For his draft on New York he would receive $1522.50. The brokerage in New York equals the discount on exchange on New Orleans; therefore the face of the draft will be $1500. Gain by indirect exchange, $37.50.

603. Problems in either Simple or Compound Arbitration are readily solved by the application of the rule of *Conjoined Proportion* or *Chain Rule.*

Rule.—Arrange the terms in the form of equations, and distinguish the members as *antecedents* and *consequents.*

1. *Write the term to which a corresponding value is sought in the solution of the problem, which is called the term of demand.*

2. *To the left of this, on a line below, enter the term which is of the same denomination as the term of demand, and on the same line, connected by the sign of equality, the given term of equal value.*

3. *Under the first antecedent, enter the term which is of the same denomination as the last consequent, and on the same line, as before, its equal in value; and so on.*

4. *Divide the product of the consequents by the product of the antecedents, and the quotient will be the answer, in the same denomination as the last consequent.*

Note.—When the final remittance must be a given sum, and brokerage is deducted for re-investing and remitting at the intermediate places, place 1, minus the rate of brokerage, on the left for every deduction, to find the amount required for the original investment; but when the proceeds of a certain sum remitted are required, place 1, minus the rate of brokerage, on the right for each deduction. To find the cost of exchange, when brokerage is *paid or charged on the face of a bill*, for re-exchange, place 1, plus the rate of brokerage, on the right for each payment, and to find the proceeds of a bill, place it on the left.

Example.

I wish to remit 5000 marks to Hamburg. I can remit to Paris at 5 francs 16 centimes to the dollar; from Paris to London at 24 francs 50 centimes per £1; from London to Hamburg at 20.4 marks per £1; or I can remit to London at $4.86 per £1, and from London to Hamburg at 20.4 marks per £1. Suppose the direct exchange on Hamburg to be 23.8 cents per mark, which is the more profitable course of making the remittance—through Paris and London, through London only, or directly to Hamburg, allowing $\frac{1}{4}$% brokerage for each re-exchange?

The question may be stated thus:—What is the value in federal money of 5000 marks, if 20.4 marks = £1, and £1 = 24.50 francs, and 5.16 francs = $1? and what is the value in federal money of 5000 marks, if 20.4 marks = £1, and £1 = $4.86, taking into consideration also the charges for brokerage? Lastly, what is the cost of 5000 marks, at 23.8 cents per mark?

Applying the rule, we have the following

STATEMENTS AND OPERATIONS.

Exchange through Paris and London.

5000 marks, term of demand.
20.4 marks = £1.
£1 = 24.50 francs.
5.16 francs = $1.

1−¼% brokerage, .9975.
1−¼% " .9975.

$$\frac{5000 \times 1 \times 24.50 \times 1}{20.4 \times 1 \times 5.16 \times .9975 \times .9975} = \frac{122500}{104.7383379} = 1169.58+$$

Exchange through London.

5000 marks, term of demand.
20.4 marks = £1.
£1 = $4.86.

1−¼% brokerage, .9975.

$$\frac{5000 \times 1 \times 4.86}{20.4 + 1 \times .9975} = \frac{24300}{20.349} = 1194.16.$$

Direct Exchange.

$$5000 \times .238 = 1190.$$

The cost of exchange through Paris and London will be $1169.58.
" " " " " London " 1194.16.
" " " direct exchange " 1190.00.

Ans. It will be more profitable to remit through Paris and London.
Gain over exchange through London, $24.58.
" " direct exchange, . . . 20.42.

PROOF.

$1169.58 will purchase a bill on Paris, at 5.16, for 6035.03 francs.
Deduct brokerage, ¼%, 15.08
Amount to be invested at Paris, 6019.95 francs.

6019.95 francs will purchase a bill on London, at 24.50, for £245.70.
Deduct brokerage, ¼%, .60.
Amount to be invested at London, £245.10.

£245.10 will purchase a bill on Hamburg, at 20.4, for 5000 marks.

INVOLUTION.

604. **Involution** is the process of finding the powers of numbers.

605. A **Power** of a number is the product obtained by multiplying the number continually by itself a certain number of times.

606. When the power is formed, the root is said to be *involved*, or *raised to the power*. The number involved is the *Root*, or *First Power*.

607. If the root be multiplied by itself, or employed twice as a factor, the product is called the *Second Power*, or *Square* of that number. Thus, 4 × 4 = 16, the second power, or square of 4.

608. If the root be taken three times as a factor, the product is called the *Third Power*, or *Cube* of that number; if four times, the *Fourth Power*, and so on.

609. A power is indicated by a superior figure written at the right of the root, called the *Exponent* or *Index*. Thus, 5^2 indicates the square of 5, or that 5 is to be raised to the second power, and 2 is the exponent.

610. The exponent of the first power is 1, but is not usually written. Hence we may write,—

$$\begin{aligned} 2 &= 2, \text{ the 1st power of 2.} \\ 2^2 = 2 \times 2 &= 4, \text{ the 2d power of 2.} \\ 2^3 = 2 \times 2 \times 2 &= 8, \text{ the 3d power of 2.} \\ 2^4 = 2 \times 2 \times 2 \times 2 &= 16, \text{ the 4th power of 2.} \\ 2^5 = 2 \times 2 \times 2 \times 2 \times 2 &= 32, \text{ the 5th power of 2.} \end{aligned}$$

Powers of the same root may be multiplied together, by adding their exponents. Thus, $7^3 \times 7^2 = 7^5$.

A power may be divided by another power of the same root, by subtracting the exponent of the divisor from that of the dividend. Thus, $13^3 \div 13^2 = 13$.

A fraction may be raised to any power, by raising each of its terms to that power. Thus, $(\frac{3}{4})^2 = \frac{9}{16}$.

EXAMPLES.

1. What is the second power of 8?
2. What is the cube of 5? The fourth power?
3. Find the value 3^2, 6^3, 4^5, $(\frac{1}{3})^3$ $(\frac{3}{4})^2$.
4. Multiply 24^6 by 24^3.
5. Divide 49^7 by 49^2.
6. What is the fourth power of 3.2?
7. What is the cube of 3^2?
8 Raise $3\frac{1}{3}$ to the fourth power.

EVOLUTION.

611. Evolution, or the extraction of roots, is the reverse of Involution. It is the process of finding the root of any number regarded as a power of the root.

612. A **Root** of any number is a factor which, multiplied by itself a certain number of times, will produce that number. It is, therefore, one of the equal factors into which a number may be resolved. Thus, 7 is the square root of 49, because $7 \times 7 = 49$; and 9 is the cube root of 729, for $9 \times 9 \times 9 = 729$.

613. A root is denoted by the *Radical Sign*, $\sqrt{}$, before the power, with the index of the root over it, or by a *Fractional Exponent*. Thus, $\sqrt[3]{27}$, or $27^{\frac{1}{3}}$, denotes the cube root of 27.

The index of the square root is usually omitted, the sign, $\sqrt{}$, being used alone. Thus, $\sqrt{36}$ indicates the square root of 36.

The root of a power may be expressed either by the radical sign, or a fractional exponent. Thus, $\sqrt[3]{8^2}$, or $8^{\frac{2}{3}}$ denotes the cube root of the square of 8.

An indicated root whose value cannot be exactly expressed in numbers is called a *Surd*, or an *Irrational Quantity*. Every quantity which is not a surd is said to be *Rational*.

Numbers under the same radical sign, or index, may be multiplied together, or divided, like rational numbers, the product, or quotient, being placed under the common radical sign, or index. Thus, $\sqrt{18} \times \sqrt{3} = \sqrt{54}$; and $\sqrt[3]{75} \div \sqrt[3]{25} = \sqrt[3]{3}$.

SQUARE ROOT.

614. The **Square Root** of a number is one of the two equal factors of which it is composed, or a number which multiplied by itself will produce the given number.

615. The method of extracting the square root of a number will be best understood by squaring a number of two figures regarded as a *binomial,* and observing the combinations produced.

616. A **Binomial** is a quantity consisting of two terms, and its square is equal to *the square of the first term, plus twice the product of the first by the second, plus the square of the second.*

Let 35 be written 30 + 5. Then, squaring this number, by multiplying each part separately, we have,

$$\begin{array}{r} 30 + 5 \\ 30 + 5 \\ \hline 150 + 25 \\ 900 + 150 \quad\;\; \\ \hline 900 + 300 + 25 = 35^2 = 1225. \end{array}$$

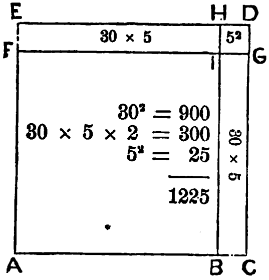
The product is composed of three parts: the first is 900, $= 30^2$; the second, 300, $= (30 \times 5) \times 2$; and the third, $25 = 5^2$.

$$\text{Hence, } (30 + 5)^2 = 30^2 + 2\,(30 \times 5) + 5^2.$$

Now, if we subtract from this power 30^2, or the square of the first term, there will remain, $2\,(30 \times 5) + 5^2$, which may be written thus:— $(\overline{2 \times 30} + 5) \times 5$.

Hence, the remainder is the product of two factors, one of which *is twice the tens plus the units, and the other the units.*

617. This involution is illustrated geometrically by the following diagram:—



Let A C be a straight line divided into two parts, A B, B C, at the point B.

Let the line A B represent the 3 tens, or 30, and B C, the 5 units.

Then, the square H D I G will be equal to a square on the unit line B C, for I G is equal to B C. The two rectangles E H F I, I G B C, are each equal in length to A B, or the tens' line, and in width to B C, the unit line, or each is equal to A B × B C. But the whole square described on A C is equivalent to the sum of the squares described on the parts, A B, B C, together with twice the rectangle contained by A B, B C; that is, $AC^2 = AB^2 + BC^2 + 2\,AB \times BC$; or

$$(30 + 5)^2 = 30^2 + 2\,(30 \times 5) + 5^2.$$

618. In the same manner it may be shown that the square of any polynomial, or quantity consisting of two or more terms, is equal to *the square of the first term, plus twice the first term into the second, plus the square of the second; plus twice the sum of the first two into the third, plus the square of the third; and so on.*

Every number may be regarded as composed of tens and units; but, if a number of three or more figures be considered a polynomial, the relation of the highest or any order to the next lower, is the same as that of tens to units; and, by squaring it, the number expressed by any two adjacent figures will be developed as a binomial. Thus, $254^2 = (200 + 50 + 4)^2 = \overline{200^2 + 2 \times 200 \times 50 + 50^2} + 2\,(200 + 50) \times 4 + 4^2$. This is equal to $(200 + 50)^2 + 2\,(200 + 50) \times 4 + 4^2$. Hence the Binomial Formula is universally applicable.

619. By reversing this process of involution, we may find the square root of any number.

First, it is necessary to determine from what parts of the power the several parts of the root are derived. This is done by separating the given number into periods of two figures each, the reason for which is explained as follows:—

The square of any number that can be expressed by one figure will always be found in the first *two places* of the power; the square of any number that can be expressed by two figures, in the first *four places;* and so on, every two places in the power will give one figure in the root. This will be apparent, if we square the lowest and highest numbers consisting of one, two, three, or more figures; as, 1 and 9, 10 and 99, etc. Thus,

Roots,	1	9	10	99	100	999	1000	9999
Squares,	$\dot{1}$	$8\dot{1}$	$\dot{1}0\dot{0}$	$9\dot{8}0\dot{1}$	$\dot{1}0\dot{0}0\dot{0}$	$9\dot{9}8\dot{0}0\dot{1}$	$\dot{1}0\dot{0}0\dot{0}0\dot{0}$	$9\dot{9}9\dot{8}0\dot{0}0\dot{1}$

The last will be a partial period, when its root is less than 4.

620. From the foregoing principles and elucidations we derive the following

RULE.—I. *Separate the number into periods of two figures each, by placing a dot over the place of units, and over every alternate figure to the left in integers, and to the right in decimals.*

II. *Find the greatest perfect square in the first period on the left, and place its root at the right, after the manner of a quotient. Subtract the square of this root from the first period, and to the remainder annex the two figures of the second period for a dividend.*

III. *Double the part of the root already found, and place it on the left for a divisor. Ascertain how many times this is contained in the dividend, exclusive of the right-hand figure, and place the quotient in the root, and also at the right of the divisor.*

IV. *Multiply the divisor thus obtained by the digit of the root last found: subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

V. *Double the part of the root already found, for a new divisor, and proceed as before; and so on, through all the periods.*

NOTES.—1. When the given number is not a perfect square, the root may be continued to as many decimal places as may be required, by annexing periods of ciphers.

2. When the trial divisor is not contained in the remainder, exclusive of the right-hand figure, place a cipher in the root, annex a cipher to the trial divisor, and bring down the next period.

3. The root of a decimal power contains as many decimal places as there are decimal periods in the power.

EXAMPLES.

1. What is the square root of 226576?

OPERATION.

$$\begin{array}{rl} & 2\dot{2}6\dot{5}7\dot{6}(476,\ \textit{Ans.} \\ & 16 \\ \hline 87) & 665 \\ 7 & 609 \\ \hline 946) & 5676 \\ & 5676 \\ \hline \end{array}$$

ANALYSIS.—Having separated the number into periods of two figures each, we find that the root will consist of a number of three figures, or of hundreds, tens, and units, and that the square of the hundreds will be found in 22, the left-hand period. We now seek for the greatest square in 22, which we find to be 16, and its root is 4. Placing this root at the right, we subtract its square, 16, from 22, and to the remainder annex the second period for a dividend. Now, this dividend must contain the product of two factors, one of which is *twice the hundreds plus the tens*, and the other the *tens*. And, since the number of tens is as yet unknown, we must find it by *trial*, and must take *twice the hundreds*, which we know is a part of the other factor, as a *trial divisor*. One place is wanting in this divisor, and for this reason, we find how many times it is contained in the dividend, exclusive of the right-hand figure. It is contained 8 times, but when 8 is annexed to the trial divisor, the divisor will be too large, for in the multiplication there will be 6 to carry. Therefore, we *try* 7, which is found to be the right number. Multiplying 87 by 7, and subtracting the product from the dividend, we obtain 56 for a remainder. Again we double the part of the root already found, or, which amounts to the same thing, add the last figure to the last divisor, to form a new trial divisor. Bringing down the last period, we ascertain how many times 94 is contained in 567, for the last figure of the root. It is contained 6 times, and placing 6 in the root, and annexing it to the trial divisor, we multiply as before. The product is equal to the dividend, and hence 476 is the root required.

2. What is the square root of 13.140625?

OPERATION.

$$\begin{array}{rl} & 1\dot{3}.1\dot{4}0\dot{6}2\dot{5}(3.625 \\ & 9 \\ \hline 66) & 414 \\ & 6396 \\ \hline 722) & 1806 \\ & 21444 \\ \hline 7245) & 36225 \\ & 36225 \end{array}$$

ANALYSIS.—The integral part of the number consists of only one period. The decimals are pointed off in periods of two figures each, beginning with the units. In other respects, the operation is the same as in whole numbers.

What is the square root of—

3. 33489 ?
4. .0196 ?
5. 22.2784 ?
6. 6700921 ?
7. 28009 ?
8. 4937284 ?

9. Find the square root of 2, correct to four decimal places.

10. Find the square root of 149.4, correct to three decimal places.

11. Find the values of the following expressions:—

1. $\sqrt{47089}$
2. $\sqrt{7.29}$
3. $\sqrt{81}$
4. $\sqrt{87^2}$
5. $\sqrt{462400}$
6. $\sqrt{.015625}$
7. $5625^{\frac{1}{2}}$
8. $\sqrt{81} \times \sqrt{25}$.
9. $\sqrt{144 \div 4}$

12. What is the square root of 44890000?

13. Find the square root of 66650896.

14. Find the square root of 8.

15. Extract the square root of 130321.

16. Extract the square root of 824464.

17. Find the square root of 366025. Of 4937284.

CONTRACTED METHOD.

621. When many decimal places are required in the root, a very good approximation may be obtained by extracting the root in the usual way to half the number of places required, or one more, and then continuing by contracted division.

EXAMPLE.

1. Find the square root of 12, correct to 8 decimal places.

OPERATION.

$\dot{1}\dot{2}$) 3.46410161 +
9
——
64)300
256
——
686)4400
4116 (over)

EXPLANATION.—We extract the root by the usual method to the fourth decimal place, and have a remainder of 1119. Then, cutting off one figure from the right of the trial divisor, we find how many times the remaining part is contained in 1119. It is contained 0 times; so, placing a cipher in the root, we cut off another figure, leaving 692, which is contained once in 1119, with a remainder of 427. Placing the quotient, 1, in the

```
      ——————
  6924)28400
       27696
      ——————
6,9,2,8,1)70400
          69281
          —————
           1119
            692
            ———
            427
            415
            ———
             12
```

root, we cut off another figure, and divide 427 by 69, observing in the multiplication to carry 1 for the figure dropped. We now have 12 remaining, which would contain 6 twice, if there were none to carry; but, for the 9 rejected 1 must be carried, and, therefore, we write 1 in the root, and the work is complete.

EXTRACTION OF THE SQUARE ROOT BY FACTORING.

622. The square root of a number which is a perfect square may be obtained by resolving it into its prime factors, and finding the continued product of one half of the several *different* factors.

EXAMPLES.

1. What is the square root of 39204?

SOLUTION.—The prime factors of this number are 2, 2, 3, 3, 3, 3, 11, 11; and 2 × 3 × 3 × 11 = 198, *Ans.*

2. Required the square root of 1764.

3. Required the square root of 131769.

4. What is the square root of 9604?

623. The root of any number whose index is any power of 2, as, the 4th, 8th, 16th root, etc., may be found by the following

RULE.—*Extract the Square Root as many times successively as 2 is contained as a factor in the index of the required root, that is, twice for the 4th root, three times for the 8th root, etc.*

EXAMPLES.

1. Extract the 4th root of 1296.

2. Find the 8th root of 5764801.

3. Find the 8th root of 20390869140625.

SQUARE ROOT OF FRACTIONS.

624. The square root of a common fraction is the square root of its numerator divided by the square root of its denominator. Hence, when both terms are perfect squares, we have the following

RULE.—*Extract the square root of each term separately; the results will be the corresponding terms of the fraction which is the required root.*

Or, *reduce the given fraction to a decimal, and then extract its root by the previous rule.*

Unless both terms of a fraction are perfect squares, the fraction is not a perfect square, and only its approximate root can be found.

Either term, however, can always be made a perfect square, without altering the value of the fraction, by multiplying both terms by the same number. For example, $\frac{7}{12} = \frac{21}{36}$; and $\sqrt{\frac{21}{36}} = \frac{\sqrt{21}}{6}$; or, $\frac{7}{12} = \frac{49}{84}$; and $\sqrt{\frac{49}{84}} = \frac{7}{\sqrt{84}}$.

APPROXIMATE SQUARE ROOT OF FRACTIONS.

625. To extract the square root of a fraction which is not a perfect square, to within less than its fractional unit.

RULE.—*Multiply the numerator by the denominator, and extract the square root of the product to within less than* 1; *divide this result by the denominator of the given fraction; the quotient will be the root required.*

NOTE.—This is equivalent to multiplying both terms by the same number, and extracting the root of both.

EXAMPLES.

1. Extract the square root of $\frac{7}{10}$ to within less than $\frac{1}{10}$.

SOLUTION. $\frac{7}{10} = \frac{70}{10^2}$, and the square root of 70, within less than 1, is 8; hence $\frac{8}{10}$ or $\frac{4}{5}$ is the required approximate root.

2. Find the square root of $\frac{5}{8}$ to within less than $\frac{1}{8}$. *Ans.* $\frac{3}{4}$.

3. Extract the square root of $\frac{7}{11}$ to within less than $\frac{1}{11}$.

Note.—To make a nearer approximation to the true root, increase the denominator to any larger square.

626. **To extract the square root of a fraction which is not a perfect square, to within less than any fractional unit.**

Rule.—*Reduce the given fraction to the same denomination as the fractional unit, and then proceed according to the previous rule.*

Examples.

1. Extract the square root of $\frac{3}{7}$ to within less than $\frac{1}{10}$.

Solution. $\frac{3}{7}=\frac{4\frac{2}{7}}{10}=\frac{42\frac{6}{7}}{10^2}$, and the square root of $42\frac{6}{7}$, to within less than .1, is 6; hence $\frac{6}{10}=\frac{3}{5}$ is the required root.

2. Extract the square root of $\frac{7}{50}$ to within less than $\frac{1}{100}$.

3. Find the square root of $\frac{2}{11}$ to within less than .001.

4. Find the square root of $\frac{1}{5}$ to within less than $\frac{1}{25}$.

5. Find the square root of $\frac{5}{8}$ to within less than .0001.

Note.—The approximate root of a fraction is usually found by reducing the fraction to a decimal, and then extracting the root by the ordinary method.

CUBE ROOT.

627. The **Cube Root** of a number is one of the three equal factors into which the number can be resolved.

628. Any number which is not the product of three equal factors is an imperfect cube, and its cube root can be found only by approximation.

629. The reasons for the several steps in the operation of extracting the cube root, will appear by cubing a number regarded as binomial.

Let 35 be written 30 + 5. Then, by cubing this number, we have—

$$\begin{array}{rrrr}
 & & 30\ + & 5\\
 & & 30\ + & 5\\ \hline
 & & 150\ + & 25\\
 & 900\ + & 150 & \\ \hline
 & 900\ + & 300\ + & 25\\
 & & 30\ + & 5\\ \hline
 & 4500\ + & 1500\ + & 125\\
27000\ + & 9000\ + & 750 & \\ \hline
27000\ + & 13500\ + & 2250\ + & 125
\end{array}$$

By examining the composition of this number, we see that

$$\begin{array}{rrl}
30^3 = & 27000 = & \text{Cube of the tens.}\\
30 \times 30 \times 3 \times 5 = & 13500 = & \text{Three times the square of the tens multiplied by the units.}\\
30 \times 3 \times 5^2 = & 2250 = & \text{Three times the tens multiplied by the square of the units.}\\
5^3 = & 125 = & \text{Cube of the units.}\\ \cline{2-2}
 & 42875 = & \text{Cube of 3 tens and 5 units.}
\end{array}$$

Or, $(30 + 5)^3 = 30^3 + (30^2 \times 3) \times 5 + (30 \times 3) \times 5^2 + 5^3$.

Hence the Binomial Formula:

The Cube of a Binomial is equal to the cube of the first term, plus three times the square of the first term multiplied by the second, plus three times the first term multiplied by the square of the second, plus the cube of the second.

And, whatever the number of terms, we have the general formula:—

The Cube of any Polynomial is equal to the cube of the first term, plus three times the square of the first term into the second, plus three times the first into the square of the second, plus the cube of the second; plus three times the square of the SUM OF THE FIRST TWO *into the third, plus three times the* SUM OF THE FIRST TWO *into the square of the third, plus the cube of the third, etc.*

Now, if from $30^3 + (30^2 \times 3) \times 5 + (30 \times 3) \times 5^2 + 5^3$, we subtract 30^3, or the cube of the first term, there will remain $(30^2 \times 3) \times 5 + (30 \times 3) \times 5^2 + 5^3$, in which we observe that 5 is a factor of every part. The remainder may, therefore, be written thus :—

$$(30^2 \times 3 + 30 \times 3 \times 5 + 5^2) \times 5.$$

Hence, the remainder is the product of two factors, one of which is composed of *three times the square of the tens, plus three times the tens multiplied by the units, plus the square of the units ;* and the other *is the units.*

Every number may be regarded as a binomial. Thus, 325 is 32 tens and 5 units. But, the development of this number, as a trinomial (300 + 30 + 5), will give—

$$300^3 + 3\,(300^2 \times 20) + 300 \times 3 \times 20^2 + 20^3 + 3\,(300 + 20)^2 \times 5 + 3\,(300 + 20) \times 5^2 + 5^3,$$

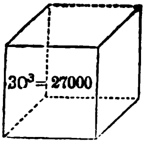
which contains the cube of the binomial, $300 + 20 = 300^3 + 3\,(300^2 \times 20) + 300 \times 3 \times 20^2 + 20^3$, and the remainder $3\,(300 + 20)^2 \times 5 + 3\,(300 + 20) \times 5^2 + 5^3$, or,

$$\left(3(300 + 20)^2 + 3\,(300 + 20) \times 5 + 5^2\right) \times 5.$$

Therefore, the remainder is the product of two factors, one of which is *three times the square of the first two terms, plus three times the first two multiplied by the third, plus the square of the third;* and the other *is the third term ;* and so of any number whatever. Hence the formula applies in all cases.

630. The involution of a binomial may be illustrated geometrically as follows :—

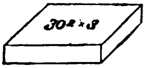
A



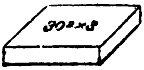
Let figure A represent a cube whose edge is 30 inches. Then, its solid contents are $30^3 = 27000$ cubic inches.

B

C

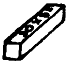


D



Now, suppose this cube to be increased to a cube of 35 inches, by additions upon three of its sides. These additions may be represented,

1st. By the three parallelopipedons, figures B, C, D, each 30 inches square and 5 inches thick. Their solid contents are $30^2 \times 5 \times 3 = 13500$ cubic inches.

E


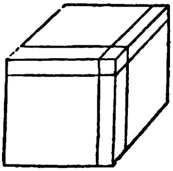
F

2d. By the three parallelopipedons, E, F, G, each 30 inches long by 5 inches square. Their solid contents are $30 \times 5^2 \times 3 = 2250$ cubic inches.

G

H

3d. By the small cube, figure H, whose edge is 5 inches, and solid contents $5^3 = 125$ cubic inches.

I


The cube thus constructed is represented by figure I, a cube of 35 inches = 42875 cubic inches.

631. By reversing this process of involution, we extract the cube root of any number.

First, as in square root, it is necessary to distinguish the parts of the power from which the several parts of the root are derived. This is done by separating the given number into periods of three figures each, the reason for which is explained as follows:—

The cube of any number expressed by one figure will always be found in the first *three places* of the power; the cube of any number expressed by two figures, in the first *six places;* and so on, every three places in the power giving one figure in the root. This may be easily demonstrated by cubing the lowest and highest numbers consisting of one, two, three, or more figures; thus—

Roots,	1	9	10	99	100	999
Cubes,	1	$72\dot{9}$	$\dot{1}00\dot{0}$	$97\dot{0}29\dot{9}$	$\dot{1}00\dot{0}00\dot{0}$	$99\dot{7}00\dot{2}99\dot{9}$

The left-hand period will be a partial period consisting of only one, or two figures, when its root is less than 5.

632. From these principles we derive the following

RULE.—I. *Separate the given number into periods of three figures each, by placing a dot over units, another over thousands,* *and so on, over every third figure to the left in integers, and to the right in decimals.*

II. *Find the greatest perfect cube in the left-hand period, and place its root on the right for the first figure of the root; subtract the cube from the left-hand period, and to the remainder bring down the next period for a dividend.*

III. *Take three times the square of the part of the root already found for a trial divisor.*

IV. *Find how many times the trial divisor is contained in the dividend, exclusive of the two right-hand figures, and write the quotient as the next figure of the root.*

V. *To complete the divisor, multiply the part of the root previously found by the last root figure, triple the product, and write it under the divisor one place to the right; square the last root figure, and write its square under the preceding product one place to the right: add these three together, and their sum will be the divisor completed.*

VI. *Multiply the complete divisor by the last root figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

VII. *Thus proceed till the whole root is obtained.*

To find each successive trial divisor, proceed as directed by part III, or, *add together the numbers added to complete the previous trial divisor, the last true divisor, and the square of the last root figure.*

NOTES.—1. When the trial divisor is greater than the dividend, exclusive of the two right-hand figures, place a cipher in the root, annex two ciphers to the trial divisor, and bring down the next period for a dividend.

2. When any divisor, completed as directed, is found to be too large, diminish the root figure till the proper divisor is obtained.

3. After having found the second figure of the root, by using the trial divisor, we may, instead of completing the divisor, cube the two figures of the root already found, and subtract the cube from the two left-hand periods; then, form a new trial divisor, find the next figure of the root,

and subtract the cube of the three figures of the root found from the three left-hand periods; and so on, till the work is completed.

EXAMPLES.

1. What is the cube root of 107171875?

OPERATION.

$10\dot{7}17\dot{1}87\dot{5}(475$

$4^3 =$ 64

1st trial divisor, $4^2 \times 3 = 48$ — 43171, 1st dividend.

$4 \times 7 \times 3 = 84$ — 39823

$7^2 = 49$

1st complete divisor, 5689 — 3348875

3348875

2d trial divisor, $47^2 \times 3 = 6627$

$47 \times 5 \times 3 = 705$

$5^2 = 25$

2d complete divisor, 669775

Ans. 475.

ANALYSIS.—Separating the given number into periods of three figures each, we see that the root will consist of three figures, and that the cube of the hundreds will be found in 107, the left-hand period. The greatest cube in 107, is 64, and therefore, its root, 4, is the first figure of the required root, which we write at the right. Next, we subtract 64 from 107, and to the remainder annex the next period to form a dividend. Now, this dividend, 43171, contains the product of two factors, one of which is *three times the square of the hundreds, or* $4^2 \times 3$, *plus three times the hundreds multiplied by the tens* (*unknown*), *plus the square of the tens;* and the other is *the tens*, or next figure of the root. The required term, being unknown, must be found by *trial.* Therefore, we take three times the square of the hundreds, or known part of one factor, as a *trial divisor;* that is $4^2 \times 3 = 48$. And since 48 represents 4800, we find how many times it is contained in the dividend, exclusive of the two right-hand figures. It is contained 8 times; but 8 is too large, because the divisor yet requires quite a large addition. Therefore, we *try* 7. Assuming this to be correct, we supply the missing parts of the factor, which we call the divisor; that is, we *add three times the hundreds multiplied by the tens, plus the square of the tens.* This completes the divisor. We now test the correctness of the work by multiplying this complete divisor (factor) by 7, the other factor, or root figure. It is found to be correct, and we subtract the product from the dividend, and annex the next period for a new dividend, with which we proceed as before, and so continue till the whole root is obtained.

633. NEW METHOD OF FORMING THE TRIAL DIVISORS

2. Required the cube root of 15126032609891.

OPERATION.

$2^3 =$

$2^2 \times 3 = 12$
$2 \times 4 \times 3 = 24$
$4^2 = 16$
1456
16
1728
$24 \times 7 \times 3 = 504$
$7^2 = 49$
177889
49
183027
$247 \times 3 \times 3 = 2223$
$3^2 = 9$
18324939
9
18347187
$2473 \times 1 \times 3 = 7419$
$1^2 = 1$
1834792891

15126032609891(24731
8
7126
5824
1302032
1245223
56809609
54974817
1834792891
1834792891

EXPLANATION. — The successive trial divisors after the first are formed by adding the numbers inclosed by the brackets. The rest of the work is performed in the same manner as in the other method. The principle of this mode of forming the trial divisors can be best explained by means of the geometrical illustration. The trial divisor is always the *surface of three sides* of the cube already found, and a completed divisor is the *surface of one side of seven additions* to be made to complete an increased cube. The surface of three sides of the cube constructed by these additions, as shown by Fig. I, page 354, is equal, in the above example (considering the first figure of the root as tens of the next lower order), to $20^2 \times 3 + (20 \times 4 \times 3) \times 2 + 4^2 \times 3 = 1728$, a result which is obtained by adding to the last complete divisor all that was added to the previous trial divisor to complete it, plus the square of the last figure of the root, as shown in the solution.

This method is at once simple and labor-saving, and will, it is believed, commend itself to general favor over any other.

EXAMPLES FOR PRACTICE.

1. What is the cube root of 48228544?

2. Find the cube root of 32461759.

3. Find the cube root of 12.977875.

4. Find the cube root of 37.8756, correct to three decimal places.

NOTE.—In cases of this kind, annex as many ciphers, or periods of ciphers, as may be required.

5. Required the cube root of 2, correct to four decimal places.

6. Required the cube root of 27.3421, correct to two decimal places.

7. Required the cube root of 3048625.

8. Find the cube roots of 531441 and 681472.

9. Find the cube roots of 456533 and 132651.

10. Find the values of $\sqrt[3]{212776173}$ and $\sqrt[3]{357911}$.

11. Find the value of $\sqrt[3]{91125} + \sqrt[3]{3645}$.

12. Find the value of $\sqrt[3]{10077696} - \sqrt[3]{15625}$.

13. Find the value of $\sqrt[3]{4096} \times \sqrt[3]{42875}$.

14. Find the value of $\sqrt[3]{681472} \div \sqrt[3]{1331}$.

15. Find the cube root of 1003003001.

16. A bushel contains 2150.42 cubic inches. What must be the inside measure of a cubical bin which will contain 100 bushels? *Ans.* 4 ft. 11.91 in. +.

17. What is the length of the side of a cube which contains 3796416 cubic feet?

CONTRACTED METHOD.

634. When the root is interminable, or many decimal places are required, a very good degree of approximation may be obtained, and the operation considerably shortened, by proceeding in the usual way to several decimal places, and then continuing the work by contracted division.

Example.

1. Required the cube root of 7.857142, correct to ten decimal places.

OPERATION.

```
      3                        7.857142(1.9880235932.
    { 27                       1
    {  81                      ----
    {  --                      6857
    { 651                      5859
    {  81                      ----
     ----                       998142
    1083                        903392
    {  456                      ------
    {    64                      94750000
    {  ----                      94470272
    { 112924                     --------
    {     64                       279728
     ------                        237128
    117612                         ------
    {    4752                       42600
    {      64                       35569
    { --------                      -----
    { 11808784                       7031
    {       64                       5928
     --------                        ----
 1,1,8,5,6,4,3,2                     1103
                                     1066
                                     ----
                                       37
                                       35
                                       --
                                        2
                                        2
                                        -
```

Explanation.—We stop here, after proceeding by the usual method to the third decimal place of the root. The remainder is 279728, and the last complete divisor, 11856432; but, instead of annexing another period of ciphers to the dividend, we cut off one figure from the right of the divisor, and see how many times the remaining part is contained in the dividend. It is contained 0 times; so placing a cipher in the quotient, we cut off another figure from the divisor, and find that the remainder is contained in the dividend 2 times. Continuing the work in this manner, cutting off one figure from the divisor at every division, we finally obtain 1.9880235932 as the required root.

METHOD BY FACTORING.

635. The cube root of a number which is a perfect cube, may be found by resolving the given number into its prime factors, and taking the continued product of one third of each of those factors.

EXAMPLE.

1. What is the cube root of 250047?

SOLUTION.—The prime factors of this number are 3, 3, 3, 3, 3, 3, 7, 7, 7; therefore, 3 × 3 × 7 = 63, the required root.

CUBE ROOT OF FRACTIONS.

636. The cube root of a fraction is the cube root of the numerator divided by the cube root of the denominator.

EXAMPLES.

1. What is the cube root of $\frac{13824}{42875}$? *Ans.* $\frac{24}{35}$.

2. What is the cube root of $\frac{729}{1331}$?

3. Required the cube root of $\frac{27}{26}$.

NOTES.—1. When one of the terms is a surd, as in this case, the fraction may be reduced to a decimal, and the root extracted to any required degree of exactness.

2. When the denominator is rational, the approximate root of the numerator may be found, and divided by the exact root of the denominator.

APPROXIMATE CUBE ROOT OF FRACTIONS.

637. To extract the Cube Root of a common fraction which is not a perfect cube, to within less than its fractional unit.

RULE.—*Multiply the numerator by the square of the denominator, and extract the cube root of the product to within less than* 1; *divide this result by the denominator, and the quotient will be the required root.*

NOTE.—This is equivalent to multiplying both terms of the fraction by the square of the denominator, and extracting the root of both terms. The denominator is thus made rational.

EXAMPLES.

1. Find the cube root of $\frac{7}{16}$ to within less than $\frac{1}{16}$.

SOLUTION. $\frac{7 \times 16^2}{16 \times 16^2} = \frac{1792}{16^3}$, and the cube root of 1792, to within less than 1, is 12; therefore $\frac{12}{16}$, or $\frac{3}{4}$, is the required root.

2. Find the cube root of $\frac{11}{13}$ to within less than $\frac{1}{13}$.

3. Required the cube root of $\frac{14}{17}$ to within less than $\frac{1}{17}$.

638. To extract the cube root of a common fraction which is not a perfect cube, to within less than any fractional unit.

RULE.—*Reduce the fraction to the same denomination as the given fractional unit, and then proceed according to the previous rule.*

EXAMPLES.

1. Required the cube root of $2\frac{4}{9}$ to within less than $\frac{1}{27}$.

SOLUTION. $2\frac{4}{9} = \frac{22}{9} = \frac{66}{27}$; and the cube root of 66, to within less than 1, is 4, and the cube root of 27 is 3; hence $\frac{4}{3}$, or $1\frac{1}{3}$, is the required root.

2. Find the cube root of .8 within .01. *Ans.* .92.

3. Find the cube root of .65 within .1.

4. Required the approximate cube root of $\frac{2}{27}$. *Ans.* $\frac{2}{3}$.

ROOTS OF ALL DEGREES.

639. Roots of higher degrees than square and cube roots may be extracted by the rules already given, when their indices contain no other factors than 2 or 3. Thus, the 6th root of a number is the square root of its cube root, or the cube root of its square root; for 6 = 2 × 3. The 9th root is the cube root of the cube root. The 12th root is the cube root of the square root of the square root of the given number; and so on.

If the index of the required root is a prime number, as 5, 7, 11, etc., or contains any other factor, than 2 or 3, the following rule may be employed:

RULE.—For extracting any root whatever.

I. *Separate the given number into periods of as many figures each as there are units in the index of the required root.*

II. *Find the greatest power in the left-hand period whose exponent corresponds with the index of the required root, and place its root as the first figure of the required root; subtract the power from the left-hand period, and to the remainder bring down the next period for a dividend.*

III. *Then, for a trial divisor, raise the part of the root already found to the power next inferior to the given power, and multiply the result by the index of the required root.*

IV. *Find how many times the trial divisor is contained in the dividend, exclusive of all the figures last brought down, except the first, and write the quotient as the next figure of the root.*

V. *Then, raise the two figures of the root found to the power whose exponent corresponds with the index of the required root, and subtract this power from the two left-hand periods of the whole given number, (unless it be found too large, in which case reduce the last root figure, and repeat the process till the right number is obtained,) and to the remainder bring down the next period for a dividend.*

VI. *Proceed in the same manner to find the next figure of the root: then subtract the power of the three figures found from the first three periods of the given number: and so continue till the required root is obtained.*

NOTE.—There are several shorter methods of extracting roots of all degrees, deduced from algebraic formulas, but they are not of sufficient importance to students of arithmetic in general to demand attention in this connection. (See Strong's Algebra, p. 313; Hutton's Mathematics, p. 64.)

640. The method by the use of logarithms is, however, superior to all others. Persons having tables of logarithms at command, and understanding their use, can apply the following

RULE.—To extract any root of a number:—

Find from a table the logarithm of the given number, and divide it by the index of the root: find from the table the number corresponding to the logarithm, and it will be the root required.

NOTE.—For practical applications of Square Root and Cube Root, see Mensuration.

PRINCIPLES AND PRACTICAL APPLICATIONS OF MECHANICAL POWERS.

641. **Mechanical Powers** are certain simple machines, or elements of machinery, which convert a small force acting through a great space into a great force acting through a small space, or *vice versa.*

They are usually considered to be six; namely, the *Lever,* the *Wheel* and *Axle,* the *Pulley,* the *Inclined Plane,* the *Wedge,* and the *Screw.* But, properly, two of these comprise the whole, —the Wheel and Axle and the Pulley being only modifications of the lever, and the Screw and the Wedge, of the inclined plane.

642. All of these machines act on the fundamental principle known as *virtual velocities.* According to this principle, the pressure or resistance is inversely as the velocities, or spaces passed through, or that would be passed through, if the piece were put in motion.

THE LEVER.

643. The **Lever** in its simplest form is a bar of metal, wood, or other substance, capable of turning on a support, called the *fulcrum.*

The mass to be raised, or resistance to be overcome, is called the *Weight.* The moving force applied at the other end of the bar is called the *Power.* The parts of the bar between the fulcrum and the points where the power and weight act are called the *Arms* of the lever.

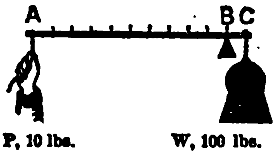
644. The Principle or Law by which the lever acts as a mechanical power is that

The power and resistance, or weight, are to each other inversely as their distances from the fulcrum, or centre of oscillation.

This is the relation between the power and weight, without reference to friction, when they *balance* each other. If the weight is to be *raised,* additional power must be applied to overcome friction, and give it motion.

645. Levers are distinguished, according to the mode in which the power is applied, as being of three kinds.

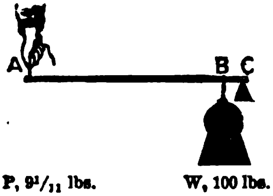
1st.



P : W :: B C : A B.
W : P :: A B : B C.
B C : A B :: P : W.
A B : B C :: W : P.

In the *first kind*, the fulcrum is placed between the power and the weight.

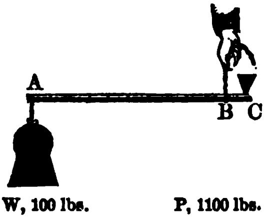
2d.



P : W :: B C : A C.
W : P :: A C : B C.
B C : A C :: P : W.
A C : B C :: W : P.

In the *second kind*, the weight is placed between the power and the fulcrum. The distance between the fulcrum and the power is greater than in the first kind, and, therefore, the effect produced by it is greater in proportion.

3d.



P : W :: A C : B C.
W : P :: B C : A C.
A C : B C :: P : W.
B C : A C :: W : P.

In the *third kind*, the power is placed between the weight and the fulcrum. This is the same as the first, except that the weight takes the place of the power, the power the place of the fulcrum, and the fulcrum the place of the weight.

646. In the levers represented by the cuts, the short arm is to the long arm as 1 to 10, and the weight is supposed to be 100 pounds.

Let A C = the length of the lever;
A B = the long arm of the lever;
B C = the short arm of the lever.

Then, in a lever of the first kind,

$$P : W :: B\,C : A\,B \cdot \text{ or, } A\,B : B\,C :: W : P.$$

Hence, 10 : 1 : : 100 lbs. : 10 lbs., the power required to balance 100 lbs.; or, $\frac{B\,C \times W}{A\,B} = P$. And, according to the principles of proportion, any one of the elements may be found, when the others are known.

EXAMPLES.

1. The long arm of a lever resting on a fulcrum, is 8 feet in length; the short arm 2 feet. What weight at the end of the short arm will balance a power of 145 lbs. applied at the end of the long arm?

SOLUTION. $2 : 8 :: 145 : W.$

$$\begin{array}{r} 8 \\ \hline 2)1160 \\ \hline 580 = W. \end{array}$$

Ans. 580 lbs.

2. At what point in a lever 15 feet long must the fulcrum be placed, so that 550 lbs. at one end may be balanced by a power or weight of 50 lbs. applied at the other end?

NOTE.—P : W : : 50 : 550; hence, the lever must be divided into two parts which shall be to each other as 50 to 550, or as 1 to 11.

3. The weight upon the short arm of a lever is 1 ton, and its distance from the fulcrum 2½ feet; what power on the long arm, 12 feet from the fulcrum, will just balance the weight?

NOTE.—In all cases of this kind, in theory, the forces which are applied are supposed to act at *right angles* to the lever, and the weight of the lever is not taken into account.

647. In a lever of the second kind,

$$P : W :: B\,C : A\,C; \text{ or, } A\,C : B\,C :: W : P.$$

Hence, 1 : 11 : : 100 lbs. : $9\frac{1}{11}$ lbs., the power required to balance 100 lbs.; or, $\frac{B\,C \times W}{A\,C} = P$.

From a lever of this kind, as shown, the greatest effect is obtained from any given amount of power.

EXAMPLES.

1. Suppose one end of a lever 8 feet long to rest upon a fulcrum, what power at the other end will sustain a weight of 350 lbs. suspended 2 feet 6 inches from the fulcrum?

Solution. $P : W :: 2\frac{1}{2} : 8$; hence, $8 : 2\frac{1}{2} :: 350 : P$.

$$\begin{array}{r} 2\frac{1}{2} \\ \hline 8)875 \\ \hline 109\frac{3}{8} = P. \end{array}$$

Ans. 109⅜ lbs.

2. Find the length of a lever of the second kind which will be required to balance a weight of 250 lbs., 10 inches from the fulcrum, by a power of 10 lbs.

3. Suppose a man wishes to raise a weight of 800 pounds with a lever of the second kind, by lifting 100 pounds, and that the distance between the fulcrum and the weight is 8 inches: at what point of the long arm must he take hold of the lever?

4. Two men are carrying a load of 120 lbs. suspended on a pole between them, 4 feet from the hands of one and 6 feet from the hands of the other: what proportion of the weight does each carry?

648. In a lever of the third kind,

$$P : W :: AC : BC; \text{ or, } BC : AC :: W : P.$$

Hence, 1 : 11 : : 100 lbs. : 1100 lbs., the power required to balance 100 lbs.; or, $\frac{AC \times W}{BC} = P$.

This kind of lever is, therefore, the least effective of all as a *power;* but, as an element of machinery, it is used where velocity is required at the expense of force, and is applied for various other purposes, as on the safety valves of steam-boilers. In the latter case, steam is the lifting power, and the weight the resistance.

EXAMPLES.

1. Required the power necessary to sustain a weight of 75 lbs. by a lever of the third kind, the distance from the fulcrum to

the weight being 7 feet, and to the point where the power is applied, 18 inches.

Solution. P : W : : 7 : 1½; hence, 1½ : 7 : : 75 : P.

$$\begin{array}{r} 7 \\ \hline P = 525 \div 1\tfrac{1}{2} = 350. \end{array}$$

Ans. 350 lbs.

2. At what distance from the fulcrum of a lever of this kind, 16 inches long, must a power of 448 lbs. be applied, to balance 56 lbs. at the end of the long arm? *Ans.* 2 inches.

3. Suppose the orifice of the safety-valve of a steam-boiler, suited to an engine of 10-horse power, to be 8 square inches, and the short arm of the lever 4 inches in length; at what distance from the fulcrum must a weight of 56 lbs. be placed, to just counteract the force of the steam, at a pressure of 30 lbs. on the square inch? *Ans.* 13¼ inches.

4. Two weights keep a horizontal lever at rest; the pressure on the fulcrum is 10 pounds, the difference of the weights is 4 pounds, and the difference of the lever arms 9 inches: what are the weights, and their lever arms?

Ans. Weights, 7lbs. and 3 lbs.
Arms, 15¾ in., and 6¾ in.

THE WHEEL AND AXLE.

649. The **Wheel and Axle** consists of a wheel fixed upon an axle, concentric with it, and is used to obtain mechanical advantage in raising great weights, by applying the power to the circumference, and attaching the weight by a rope or chain to that of the axle. It is a modification of a lever of the first kind, and, from its properties, is sometimes called a *Perpetual Lever.*

650. A portion of the wheel, which may be represented by a line drawn from the circumference to the centre, or by a spoke, is the long arm of lever, and the same in the axle is the short arm. The centre is the fulcrum.

The common windlass, used for drawing water, coal, etc., from wells and pits, and the capstan of a ship, are among the simplest forms of this machine, in which a rope or chain, with the weight attached, is wound about a cylinder or roller, moved by a crank, shifting levers, or similar means.

PRINCIPLE.—*The power is to the weight as the radius of the axle is to the radius of the wheel.*

EXAMPLES.

1. The diameter of a wheel is 4 feet, and of its axle 8 inches: what power applied to the circumference of the wheel would be required to raise a weight of 120 lbs., attached to a rope wound round the axle?

SOLUTION. P : W :: 4 : 24; hence, 6 : 1 :: 120 : P = 20 lbs., *Ans.*

NOTE.—The rule gives, of course, only the power required to hold the weight in *equilibrium*, not considering friction. Therefore, in the practical working of the machine, let it be understood that, to *raise* the weight, the power must be increased sufficiently to overcome the friction and put the wheel in motion.

2. Required the weight that can be raised by a power of 160 pounds applied at the circumference of a wheel, whose diameter is 4 feet 6 inches, the rope attached to the weight being wound round the axle whose radius is 6 inches.

3. What power will be required to raise a load of 4 tons from a pit, by a common windlass, the cylinder or roller being 6 inches in diameter, and the crank to which the power is applied 3 feet long?

4. Suppose an anchor weighing one ton is to be raised by four sailors working at the capstan of a ship, which is 15 inches in diameter, what force must each exert on his bar 4 feet from the centre of the capstan? *Ans.* 78⅛ lbs.

5. What must be the radius of the axle, if 75 lbs. on a wheel of 3 feet radius balances 1200 lbs. on the axle?

THE PULLEY.

651. The **Pulley** consists of a wheel turning on a pin in a block or frame, with a groove in its circumference. It is used in raising and lowering weights, by means of a rope which runs in the groove, the weight being attached to one end, and the power applied at the other.

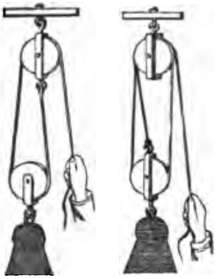


FIG. A. FIG. B.

652. A single pulley of this description affords no mechanical advantage, except that derived from the flexibility of the rope, and the facility with which the power can be applied in different directions. But when two or more are combined, as in the Block and Tackle, Figs. A and B, or as represented by Fig. C, an advantage is obtained by a distribution of the weight on the various points of support. Thus, if two only be used, one being movable, the weight will rest on two points, and therefore one half will be sustained by the power. In a system of three pulleys, two of them movable, as shown by Fig. C, the whole weight is suspended from the third; the second supports one half of it, and the first one fourth of it, which is equal to the power required to keep the whole in equilibrium; and so on for any number. Hence, *the power is equal to the weight divided by twice the number of movable pulleys.*

The number of movable and fixed pulleys in a Block and Tackle may be equal or unequal. When the end of the rope or cord is attached to the movable block, one fixed pulley more is required, in order that the power may be applied downward, as will be seen by inspecting Figs. A and B.

In a Block and Tackle, the weight bears equally on all the cords which support it, or parts of the entire cord passing between the fixed and the movable block. Hence, to find either the power or weight we have the following

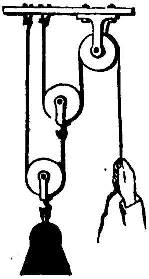


FIG. C

PRINCIPLE.—*The power is equal to the weight divided by the number of parts of the cord which support the movable block.*

NOTE.—The pulleys in a Block and Tackle are called *sheaves.*

EXAMPLES.

1. Required the power necessary to raise 2000 pounds by a Block and Tackle, having two pulleys in each block.

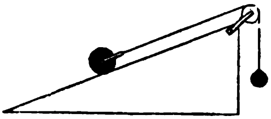
SOLUTION.—There must be, in this case, 4 supporting cords, or parts; hence, 2000 lbs. ÷ 4 = 500 lbs., *Ans.*

2. What power will be required to raise 2000 pounds by a Block and Tackle, with three sheaves in the fixed block, and two in the movable one? *Ans.* 400 lbs.

3. Required the weight that may be raised by a power of 100 pounds, with a Block and Tackle, having four sheaves in the fixed block, and three in the movable one.

THE INCLINED PLANE.

653. An **Inclined Plane** is a plane that makes an oblique angle with the plane of the horizon; a sloping plane.



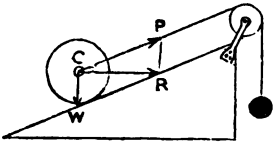
654. The principle upon which an inclined plane acts as an elementary power, may be explained as follows:—

1. When a body is raised vertically by direct force, its whole weight is supported by the elevating power, which is consequently equal to the weight elevated.

2. When a body is drawn along a horizontal plane, no part of its weight is sustained by the tractive power, which is used only to overcome the friction and other resistances opposed to its motion.

3. Therefore, if a body be drawn up an inclined plane, the ratio of the power to the weight will vary with the inclination of the plane.

Now, according to a principle known as the *Parallelogram of Forces*, if a body be held in equilibrium on an inclined plane, by a power acting parallel to the plane, a line drawn horizontally from its centre of gravity to the plane, will represent the resultant of the downward pressure of the body and the power so exerted, and will be the diagonal of a parallelogram whose adjacent sides represent these two forces when friction is not taken into account.



Let C P in this diagram represent the power, and C W the weight; then C R represents the resultant, or the resistance of the plane when these forces are in equilibrium. And, since the power used in raising a weight up the plane will be exerted through a space equal to the length of the plane, while the weight will be raised vertically only to its height at the point reached, it follows, according to the principle of *virtual velocities*, that *the power is to the weight as the height of the plane is to its length.* Hence the

RULE.—I. *Divide the weight or resistance by the ratio of inclination, or multiply it by the height of the plane, and divide the product by the length; the quotient will equal the power required to hold the weight in equilibrium on the plane.*

II. *Multiply the power by the length of the plane, and divide the product by the height, or multiply by the ratio of inclination, and the result will equal the weight, or resistance.*

EXAMPLES.

1. Required the power, or equivalent weight, capable of holding in equilibrium a load of 450 pounds upon a plane 1 in 4, or 3 feet in height and 12 feet long.

SOLUTION. $450 \div 4 = 112\frac{1}{2}$.

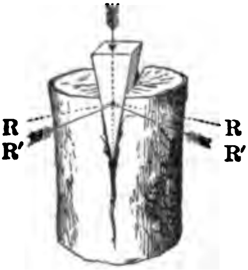
Or, $\frac{450 \times 3}{12} = 112\frac{1}{2}$. *Ans.* $112\frac{1}{2}$ lbs.

2. An inclined plane is 18 feet long, and 6 feet high: what is the weight that will be supported on it by a power of 140 pounds?

3. Required the power necessary to raise one ton up an inclined plane 12 feet in height and 90 feet in length.

4. The height of a plane is $4\frac{1}{2}$ feet, and the length 14 feet: what weight upon it will a power of 50 pounds support?

THE WEDGE.



655. The **Wedge,** as a mechanical power, is a movable inclined plane. If in the form of a right-angled triangle, the back to which the power is applied is the height of the plane, and its sloping surface the length. In a wedge of this kind, on the supposition that there is but one yielding resistance, *the power is to the resistance as the thickness or head of the wedge is to the length of the inclined side.*

656. An **Isosceles Wedge** is a double inclined plane, and the force it exerts, when acting against two yielding resistances, is divided into two equal parts. Hence, the principle is, *the power is to the resistance as one half the thickness of the back or head of the wedge is to the length of one of the inclined sides.*

657. A wedge is driven by pressure or impact. In either case, there is an equilibrium between the driving and resisting forces, when the *resultants*, R R, of these forces are at right angles to the acting surfaces of the wedge. When the wedge is driven forward, the directions of the resultants are inclined to R′ R′.

Theoretically, the resistances act at right angles to the *axis* of the wedge. Therefore, the power corresponds with the resultant of the power and weight in an inclined plane.

EXAMPLES.

1. The breadth of the back or head of an isosceles wedge is 3 inches, and its inclined sides each 10 inches long: what pressure is required to act upon the wedge to drive it between two resisting surfaces whose force is equal to 180 pounds?

SOLUTION. $\frac{180 \times 1\frac{1}{2}}{10} = 27.$ *Ans.* 27 lbs.

2. Suppose a pressure of 75 pounds to be applied to a triangular wedge, the head of which is 2 inches thick, and the side 12 inches long, what resistance on the sloping side will it counteract, if the other side remain fixed, without regarding friction?

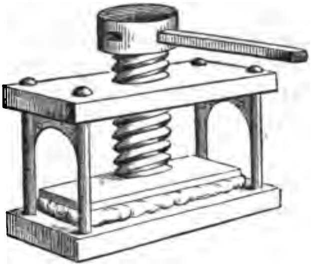
Ans. 450 lbs.

658. A wedge is usually driven by a hammer, mallet, or beetle, and in this manner its effectiveness in cleaving solid substances becomes very great. But, the conditions under which such forces are applied are so various, that an explanation of the principles involved would require more space than can be devoted to the subject here. Therefore, in each of the following examples, the measure of the "accumulated units of work," at the moment of impact, is given in terms of pressure.

3. The head of an isosceles wedge is 3 inches thick, the length of the side 6 inches: what will be the measure of its effect, if it be struck by a force equal to a pressure of 350 pounds?

4. If the head of a triangular wedge be 4 inches thick, and the length of the inclined side 7 inches, what resistance will it overcome, if struck by a hammer whose impact upon it is equal to a force of 200 pounds?

THE SCREW.



659. The **Screw,** as a mechanical power, is a modification of the inclined plane, and may be viewed as an inclined plane winding spirally round a cylinder, at a constant inclination to its axis.

660. The distance between two helices, or threads, measured on a line parallel to the axis, and called the pitch of the screw, is the height of the plane formed by one convolution, and the circumference is its base. The pitch and circumference are, therefore, two sides of a right-angled triangle, of which the helix, or thread, is the hypothenuse. Hence, the length of the plane may be found by the rule for finding the hypothenuse of a right-angled triangle, the other two sides being given; and the effective power of the screw, when the pressure is applied to its circumference, may be calculated from this, as a movable inclined plane, or wedge.

661. The power is, however, usually applied by means of a lever, which introduces another element, but does not change the principle, deduced as above, that the mechanical advantage gained by the screw, depends upon its pitch, and the length of the lever, or the circumference described by the power. Hence, we have the following general

PRINCIPLE.—*As the common distance between the threads of the screw is to the circumference of the circle described by the power, so is the power to the pressure exerted.*

NOTE.—It is immaterial to the result what the diameter of the screw is, for if it be increased or diminished, the ratio of inclination of the thread will be diminished or increased in the same proportion.

662. In the ordinary applications of this power to produce pressure, as in the common press, vice, or similar machine, both an external and an internal screw are required. An internal screw is a screw made on the interior surface of a hollow cylinder, or perforation in a block, and is called a nut or burr. It is the reverse of the external screw which is used with it. The two combined form a double inclined plane, one sliding upon the other, producing and transmitting pressure according to the mode of their arrangement.

EXAMPLES.

1. Required the effective power obtained by a screw of $\frac{7}{8}$ inch pitch, and moved by a force equal to 50 pounds at the extremity of a lever 30 inches in length.

SOLUTION. $\frac{7}{8} : (30 \times 2 \times 3.1416) :: 50 : P.$

$$\frac{30 \times 2 \times 3.1416 \times 50}{.875} = 10771 + .$$

Ans. 10771 lbs.

2. Required the power necessary to overcome a resistance equal to 7000 pounds, by a screw of $1\frac{1}{4}$ inches pitch, and moved by a lever 25 inches in length. *Ans.* 55.7 + lbs.

3. If the threads of a screw be one half an inch apart, and a power of 100 pounds be exerted at the end of a lever 36 inches long, what pressure will be produced at the end of the screw?

4. What power is necessary to raise a weight of 5 tons, by a screw whose pitch is 1 inch, if the power be applied at the end of a lever 10 feet long?

NOTE.—No account is here taken of friction, for which, in practice, considerable allowance must be made.

PROGRESSION.

663. Progression is a regular increase or decrease in a succession of numbers, called a series.

664. A **Series of Numbers in Arithmetical Progression**, or **An Arithmetical Series,** is a series increasing or decreasing by a *common difference;* as, 3, 5, 7, 9, 11, etc.; 19, 15, 11, 7, and 3.

An arithmetical series is formed by continually adding the same number to, or subtracting it from, any given number.

665. A **Series of Numbers in Geometrical Progression**, or a **Geometrical Series,** is a series increasing or decreasing by a *common ratio;* as, 2, 6, 18, 54, etc.; 64, 32, 16, 8, 4, and 2.

A geometrical series is formed by continually multiplying or dividing any given number by the same number.

666. The **Terms** of a series are the several numbers which compose it.

667. The **Extremes** are the first and last terms; the **Means,** the intermediate terms.

A series is called *Increasing* or *Ascending,* when the terms successively increase, and *Decreasing* or *Descending,* when they successively decrease.

ARITHMETICAL PROGRESSION.

In the solution of problems in arithmetical progression, there are five quantities, or elements, to be considered; viz.,

1. The first term;
2. The last term;
3. The common difference;
4. The number of terms;
5. The sum of all the terms.

Any three of these being known, the remaining two may be found.

PROBLEM I.

668. One of the extremes, the common difference, and the number of terms being given, to find the other extreme.

EXAMPLE. The first term of an arithmetical series is 2, the common difference, 3, and the number of terms, 9: what is the last term?

SOLUTION.

8, No. of terms less 1.
3, Com. difference.
—
24, Product.
2, First term added.
—
26, Last term.

ANALYSIS.—The series begins with 2; the second term is $2 + 3 = 5$; the third term is $5 + 3 = 8$; the fourth term is $8 + 3 = 11$; and so on to the last, which is formed by adding 3 for the *eighth* time, or *the number of additions is one less than the number of terms.* Hence, the last, or 9th term, will be equal to 2, the first term, plus 8 times 3, the common difference. $2 + 8 \times 3 = 26$, *Ans.*

A decreasing series is produced by continual subtraction; therefore, when the given extreme is the first term of a descending series, the product found as above must be subtracted from that term, to find the last term. Hence the

RULE.—*Multiply the common difference by the number of terms less* 1; *add the product to the first term in an ascending series; but subtract it from the first term of a descending series.*

EXAMPLES FOR PRACTICE.

1. The first term of an increasing series is 3, the common difference $\frac{1}{2}$, and the number of terms 16: what is the last term?

2. A laborer agreed to mow 20 acres of grass at 25 cents for the first acre, 50 cents for the second, and so on to the last, in arithmetical progression: what would he receive for the last acre?

3. What is the fiftieth term of the decreasing series of even numbers, commencing with 100; as 100, 98, 96, etc.?

PROBLEM II.

669. The extremes and number of terms being given, to find the common difference.

EXAMPLE. The first term of a series is $\frac{1}{4}$, the last 5, and the number of terms, 20: what is the common difference?

SOLUTION.

$5 - \frac{1}{4} = 4\frac{3}{4}$.

$4\frac{3}{4} \div 19 = \frac{1}{4}$, *Ans.*

ANALYSIS.—Since, as was shown by Problem I, the difference of the extremes is equal to the common difference multiplied by the number of terms less 1, the difference of the extremes *divided* by the number of terms less 1, will be the common difference. Hence the

RULE.—*Divide the difference of the extremes by the number of terms less* 1.

EXAMPLES FOR PRACTICE.

1. The extremes of an arithmetical series are 2 and 22, and the number of terms 25: what is the common difference?

2. A man traveled a certain distance in 5 days, going 8 miles the first day, and increasing every day by the same number of miles. His last day's journey was 48 miles: what was the daily increase?

3. Insert 8 arithmetical means between 9 and 33.

PROBLEM III.

670. To find the sum of the terms of an arithmetical series.

EXAMPLE. Find the sum of the series the first term of which is 3, the last 27, and the common difference 4.

To obtain a rule for finding the sum, let us write the series, and under it place the same with the order of the terms reversed. Then, by adding corresponding terms, we shall have :—

$$\begin{array}{r}
3 + \ \ 7 + 11 + 15 + 19 + 23 + 27 = 105. \\
27 + 23 + 19 + 15 + 11 + \ \ 7 + \ \ 3 = 105. \\
\hline
30 + 30 + 30 + 30 + 30 + 30 + 30 = 210.
\end{array}$$

Now, the successive sums are each equal to the sum of the first and last terms, and their entire sum is equal to twice the sum of the series. Therefore the sum of the series is equal to the sum of the extremes multiplied by one half the number of terms; or $\frac{27 + 3 \times 7}{2} = 105$, *Ans.* Hence the

RULE.—*Multiply the sum of the extremes by the number of terms, and divide the product by 2.*

1. The extremes of an arithmetical series are 80 and 4, and the number of terms 20: what is the sum of the series?

2. How many times does a clock strike in 12 hours?

3. Suppose 50 eggs to be laid one yard apart in a straight line, the first one yard from a basket: how far must a person travel to pick them up singly, and lay them in the basket, taking that as the starting-point?

4. A falling body falls $16\frac{1}{12}$ feet the first second, and increases in velocity at the rate of $32\frac{1}{6}$ feet per second: how far will it fall in 15 seconds? *Ans.* 3600 feet.

NOTE.—The acquired velocity at the end of 15 seconds is $15 \times 32\frac{1}{6}$. This is the last term, and 15 is the number of terms. (See page 418.)

5. What is the difference between simple and annual interest on $1000, at 6%, for 10 years, nothing having been paid till the end of the term?

NOTE.—At annual interest, the first term, regarding the series as ascending, or reversing its natural order, is the principal, and the last term, its amount at simple interest for 9 years. The number of terms is 10. Thus the series is $1000, $1060, $1120, etc.

GEOMETRICAL PROGRESSION.

671. In **Geometrical Progression**, as in Arithmetrical Progression, there are five quantities to be considered in the solution of Problems; viz.,

1. The first term;
2. The second term;
3. The common ratio;
4. The number of terms;
5. The sum of all the terms.

Any three of these being known, the other two can be found.

PROBLEM I.

672. One of the extremes, the ratio, and the number of terms being given, to find the other extreme.

EXAMPLE. The first term of an ascending geometrical series is 2, the ratio 3, and the number of times 7: what is the last term?

SOLUTION.
$3^6 = 729$
2, 1st term.
——
1458, *Ans.*

ANALYSIS.—The series begins with 2; the second term is 2 × 3 = 6; the third term is 6 × 3 = 18; and so on to the seventh term, which is found by multiplying by 3 for the *sixth* time; that is, 3 is taken as a factor one time less than the number of terms. Therefore, the last term is,

$$2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3, \text{ or } 2 \times 3^6 = 729.$$

In a descending series, the ratio is a whole or mixed number taken as a *divisor*, or a proper fraction taken as a *multiplier*.

An ascending series can be produced only by a common multiplier which is a whole or mixed number, or its equivalent.

Hence, whether the ratio be an integer or a fraction, regarding it always as a multiplier of the first and successive terms, we have the following

RULE.—I. When the first term is given, *multiply it by the ratio raised to a power one degree less than the number of terms; the product will be the last term.*

II. When the last term is given, *divide it by the ratio raised to a power one degree less than the number of terms; the quotient will be the first term.*

NOTE.—When the ratio of a descending series is given as a whole number, it is evident that its reciprocal may be employed, and the first part of the rule applied.

EXAMPLES FOR PRACTICE.

1. The first term of a geometrical series is 64, the ratio $\frac{1}{2}$, and the number of terms 7: what is the last term? *Ans.* 1.

2. The last term of a geometrical series is 18750, the ratio 5, and the number of terms 6: required the first term.

3. What is the 10th term of the series, $\frac{1}{2}$, $1\frac{1}{2}$, $4\frac{1}{2}$, $13\frac{1}{2}$, etc.?

4. What is the amount of $100, for 10 years, at 6 per cent. per annum, compound interest?

Note.—The principal is the first term, 1.06 the ratio, and 11 the number of terms.

PROBLEM II.

673. The extremes and number of terms being given, to find the ratio.

From the last rule we deduce the proposition that, if the greater extreme be divided by the less, the quotient will be the common ratio raised to a power one degree less than the number of terms. Hence the

Rule.—*Divide the greater extreme by the less, and extract that root of the quotient which is indicated by the number of terms less* 1.

EXAMPLES.

1. The first term of a geometrical series is 1, and the last 64: what is the ratio? *Ans.* 2.

2. Insert two mean proportionals between 7 and 56.

Note.—Since two mean proportionals are required, the number of terms will be 4.

3. The extremes of a series are $\frac{1}{3}$ and $\frac{1}{192}$, and the number of terms 7: required the ratio.

PROBLEM III.

674. To find the sum of the terms of a geometrical series.

Example. Find the sum of the series, 1, 3, 9, 27, 81.

OPERATION.

$$81 \times 3 = 243$$

$$\frac{243-1}{2} = 121, \textit{ Ans.}$$

Analysis.—The sum of the series is equal to 3 times the last term minus the first term, divided by the ratio less 1. To prove this, let the terms of the given series be multiplied by the ratio 3: the result will be a new series 3 times as great as the original one. Now, if from this derived series we subtract the given series, the remainder will be equal to twice the sum of the latter, as shown below :

$$\begin{array}{rl} 3,\ 9,\ 27,\ 81,\ 243 = & 3 \text{ times the series.} \\ \underline{1,\ 3,\ 9,\ 27,\ 81,} = & 1 \quad \text{"} \quad \text{"} \quad \text{"} \\ 243 - 1 = & 2 \quad \text{"} \quad \text{"} \quad \text{"} \end{array}$$

But, 243 is 3 times the last term, 81; and 243 minus 1, the first time, equals, by the above demonstration, twice the sum of the series.

Therefore, $\overline{243 - 1} \div 2 = 121$, the sum of the series. Hence the

Rule.—*Multiply the greater extreme by the ratio, and divide the difference between this product and the less extreme by the difference between* 1 *and the ratio.*

Examples for Practice.

1. The first term of a descending geometrical series is 800, the ratio 2, and the last term 50: what is the sum of the series?

2. A man agreed to labor for ten days upon the following terms:—one mill for the first day, one cent for the second, one dime for the third, and so on in geometrical progression: what should he receive for his ten days' labor?

Ans. $1,111,111.111.

3. What is the sum of a series of 10 terms, of which 40 is the first term, 4 the common ratio, and $\frac{5}{32768}$ the last term?

4. What is the sum of the first ten terms in the series, 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, etc.?

Problem IV.

675. The first term, the ratio, and the number of terms being given, to find the sum of the series.

The product of the last term by the ratio, is the same as the product found by multiplying the first term by the ratio raised to the power indicated by the number of terms. Hence, from Problems I and III, we derive the following

Rule.—*Raise the ratio to a power whose index is equal to the number of terms, and multiply this power by the first term; from the product subtract the first term, and divide the remainder by the ratio less* 1.

Note.—This rule is employed in computing the values of annuities.

EXAMPLES.

1. The first term of a geometrical series is 5, the ratio 3, and the number of terms 6: what is the sum of the series?

SOLUTION. $$\frac{\overline{3^6 \times 5} - 5}{3 - 1} = 1820, \textit{Ans.}$$

2. The first term is 4, the ratio 5, and the number of terms 7: required the sum of the series.

3. A man receives an annual income of $1000, and invests it at 6 per cent. compound interest: how much will it amount to at the end of 5 years?

SOLUTION.—The first term (or the last, taking the series as descending), is $1000; the common ratio is 1.06, and the number of terms 5. Hence,

$$\frac{\$1000 \times 1.06^5 - \$1000}{.06} = \$5637.10, \textit{Ans.}$$

DESCENDING INFINITE SERIES.

676. Infinite in mathematics, is a term applied to quantities which are greater than any assignable quantity.

An **Infinite Descending** series of numbers is a series decreasing *without limit,* or to infinity. In such a series the terms, though mathematically never ending, become *infinitely small,* or less than any assignable quantity; so that, for the purpose of finding the sum of the series, 0 may be regarded as the last term. Hence,

To find the sum of an infinite decreasing series, we have the following

RULE.—*Multiply the first term by the ratio, and divide the product by the ratio less* 1.

EXAMPLES.

1. What is the sum of the infinite decreasing series, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, etc.? *Ans.* 16.

2. Find the sum of the series 1, $\frac{1}{3}$, $\frac{1}{9}$, etc., carried to infinity.

3. What is the sum of the infinite decreasing series, of which .25 is the first term, and 100 the ratio?

ANNUITIES.

677. An **Annuity** is a sum of money payable annually. The term is also applied to payments to be made at any regular intervals of time; as, semi-annually, quarterly, monthly, etc.

678. A **Certain Annuity** is one payable for a definite length of time.

679. A **Contingent Annuity** is one whose commencement, continuance, or termination is uncertain, or depends on the occurrence of some contingent event, as the death of an individual or individuals.

680. A **Perpetual Annuity** is one which continues forever.

An annuity, whether certain, contingent, or perpetual, may be *immediate or in possession*, *deferred or reversionary*.

An annuity *in possession*, or an *Immediate annuity*, is one already entered on, or which commences at once.

A *Deferred* annuity is one whose term begins at some specified future time.

An annuity *in reversion* is one that reverts, or descends, to one person on the death of another, or whose term begins on the occurrence of some event.

A Contingent annuity, dependent on the life of an individual, is also called a *Life annuity.*

681. An **Annuity Foreborne,** or in arrears, is one whose payments have not been made when due.

682. The **Amount** of an **Annuity Foreborne** is the sum of all the payments due, and interest on them severally, from the time they become due to the end of the given time.

683. The **Final Value** of an *Annuity Certain* is what it would amount to at its termination, if nothing should be paid till that time.

684. The **Final Value** of a *Life Annuity* is the sum to which it would amount at the end of a term called the "Expectation of Life" of the individual on whose life it depends.

685. The **Present Value,** or **Present Worth** of an annuity, is such a sum as put at interest at the given rate, would, in the given time, amount to its *Final Value.*

686. The **Present Worth** of a *Perpetual Annuity* is a sum which will yield an amount of interest equal to annuity.

TABLE,

SHOWING THE EXPECTATION OF LIFE IN YEARS AND HUNDREDTHS OF A YEAR, ACCORDING TO THE CARLISLE AND WIGGLESWORTH TABLES OF MORTALITY. SEE PAGE 226.

Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.
0	38.72	28.15	24	38.59	32.70	48	22.80	22.27	72	8.16	9.14
1	44.68	36.78	25	37.86	32.33	49	21.81	21.72	73	7.72	8.69
2	47.55	38.74	26	37.14	31.93	50	21.11	21.17	74	7.33	8.25
3	49.82	40.01	27	36.41	31.50	51	20.39	20.61	75	7.01	7.83
4	50.76	40.73	28	35.69	31.08	52	19.68	20.05	76	6.69	7.40
5	51.25	40.88	29	35.00	30.66	53	18.97	19.49	77	6.40	6.99
6	51.17	40.69	30	34.34	30.25	54	18.28	18.92	78	6.12	6.59
7	50.80	40.47	31	33.68	29.83	55	17.58	18.35	79	5.80	6.21
8	50.24	40.14	32	33.03	29.43	56	16.89	17.78	80	5.51	5.85
9	49.57	39.72	33	32.36	29.02	57	16.21	17.20	81	5.21	5.50
10	48.82	39.23	34	31.68	28.62	58	15.55	16.63	82	4.93	5.16
11	48.04	38.64	35	31.00	28.22	59	14.92	16.04	83	4.65	4.87
12	47.27	38.02	36	30.32	27.78	60	14.34	15.45	84	4.39	4.66
13	46.51	37.41	37	29.64	27.34	61	13.82	14.86	85	4.12	4.57
14	45.75	36.79	38	28.96	26.91	62	13.31	14.26	86	3.90	4.21
15	45.00	36.17	39	28.28	26.47	63	12.81	13.66	87	3.71	3.90
16	44.27	35.76	40	27.61	26.04	64	12.30	13.05	88	3.59	3.67
17	43.57	35.37	41	26.97	25.61	65	11.79	12.43	89	3.47	3.56
18	42.87	34.98	42	26.34	25.19	66	11.27	11.96	90	3.28	3.73
19	42.17	34.59	43	25.71	24.77	67	10.75	11.48	91	3.26	3.32
20	41.46	34.22	44	25.09	24.35	68	10.23	11.01	92	3.37	3.12
21	40.75	33.84	45	24.46	23.92	69	9.70	10.50	93	3.48	2.40
22	40.04	33.46	46	23.82	23.37	70	9.18	10.06	94	3.53	1.98
23	39.31	32.08	47	23.17	22.83	71	8.65	9.60	95	3.53	1.62

INTEREST ON ANNUITIES.

687. Interest on annuities is either simple or compound. Annuities in arrears are properly chargeable with simple interest only, like other debts.

In estimating the *Present* and *Final Values* of annuities, the interest is generally compounded.

PROBLEM I.

688. To find the amount of an annuity in arrears, at simple interest.

EXAMPLE. Required the amount of an annual annuity of $500, in arrears for 5 years, at 6 per cent. simple interest.

SOLUTION.

Sum of the five payments .... $500 × 5 = $2500

$$\left.\begin{array}{lllll}\text{Interest of \$500} & \text{for} & 4 & \text{years} \\ \text{“} \quad\quad\quad \text{“} & \text{“} & 3 & \text{“} \\ \text{“} \quad\quad\quad \text{“} & \text{“} & 2 & \text{“} \\ \text{“} \quad\quad\quad \text{“} & \text{“} & 1 & \text{“}\end{array}\right\} = \text{Int. of \$500 for 10 years} = \underline{300}$$

Amount, $2800

ANALYSIS.—The first payment becomes due at the end of the first year, and, consequently, draws interest to the date of settlement, or four years. The second payment is in arrears three years, and, therefore, is chargeable with interest for that time. For the same reason, the third payment draws interest for two years, and the fourth, for one year. But, 4 + 3 + 2 + 1 years' interest equals 10 years' interest; and hence, we may find the interest on $500 for 10 years at once, instead of on each of the payments separately. The amount is computed in the same manner as *annual interest.* Hence the

RULE.—*Multiply the annuity by the number of payments due, and to the product add the interest on the several payments to the end of the term; the sum will be the required amount.*

The Rule for finding the sum of a series in Arithmetical Progression, also applies. (See Problem III, page 377, Example 4, note.)

SOLUTION BY PROGRESSION.

$500, First term.
620, Last term, amount of $500 in 4 years.

1120, Sum of the extremes.
5, Number of terms.

2)5600, Product.

$2800, Amount of the annuity.

EXAMPLES FOR PRACTICE.

1. What is the amount of an annual rent of $800, unpaid for 7 years, at 7 per cent. interest?

2. A gentleman deposits, for the benefit of his son, $200 annually, for 15 years, in a savings bank, which pays 5 per cent.: what will the deposits amount to at the end of the fifteenth year?

3. Required the amount of an annuity of $1500, in arrears for 7 years, at 7% simple interest.

ANNUITIES AT COMPOUND INTEREST.

689. An annuity accumulating at compound interest forms a geometrical series, of which the first term (considering the series as ascending) is the first payment; the common ratio is 1 plus the rate of interest expressed decimally, and the last term, the amount of the first payment for the given time. Hence, problems under this head may be solved by the Rules in Geometrical Progression, though, on account of the tediousness of the calculations, Tables are generally employed in practice.

PROBLEM II.

690. To find the amount, or Final Value of an annuity, at compound interest for any given time.

EXAMPLE. Find the Final Value of an annuity of $1000, running 10 years, at 6% compound interest.

SOLUTION.—The Final Value is the sum of the series found by the rule for Problem IV in Geometrical Progression. Hence,

$$\frac{\$1000 \times 1.06^{10} - \$1000}{.06} = 13180.80, \textit{Ans.}$$

Hence the

RULE.—*Find the sum of a Geometrical Series, of which the first term is the annuity, the ratio the amount of a unit for one interval, and the number of terms the number of payments due.*

NOTE.—To save the labor of involving the ratio, take the amount of $1 for 10 years, found in the Compound Interest Table, page 203. This corresponds with the 10th power of the ratio.

EXAMPLES FOR PRACTICE.

1. What will an annuity of $450 amount to in 8 years, at 4 per cent. compound interest?

2. A widow has a life interest in an estate, amounting to $1300 annually: what is the final value of her interest, at 6 per cent. per annum, if her "Expectation of Life" is 15 years?

3. If a person should save $100 per annum, and invest it at 7% compound interest, what would he be worth in 20 years?

PROBLEM III.

691. To find the present worth of a certain annuity.

RULE.—*First find the Amount or Final Value of the annuity by the previous rules; then divide this amount by the amount of* $1 *for the given time.*

NOTES.—1. When compound interest is allowed, take the amount of $1 for the divisor from the Table, page 203.

2. The Present Worth of a deferred or reversionary annuity, is a sum which, if put at interest *at the present time*, will amount to its Final Value at its expiration.

EXAMPLES.

1. What is the Present Value of an annuity of $600 for 15 years, allowing 6 per cent. compound interest?

SOLUTION.—The Final Value found by Prob. II, is $13965.58; and $13965.58 ÷ 2.396558, the amount of $1 for 15 years = $5287.34, *Ans.*

2. A widow is entitled to a pension of $96 per annum for life. Suppose her "Expectation of Life" to be 20 years, what is the present value of the pension, at 6 per cent. compound interest?
Ans. $1121.06.

3. What is the Present Value of an annuity of $300, to commence 5 years hence, and continue 10 years, at 5 per cent. compound interest?

Solution.—The final value for 10 years = \$3773.367.
Amount of \$1 for 15 years = \$2.078928.
\$3773.367 ÷ 2.078928 = \$1815.05, *Ans.*

4. What is the present worth of a Perpetual Annuity of \$1000 per annum, at 5 per cent.?

Solution.—The Present Worth is a sum which will produce \$1000 interest per annum, and hence, by dividing \$1000 (the interest) by the rate per cent., we obtain the principal, \$1000 ÷ .05 = \$20,000, *Ans.*

ANNUITY TABLES.

692. By the use of Annuity Tables, constructed upon the principles of the foregoing rules, the solution of problems is rendered very simple.

The following Table exhibits the Amount and Present Value of an annuity of \$1, at various rates of compound interest, for any number of years from 1 to 50 inclusive. Hence,

693. To find either the Amount or Present Worth of an annuity, we have the following

Rule.—*Multiply the Amount or Present Value of* \$1 *for the given time, as found in the Table, by the given annuity.*

694. To find the Present Value of an annuity deferred or in reversion.

Rule.—*Find, from the Table, the Present Worth of an annuity of* \$1 *until the annuity commences, and also until it terminates. Then multiply the difference of these values by the annuity.*

695. To find the annuity, the time, rate per cent., and Amount or Present Worth being given.

Rule.—*Divide the Amount or Present Worth, by the Amount or Present Worth of* \$1 *for the given time.*

Note.—When the payments are due semi-annually, quarterly, etc., take the number of years in the Table which is equal the number of intervals, and find the amount at a rate of interest less than the given rate in the ratio of one year to one interval. Thus, the amount of \$100 for 5 years, at 6 per cent. per annum, payable semi-annually, is equal to the amount of \$100 for 10 years, at 3 per cent. per annum, payable annually.

TABLE,

SHOWING THE AMOUNT OR FINAL VALUE, AND ALSO THE PRESENT WORTH OF AN ANNUITY OF ONE DOLLAR, TO CONTINUE FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY, AT 4, 5, 6, AND 7 PER CENT. COMPOUND INTEREST.

Years.	Amount of an Annuity of $1.				Present Worth of an Annuity of $1.			
	4 per cent.	5 per cent.	6 per cent.	7 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.000 000	1.000 000	1.000 000	1.000 000	0.961 538	0.952 381	0.943 396	0.934 579
2	2.040 000	2.050 000	2.060 000	2.070 000	1.886 095	1.859 410	1.833 393	1.808 017
3	3.121 600	3.152 500	3.183 600	3.214 900	2.775 091	2.723 248	2.673 012	2.624 314
4	4.246 464	4.310 125	4.374 616	4.439 943	3.629 895	3.545 951	3.465 106	3.387 209
5	5.416 323	5.525 631	5.637 093	5.750 739	4.451 822	4.329 477	4.212 364	4.100 195
6	6.632 975	6.801 913	6.975 319	7.153 291	5.242 137	5.075 692	4.917 324	4.766 537
7	7.898 294	8.142 008	8.393 838	8.654 021	6.002 055	5.786 373	5.582 381	5.389 286
8	9.214 226	9.549 109	9.897 468	10.259 803	6.732 745	6.463 213	6.209 744	5.971 295
9	10.582 795	11.026 564	11.491 316	11.977 989	7.435 332	7.107 822	6.801 692	6.515 228
10	12.006 107	12.577 893	13.180 795	13.816 448	8.110 896	7.721 735	7.360 087	7.023 577
11	13.486 351	14.206 787	14.971 643	15.783 599	8.760 477	8.306 414	7.886 875	7.498 669
12	15.025 805	15.917 127	16.869 941	17.888 451	9.385 074	8.863 252	8.383 844	7.942 671
13	16.626 838	17.712 983	18.882 138	20.140 643	9.985 648	9.393 573	8.852 683	8.357 635
14	18.291 911	19.598 632	21.015 066	22.550 488	10.563 123	9.898 641	9.294 984	8.745 452
15	20.023 588	21.578 564	23.275 970	25.129 022	11.118 387	10.379 658	9.712 249	9.107 898
16	21.824 531	23.657 492	25.670 528	27.888 054	11.652 296	10.837 770	10.105 895	9.446 632
17	23.697 512	25.840 366	28.212 880	30.840 217	12.165 669	11.274 066	10.477 260	9.763 206
18	25.645 413	28.132 385	30.905 653	33.999 033	12.659 297	11.689 587	10.827 603	10.059 070
19	27.671 229	30.539 004	33.759 992	37.378 965	13.133 939	12.085 321	11.158 116	10.335 578
20	29.778 079	33.065 954	36.785 591	40.995 492	13.590 326	12.462 210	11.469 421	10.593 997
21	31.969 202	35.719 252	39.992 727	44.865 177	14.029 160	12.821 153	11.764 077	10.835 527
22	34.247 970	38.505 214	43.392 290	49.005 739	14.451 115	13.163 003	12.041 582	11.061 241
23	36.617 889	41.430 475	46.995 828	53.436 141	14.856 842	13.488 574	12.303 379	11.272 187
24	39.082 604	44.501 999	50.815 577	58.176 671	15.246 963	13.798 642	12.550 358	11.469 334
25	41.645 908	47.727 099	54.864 512	63.249 030	15.622 080	14.093 945	12.783 356	11.653 583
26	44.311 745	51.113 454	59.156 383	68.676 470	15.982 769	14.275 185	13.003 166	11.825 779
27	47.084 214	54.669 126	63.705 766	74.483 823	16.329 586	14.643 034	13.210 534	11.986 709
28	49.967 583	58.402 583	68.528 112	80.697 691	16.663 063	14.898 127	13.406 164	12.137 111
29	52.966 286	62.322 712	73.639 798	87.346 529	16.983 715	15.141 074	13.590 721	12.277 674
30	56.084 938	66.438 848	79.058 186	94.460 786	17.292 033	15.372 451	13.764 831	12.409 041
31	59.328 335	70.760 790	84.801 677	102.073 041	17.588 494	15.592 811	13.929 086	12.531 814
32	62.701 469	75.298 829	90.889 778	110.218 154	17.873 552	15.802 677	14.084 043	12.646 555
33	66.209 527	80.063 771	97.343 165	118.933 425	18.147 646	16.002 549	14.230 230	12.753 790
34	69.857 909	85.066 959	104.183 755	128.258 765	18.411 198	16.192 204	14.368 141	12.854 009
35	73.652 225	90.320 307	111.434 780	138.236 878	18.664 613	16.374 194	14.498 246	12.947 672
36	77.598 314	95.836 323	119.120 867	148.913 460	18.908 282	16.546 852	14.620 987	13.035 208
37	81.702 246	101.628 139	127.268 119	160.337 400	19.142 579	16.711 287	14.736 780	13.117 017
38	85.970 336	107.709 546	135.904 206	172.561 020	19.367 864	16.867 893	15.846 019	13.193 473
39	90.409 150	114.095 023	145.058 458	185.640 292	19.584 485	17.017 041	14.949 075	13.264 928
40	95.025 516	120.799 774	154.761 966	199.635 112	19.792 774	17.159 086	15.046 297	13.331 709
41	99.826 536	127.839 763	165.047 684	214.609 570	19.993 052	17.294 368	15.138 016	13.394 120
42	104.819 598	135.231 751	175.950 645	230.632 240	20.185 627	17.423 208	15.224 543	13.452 449
43	110.012 382	142.993 339	187.507 577	247.776 496	20.370 795	17.545 912	15.306 173	13.506 962
44	115.412 877	151.143 006	199.758 032	266.120 851	20.548 841	17.662 773	15.383 182	13.557 908
45	121.029 392	159.700 156	212.743 514	285.749 311	20.720 040	17.774 070	15.455 822	13.605 522
46	126.870 568	168.685 164	226.508 125	306.751 763	20 884 654	17.880 067	15.524 370	13.650 020
47	132.945 390	178.119 422	241.098 612	329.224 386	21.042 936	17.981 016	15.589 028	13.691 608
48	139.263 206	188.025 393	256.564 529	353.270 098	21.195 131	18.077 158	15.650 027	13.730 474
49	145.833 734	198.426 663	272.958 401	378.999 000	21.341 472	18.168 722	15.707 572	13.766 799
50	152.667 084	209.347 976	290.335 905	406.528 929	21.482 185	18.255 925	15.761 861	13.800 746

EXAMPLES.

1. What will an annuity of $700 amount to in 15 years, at 5 per cent. compound interest?

2. What is the present worth of an annuity of $150 for 20 years, at 7 per cent. compound interest?

3. A will be entitled, after 3 years, to the rent of certain premises which are leased for 10 years from the present time, at $1000 per annum: what is his interest worth now, at 6 per cent. compound interest?

SOLUTION. $7.360087, Present worth of $1 for 10 years.
2.673012 " " " " 3 "
——— —
$4.687069 " " " " 7 yrs., def. 3 yrs
1000
———
$4687.069, Present worth of the annuity.

The present value of $1000 for 7 years, to commence in 3 years, is equal to its present worth for 10 years, less its present worth for 3 years. To prove this, find the *final value* of $1000 for 7 years, and divide it by the amount of $1 for 10 years.

4. A gentleman wishes to invest annually, for the benefit of his son, a sum of money sufficient to amount to $27000 at the end of 10 years: what must he invest every year, allowing 6 per cent. compound interest? *Ans.* $2048.43.

5. On the death of A, who is 55 years of age, B will be entitled to a legacy of $8000: what is its present value, at 6 per cent. compound interest? *Ans.* $2873.376.

6. Suppose C is entitled in perpetuity to the interest of $1000 at 3 per cent., payable annually, what is the value of the annuity, reckoning interest at 6 per cent.? *Ans.* $5000.

NEW YORK RULE,

For computing the value of life estates and annuities.

696. In New York, the following method of estimating the value of life estates is prescribed by the 84th rule of the Supreme Court, which is as follows:—

Rule 84.—*Gross sum, in payment of life estates, how ascertained.*

Whenever a party, as a tenant for life, or by the courtesy, or in dower, is entitled to the annual interest or income of any sum paid into court and invested in permanent securities, such party shall be charged with the expense of investing such sum, and of receiving and paying over the interest or income thereof; but if such party is willing and consents to accept a gross sum in lieu of such annual interest or income for life, the same shall be estimated according to the then value of an annuity at six per cent. on the principal sum, during the probable life of such person according to the Portsmouth or Northampton Tables.

ANNUITY TABLE.

697. A table corresponding with the Northampton Tables, referred to in the above rule, showing the value of an annuity of one dollar, at six per cent., on a single life, at any age from one year to ninety-four, inclusive.

Age.	No. of years' purchase the annuity is worth.	Age.	No. of years' purchase the annuity is worth.	Age.	No. of years' purchase the annuity is worth.	Age.	No. of years' purchase the annuity is worth.
1	10.107	25	12.063	49	9.563	73	4.781
2	11.724	26	11.992	50	9.417	74	4.565
3	12.348	27	11.917	51	9.273	75	4.354
4	12.769	28	11.841	52	9.129	76	4.154
5	12.962	29	11.763	53	9.980	77	3.952
6	13.156	30	11.682	54	8.827	78	3.742
7	13.275	31	11.598	55	8.670	79	3.514
8	13.337	32	11.512	56	8.509	80	3.281
9	13.335	33	11.423	57	8.343	81	3.156
10	13.285	34	11.331	58	8.173	82	2.926
11	13.212	35	11.236	59	7.999	83	2.713
12	13.130	36	11.137	60	7.820	84	2.551
13	13.044	37	11.035	61	7.637	85	2.402
14	12.953	38	10.929	62	7.449	86	2.266
15	12.857	39	10.819	63	7.253	87	2.138
16	12.755	40	10.705	64	7.052	88	2.031
17	12.655	41	10.589	65	6.841	89	1.882
18	12.562	42	10.473	66	6.625	90	1.689
19	12.477	43	10.356	67	6.405	91	1.422
20	12.398	44	10.235	68	6.179	92	1.136
21	12.329	45	10.110	69	5.949	93	0.806
22	12.265	46	9.90	70	5.716	94	0.518
23	12.200	47	9.846	71	5.479		
24	12.132	48	9.707	72	5.241		

Rule.—*Calculate the interest at six per cent. for one year, upon the sum to the income of which the person is entitled. Multiply this interest by the*

number of years' purchase set opposite the person's age in the Table, and the product is the gross value of the life estate of such person in said sum.

EXAMPLES.

1. Suppose a widow's age is 37, and she is entitled to dower in real estate worth $350.75, what is the gross present value of her right of dower?

SOLUTION. $\frac{1}{3}$ of $350.75 is 116.91\frac{2}{3}$.
Interest on $116.91 for 1 year, $7.01.
The number of years' purchase which an annuity of $1 is worth at the age of 37, is 11.035, and $7.01 × 11.035 = $77.35 +, *Ans.*

2. Suppose a man whose age is 50, is tenant by the courtesy in the whole of an estate worth $9000, what is the gross value of his life estate in the premises? *Ans.* $5085.18.

NOTE.—The values in this Table are calculated on the supposition that the annuities are payable yearly; if payable half-yearly, one fifth of a year's purchase should be added to those values.

MENSURATION.

698. Mensuration is that branch of applied geometry which treats of finding the lengths of lines, the areas of surfaces, or volumes of solids, from certain simple data of lines and angles.

GEOMETRICAL DEFINITIONS.

699. A **Point** is that which has position, but no magnitude.

Extension has three dimensions; *Length, Breadth,* and *Thickness.*

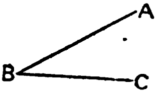
A **Line** has only one dimension; *Length.*

A **Surface** has two dimensions only; *Length* and *Breadth.*

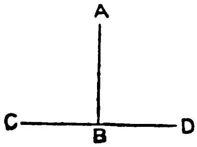
A **Solid** or **Body** has three dimensions; *Length, Breadth,* and *Thickness.* Its bounds are plane or curved surfaces.

ANGLES.

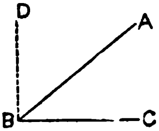
700. An **Angle** is the inclination, or opening, of two straight lines meeting in a common point; as A, B, C. The two straight lines are called sides of the angle, and the common point the *vertex*.



When one straight line meets another so that the two adjacent angles formed are equal to each other, both are called *Right Angles*, and the lines are said to be *perpendicular* to each other. Thus, A B C and A B D are right angles; A B is the perpendicular to C D, and C B and B D are perpendicular to A B.

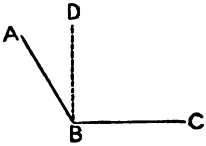


An Angle less than a right angle is called an *Acute Angle;* as, A B C.



An Angle greater than a right angle is called an *Obtuse Angle;* as, A B C.

All other angles are called *Oblique Angles.*



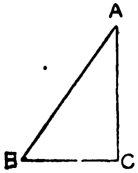
PLANE AREAS.

701. A **Plane Figure** is a limited portion of a plane, and is terminated on all sides by lines, either straight or curved.

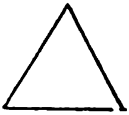
When the boundary lines are straight, the space they inclose is called a *Rectilinear Figure*, or *Polygon.*

A Polygon having only three sides is called a *Triangle;* one having four sides, a *Quadrilateral;* one of five sides, a *Pentagon;* one of six sides, a *Hexagon;* one of seven sides, a *Heptagon,* one of eight sides, an *Octagon;* one of nine sides, a *Nonagon;* one of ten sides, a *Decagon,* etc.

702. A **Right-angled Triangle** is a triangle which has one right angle. The side opposite the right-angle is called the *Hypothenuse,* and the other two sides the *Legs.* Thus A B C is a right-angled triangle; A B is the hypothenuse, and A C and B C are the legs.

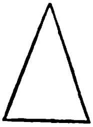


A



An **Equilateral Triangle** is a triangle having three equal sides; as, Figure A.

B



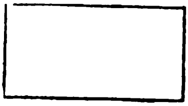
An **Isosceles Triangle** is a triangle which has only two equal sides; as, Figure B.

A **Scalene Triangle** is a triangle having all its sides unequal.

A Triangle having one obtuse angle is called an *Obtuse-angled Triangle*; one which has all its angles acute, an *Acute-angled Triangle*.

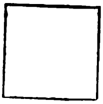
703. A **Parallelogram** is a quadrilateral which has its opposite sides parallel; as, Figures C, D, E, F.

C



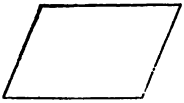
A **Rectangle** is any parallelogram whose angles are right angles; as, Figures C, D.

D



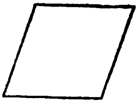
A **Square** is a rectangle whose sides are equal; as, Figure D.

E



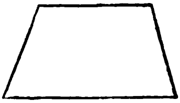
A **Rhomboid** is a parallelogram whose angles are not right angles; as, Figure E.

F



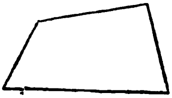
A **Rhombus** is a Rhomboid whose sides are equal; as, Figure F.

G



A **Trapezoid** is a quadrilateral which has only two of its sides parallel; as, Figure G.

H



A **Trapezium** is a quadrilateral which has no two of its sides parallel; as, Figure H.

An **Equilateral Polygon** has all its sides equal. An *Equiangular Polygon* has all its angles equal. A *Regular Polygon* has equal sides and equal angles.

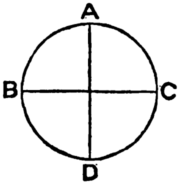
A **Diagonal** is a straight line joining the vertices of two angles of a polygon which are not adjacent.

The **Base** of any plane figure is the side upon which it is supposed to stand. In a triangle the base lies opposite the angular point chosen as the vertex. The base of a conic or cylindrical surface is the intersection of the surface by a plane.

The **Altitude** of a triangle is the perpendicular distance from the base to the vertex of the opposite angle.

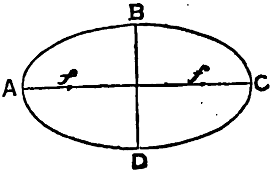
The **Altitude** of a parallelogram, or trapezoid, is the perpendicular distance between its parallel bases.

704. A **Circle** is a plane figure bounded by a curved line called the circumference, which is everywhere equidistant from a point within called the centre; as, A B C D.



The **Radius** of a circle is a right line drawn from the centre to the circumference; as, A E. The *Diameter* is a right line passing through the centre and terminated both ways by the circumference; as, A C.

705. An **Ellipse** is an oblong curvilinear plane figure, which corresponds to an oblique projection of a circle, or a section of a cone cut by a plane which passes obliquely through its two slant sides; as, A B C D.

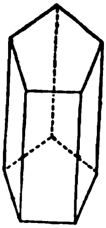


It has two fixed points (*f*, *f*,) called the foci, from which to every point of the circumference the *sum* of the distances is the same, and is equal to the transverse diameter.

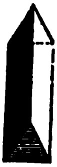
The *Transverse Diameter*, or axis, is a line passing through the *foci*. A line perpendicular to this, passing through the centre, is the *Conjugate Diameter* or *Axis*.

SOLIDS.

A



B

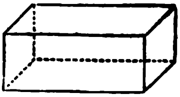


706. A **Prism** is a solid whose ends are two equal, parallel, and similar plane figures, and its sides parallelograms; as, Figures A, B.

A prism is called a triangular prism when its ends are triangles; a square prism, when its ends are square, etc.

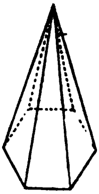
A **Cube** is a square prism having six sides which are all squares.

C



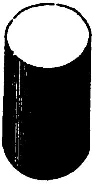
A **Parallelopipedon** is a solid having six rectangular sides, every opposite two of which are equal and parallel; as, Figure C.

D



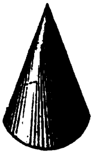
A **Pyramid** is a solid whose sides are all triangles, meeting at a common point called the *vertex*, and whose base is a polygon; as, Figure D

E



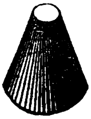
A **Cylinder** is a solid described by the revolution of a rectangle about one of its sides as an axis. It as also been defined as a *round prism;* as, Figure E.

F



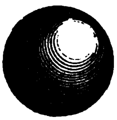
A **Cone** is a solid described by the revolution of a right-angled triangle about one of its legs as an axis. It is also defined as a solid which has a circular base, and tapers regularly to a point called the vertex; as, Figure F.

G



A **Frustum** of a cone or pyramid is the part that remains after cutting off the top by a plane parallel to the plane of its base; as, Figure G.

A **Sphere** is a solid bounded by one continued surface, every point of which is equally distant from a point within called the centre; as, Figure H.



The **Altitude** of a prism, cube, or parallelopipedon, is the perpendicular distance between its bases.

The *Altitude* of a pyramid or cone is the perpendicular distance from the vertex to the plane of the base.

The *Axis* of a solid is a line passing from its extremities through the central point. The axis of a sphere is a line passing through the centre, and terminating in two opposite points of its surface.

MENSURATION OF SURFACES.

707. Mensuration of Surfaces is the process of determining their areas.

The **Area** of a plane figure is its quantity of surface, expressed in square measure.

The **Unit of Measure** is a square whose side is some known length; as, an inch, a foot, etc.

PROBLEM I.

708. To find the area of any parallelogram, whether it be a Rectangle, Square, Rhomboid, or Rhombus.

RULE.—*Multiply the base by the altitude.*

EXAMPLES.

1. Required the area of a square whose side is 8 feet. *Ans.* 64 sq. feet.

2. How many square yards are there in the floor of a room 20 feet long, and 15 feet wide? *Ans.* $33\frac{1}{3}$.

3. The base of a rhombus is 7 feet, and its perpendicular height 5 feet: required the area. *Ans.* 35 sq. feet.

PROBLEM II.

709. To find the area of a triangle when the base and altitude are given.

RULE.—*Take one half the product of the base and altitude.*

EXAMPLES.

1. What is the area of a triangle whose altitude is 14 feet, and base 27 feet? *Ans.* 189 sq. ft.

2. Find the area of a triangle whose base is 13 inches, and altitude 10 inches.

3. What is the area of a triangle whose base is 42 rods, and altitude 28 rods?

4. What is the area of a field in the form of a right-angled triangle, of which the base is 27.5 chains, and the perpendicular 18.25 chains?

PROBLEM III.

710. To find the area of a triangle when three sides are given.

RULE.—*From half the sum of the three sides subtract each side separately. Then multiply the half sum and the three remainders continually together, and the square root of the product will be the area.*

EXAMPLES.

1. Required the area of a triangle whose three sides are, respectively, 24, 36, and 40 feet. *Ans.* 426.62 + sq. ft.

2. Find the area of a triangle whose three sides are, respectively, 8, 10, and 13 feet.

3. What is the area of a field in the form of an equilateral triangle, each side being 75 rods?

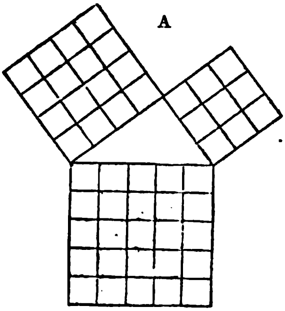
NOTE.—The area of an equilateral triangle is equal to the square of the side multiplied by .433013.

PROBLEM IV.

711. Any two sides of a right-angled triangle being given, to find the third side.

THEOREM.—The square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.

COROLLARY.—*The square of either side about the right angle is equal to the square of the hypothenuse diminished by the square of the other side.*



From this theorem, which is demonstrated by geometry, and illustrated by figure A, the following rules are derived:—

I. To find the hypothenuse:

RULE.—*Extract the square root of the* SUM *of the square of the base and the square of the perpendicular.*

II. To find the base or perpendicular:

RULE.—*Extract the square root of the* DIFFERENCE *between the square of the hypothenuse and square of the given side.*

NOTE.—The square of the Difference may be found by the following simple rule; *Multiply the sum of the given sides by their difference;* or, *multiply the smaller of the two numbers by twice their difference, and add the square of their difference.*

EXAMPLES.

1. The base of a right-angled triangle is 24 feet, and the perpendicular 18 feet: what is the hypothenuse? *Ans.* 30 feet.

2. The hypothenuse of a right-angled triangle is 41 rods, and its base 9 rods: what is the perpendicular? *Ans.* 40 rods.

3. There is a park 80 rods square: what is the distance from the centre to one of the four corners? *Ans.* 56.57 — rods.

4. The height of St. Paul's Cathedral, London, is 386 feet: what is the length of a rope which reaches from the top of the cross to a point on the ground 300 feet distant from the centre of the circular pavement under the dome?

5. A tower 80 feet high stands on a hill near a lake. The perpendicular distance from the tower to the edge of the water is 240 feet; and, from the same point, at the water's edge, to the top of the tower, 280 feet: required the height of the hill.

6. The top of a flag-pole, broken off by a blast of wind, struck the ground at a point 12 feet distant from the foot of the pole. The part broken off was 20 feet long: what was the length of the pole? *Ans.* 36 feet.

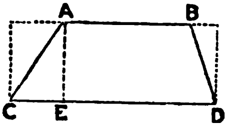
PROBLEM V.

712. **To find the area of a trapezoid.**

RULE.—*Multiply the sum of the parallel sides by the altitude, and take half the product.*

EXAMPLES.

1. Required the area of the trapezoid A B C D, whose parallel sides A B and C D are 15, and 20 feet, and the perpendicular, A E, 10 feet. *Ans.* 175 sq. ft.



2. What is the area of a plat of ground whose parallel sides are 20 and 40 rods, and the perpendicular distance between them, 75 rods?

3. Required the superficial contents of a board 18 feet long, 9 inches wide at one end, and 15 inches wide at the other.

PROBLEM VI.

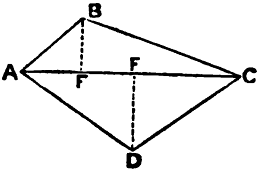
713. **To find the area of a trapezium.**

RULE.—*Multiply the diagonal by the sum of the two perpendiculars falling upon it from opposite angles; half the product will be the area.*

EXAMPLES.

1. What is the area of the trapezium A B C D, whose diagonal A C is 25 feet, the perpendicular, B E, 7 feet, and the perpendicular, F D, 10 feet?

Ans. 212½ sq. ft.



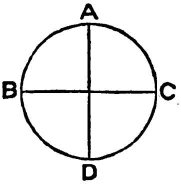
2. How many acres are there in a piece of land in the form of a trapezium whose diagonal is 35.5 chains, and the two perpendiculars falling on it from opposite angles, 9 and 12.5 chains, respectively?

3. What is the area of a trapezium whose diagonal is 10 feet 3 inches, and the perpendiculars, 3 feet 4 inches, and 4 feet 6 inches?

NOTES.—1. The area of a regular polygon is equal to half the product of the perimeter, or sum of all its sides, by the perpendicular falling from its centre upon one of the sides.

2. The perimeters of all similar figures are proportional to any homologous lines, and their areas are as the squares of those lines.

THE CIRCLE.



714. The ratio of the diameter of a circle to its circumference, is, very nearly, as 1 to 3.1416.

NOTE.—The ratio cannot be exactly expressed in figures. The decimal has been extended to two hundred and fifty places, twenty of which are contained in the following number,—3.14159265358979323846. The above is, however, sufficiently accurate for practical purposes.

The area of a circle is equal to the circumference multiplied by one half the radius; or, the square of the radius multiplied by 3.1416.

The circumferences of circles are as their radii or diameters; and the areas as the squares of their radii or diameters.

PROBLEM I.

715. To find the circumference of a circle when the diameter is given.

RULE.—*Multiply the diameter by* 3.1416.

EXAMPLES.

1. What is the circumference of a circle whose diameter is 8 inches? *Ans.* 2 ft. 1.1328 in.

2. Required the circumference of a circle, the diameter of which is 15 feet.

3. What is the circumference of a circle whose diameter is 5 rods 10 feet?

PROBLEM II.

716. To find the diameter of a circle when the circumference is given.

RULE.—*Divide the circumference by* 3.1416.

EXAMPLES.

1. What is the diameter of a circle whose circumference is 20 feet? *Ans.* 6.3661 + feet.

2. Find the diameter of a circle of which the circumference is 2 feet 4.2744 inches.

3. What is the radius of a circle whose circumference is 125.664 feet?

PROBLEM III.

717. To find the area of a circle.

RULE.—I. When both diameter and circumference are given:—*Multiply the circumference by one fourth of the diameter.*

II. When the diameter is given:—*Multiply the square of the diameter by* .7854.

III. When the circumference is given:—*Multiply the square of the circumference by* .07958.

NOTE.—The decimal .7854 is $\frac{1}{4}$ of 3.1416, which is the area of a circle whose circumference is 1; and .07958 — is $\frac{1}{4}$ of $\overline{1 \div 3.1416}$; that is, $\frac{1}{4}$ of the diameter of a circle whose circumference is 1.

EXAMPLES.

1. What is the area of a circle of which the circumference is 15.708 feet, and the diameter 5 feet?

2. The diameter of a circle is $2\frac{1}{2}$ feet: what is its area?

3. The distance around a circular park is one mile: required the area of that park.

4. A circle 10 feet in diameter is circumscribed by another circle 15 feet in diameter: what is the area of the space included between the circumferences?

5. What is the length of an arc of 75° on the circumference of a circle whose radius is 5 feet?

PROBLEM IV.

718. To find the diameter or circumference of a circle from the area.

RULE.—I. *Divide the area by .7854, and the square root of the quotient will be the diameter.*

II. *Divide the area by .07958, and the square root of the quotient will be the circumference.*

EXAMPLES.

1. The area of a circle is 19.635 feet: what is its diameter? *Ans.* 5 feet.

2. Required the circumference of a plat of ground containing one acre.

3. The diameter of a circle is 36 feet: required the diameter of a circle whose area is 4 times as great.

4. The area of a circle is 490.875 square inches: required the diameter and circumference.

5. The area of a circle is 706.86 inches: what is the diameter of a circle one third as large?

PROBLEM V.

719. To find the side of a square equal in area to a given circle.

RULE.—*Multiply the diameter of the circle by* .8862, *or the circumference by* .2821.

NOTE.—The decimal .8862, or, approximately, the square root of .7854, is the side of a square equivalent in area to a circle whose diameter is 1; and .2821 is the square root of .07958 +, or the side of a square equal in area to a circle whose circumference is 1.

EXAMPLES.

1. If the diameter of a circle be 90 feet, what is the side of a square of equal area?

2. A circular field is 150 rods in circumference: what is the side of a square field of the same area?

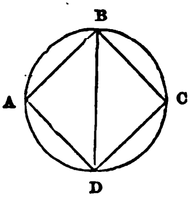
3. A has a circular garden, and B a square one; the distance around each is 16 rods: which contains the most land, and how much?

PROBLEM VI.

720. To find the side of a square inscribed in a given circle.

RULE.—*Extract the square root of one half the square of the diameter; or, divide the diameter by* 1.4142.

NOTE.—A square is said to be inscribed in a circle when the vertices of its angles are in the circumference. It may be conceived, as shown in the annexed figure, to be composed of two right-angled triangles, having the diameter of the circle as a common hypothenuse, and the sides of the square as their bases and perpendiculars, which are, of course, equal. Now, according to the Theorem, Problem IV, page 399, $AC^2 = AB^2 + BC^2$. But, $AB^2 = BC^2$; hence, $\frac{AC^2}{2} = AB^2$. Therefore, $\sqrt{\frac{AC^2}{2}} = AB$. This also explains the second rule: 1.4142 is, approximately, the square root of 2.



The same result is obtained by multiplying the diameter by .707106, which is the side of an inscribed square, when the diameter of the circle 1.

THE ELLIPSE.

PROBLEM I.

721. To find the area of an ellipse.

RULE.—*Multiply the product of the two diameters by* .7854.

EXAMPLES.

1. Required the area of an ellipse of which the diameters are 6 and 8 feet.

2. What is the area of an ellipse of which the transverse diameter is 12 feet, and the conjugate diameter 16 feet?

PROBLEM II.

722. To find the circumference of an ellipse from its two diameters.

RULE.—*Multiply the sum of the two diameters by* 1.5708.

EXAMPLES.

1. What is the circumference of an ellipse whose diameters are 32 and 25 inches?

2. Required the circumference of an ellipse, the conjugate axis of which is 20 feet, and the transverse axis 24 feet.

MENSURATION OF SOLIDS.

723. A **Solid**, or **Volume**, is a magnitude which has length, breadth, and thickness.

Mensuration of Solids is the process of determining their surfaces and contents.

The **Contents** of a solid body or volume, is its bulk or capacity, expressed in cubic measure.

The **Unit of Measure** is a cube whose side is some known length; as an inch, a foot, etc.

The **Surface** of a solid bounded by plane figures, is the area of its several sides, or faces. Its *Linear Measure* is the length and breadth of those sides.

The **Surface** of a **Sphere** is the area of its curved boundary. The linear dimensions are its diameter and circumference.

The **Convex Surface** of a solid is that part of a surface which is formed by curved lines.

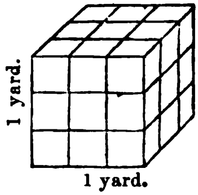
PROBLEM I.

724. To find the contents of a cube or parallelopipedon.

RULE.—*Multiply together the length, breadth, and thickness.*

EXAMPLES.

1. How many solid feet are there in a cubic block, of which the side is 9 feet? *Ans.* 729 feet.



2. Required the contents of a block of marble 8 feet long, 3 feet wide, and 2 feet thick.

3. How many cords of wood are there in a pile 27 feet long, 4 feet wide, and 4 feet high?

PROBLEM II.

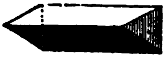
725. To find the surface of a prism:

RULE.—*Multiply the perimeter of the base by the altitude, and to this product add the area of the two ends.*

EXAMPLES.

1. Required the surface of a square prism whose base is 5 feet square, and its length 10 feet. *Ans.* 250 sq. ft.

2. What is the surface of a cube whose edge is 30 inches?



3. Find the surface of a triangular prism, the three sides of whose base are, respectively, 3, 4, and 5 feet, and the height 12 feet 6 inches.

PROBLEM III.

726. To find the contents of a right prism:

RULE.—*Multiply the area of the base by the altitude.*

EXAMPLES.

1. Required the contents of a triangular prism, of which each side is 30 inches wide and 4 feet long.

2. Find the solidity of a cube whose edge is 6 feet 4 inches.

3. Find the contents of a prism 2 inches square, and 3½ feet long.

THE CYLINDER.

727. The surface and contents of a cylinder are found by the same rules as those given for the prism.

To find the convex surface of a cylinder.

RULE.—*Multiply the base by the altitude.*

If the entire surface be required, add the areas of the two ends.

To find the contents of a cylinder.

RULE.—*Multiply the area of the base by the altitude.*

EXAMPLES.

1. What is the convex surface of a cylinder 3 feet long, and 1½ feet in diameter? What is its entire surface?

2. Required the contents of a cylinder whose length is 15 feet, and its diameter 4 feet.

3. What are the contents of a log 18 feet long, and averaging 1 foot in diameter?

4. The United States bushel is the capacity of a cylinder 18½ inches in diameter, and 8 inches deep: how many cubic inches does it contain?

THE PYRAMID AND CONE.

PROBLEM I.

728. To find the surface and contents of a pyramid or cone.

RULE.—I. *Multiply the perimeter, or the circumference of the* *base, by half the slant height, and the product will be the slant or convex surface.*

If the whole surface be required, add the area of the base.

II. *Multiply the area of the base by one third of the altitude, and the product will be the contents.*

EXAMPLES.

1. What is the slant surface of a pyramid whose slant height is 75 feet, and the base 35 feet square?

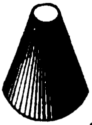
2. Find the contents of a triangular pyramid whose sides at the base are 10, 12, and 15 feet, and the altitude 40 feet.

3. Required the convex surface of a cone, the diameter of the base being 7 feet, and the altitude 12 feet.

4. Find the surface and contents of a cone whose slant height is 20 feet, and the diameter of the base 8 feet.

Ans. Surface, 251.328 + sq. ft.
Contents, 328.33 + cu. ft.

THE FRUSTUM OF A PYRAMID AND CONE.



729. To find the surface of a frustum of a pyramid, or cone.

RULE.—*Find the sum of the perimeters, or circumferences of the two ends, and multiply it by half the slant height. This product will be the slant, or convex surface. To find the entire surface, add the areas of the two ends.*

EXAMPLES.

1. Find the slant surface of the frustum of a square pyramid whose slant height is 20 feet, the side of the base 9 feet, and of the top, 7 feet.

2. There is a frustum of a cone whose slant height is 15 feet, the circumference of the base 20 feet, and of the upper end, 24 feet: what is its entire surface?

PROBLEM II.

730. **To find the contents of a frustum of a pyramid, or cone.**

RULE.—*To the sum of the areas of the two ends add the square root of their product, and multiply this sum by one third of the altitude.*

For the frustum of a cone, the following is a simpler rule:

RULE.—*To the sum of the squares of the radii of the two ends add their product, and multiply this sum by* 3.1416, *and again by one third of the altitude.*

EXAMPLES.

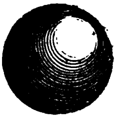
1. Required the contents of the frustum of a square pyramid, the side of the greater end being 8 feet, that of the less 4 feet, and the altitude 20 feet.

2. Find the contents of a stick of round timber 36 feet long, 18 inches in diameter at the smaller end, and 24 inches at the larger end. *Ans.* 87.1792 cu. ft.

3. What are the contents of a log 18 feet long, 12 inches in diameter at the larger end, and 6 inches at the smaller end?

THE SPHERE.

PROBLEM I.



731. **To find the surface of a sphere.**

RULE.—*Multiply the diameter by the circumference; or, the square of the radius by* 4 *and* 3.1416.

EXAMPLES.

1. What is the surface of a sphere whose diameter is 18 inches?

2. How many square miles are there on the surface of the earth, the diameter being about 7912 miles? *Ans.* 19666355.7504 sq. miles.

Note.—The surface of a zone may be found by multiplying its height by the whole circumference of the sphere.

Problem II.

732. To find the solidity or volume of a sphere.

Rule.—*Multiply the surface by* $\frac{1}{6}$ *of the diameter, or* $\frac{1}{6}$ *of the cube of the diameter by* 3.1416.

The solidities of spheres are to each other as the cubes of their radii or diameters.

Examples.

1. What is the solidity of a sphere whose diameter is 18 inches?

2. The diameter of a globe is 2 feet 6 inches: what is its volume or solidity?

3. If the diameter of the earth be 7912 miles, how many cubic miles does it contain?

4. Required the diameter of a globe which contains 6 times as many cubic inches as a globe 6 inches in diameter.

Note.—The volumes of all similar solids are to each other as the cubes of their like dimensions.

Miscellaneous Examples.

1. A board, in the form of an isosceles triangle, is 14 feet long, and 2 feet wide at one end. What must be the length of a piece, to be cut from the wide end, which will contain 6 square feet?

2. What is the area of the space included between two concentric circles, one of which is 12 inches, and the other 8 inches in diameter?

3. The diagonal of a rectangle is 20 feet long, and its base 16 feet: what is its area?

4. Find the area of a parallelogram, and also of a triangle, whose base is $9\frac{1}{2}$ feet, and altitude 12 feet.

5. Find the solidity of a prism, of which the area of the base is 48 square inches, and the altitude 24 inches.

MEASUREMENT OF LUMBER.

733. The following practical methods of calculating the contents of lumber are generally employed by lumber dealers.

CASE I.

734. To find the superficial contents of a board one inch thick.

EXAMPLE. How many square feet are there in a board 12 feet long, and 9 inches wide?

$\frac{12 \times 9}{12} = 9$, *Ans.*

ANALYSIS.—The contents equal the length in feet multiplied by the breadth in inches and divided by 12; or, since 9 inches equal $\frac{3}{4}$ of a foot, $12 \times \frac{3}{4} = 9$.

It will be observed that, in any case, if one of the dimensions be given in feet, and the other in inches, their product must always be divided by 12. Therefore, 12 may be cancelled from either factor, before multiplying. Hence, we have the following

RULE.—When the length is given in feet, and the breadth in inches, *divide either dimension by* 12, *and multiply the quotient by the other; the product will be the contents in square feet.*

GENERAL APPLICATIONS.

1. A board 12 feet long and 12 inches wide, contains 12 square feet.

2. A board 12 feet long and of any width, contains as many square feet as it is inches wide.

3. A board of *any length* contains as many square feet as it is inches in width, increased or diminished in the same proportion as its length is greater or less than 12 feet.

4. A board of *any width* contains as many square feet as it is feet in length, increased or diminished in the same proportion as its width is greater or less than 12 inches.

EXAMPLES.

1. How many square feet are there in a board 8 feet long, and 18 inches wide?

$8 + 4 = 12$; or $18 - 6 = 12$; or $\frac{2}{3}$ of $18 = 12$. *Ans.* 12.

2. Find the contents of a board 15 feet long, and 8 inches wide.

3. Required the contents of a board 10 feet long, and 9 inches wide.

4. Calculate in the same manner the contents of the following:

1. 12 Boards .... 16 × 9
2. 10 " .... 14 × 10
3. 20 " .... 18 × 9
4. 5 " .... 10 × 8
5. 8 " .... 12 × 5
6. 5 Boards .... 14 × 6
7. 10 " .... 12 × 7
8. 7 " .... 14 × 11
9. 16 " .... 15 × 8
10. 13 " .... 12 × 13

NOTE.—Since the number of boards, the length, and the breadth, are all factors of a product to be divided by 12, we may divide any one of them by 12, before multiplying.

Thus, in the first example, cancel 12, and multiply 16 by 9. In the second example, multiply 10 by 10, and add to the product $\frac{1}{6}$ of itself, because 14 is $\frac{1}{6}$ greater than 12. In the third example, increase 20 by $\frac{1}{2}$ of itself and multiply by 9. An expert will perform such calculations, mentally, with great rapidity.

CASE II.

735. To find the contents of planks, scantlings, joists, and square timber.

Lumber of this description is usually reduced to its equivalent in boards one inch thick. A plank 13 inches wide is measured as a board 26 inches wide, and a piece of scantling 3 by 2 inches, as a board 6 inches wide. Hence, we have the following

RULE.—*Multiply the width by the thickness, and then proceed as in Case* I.

736. For any lumber 3 inches thick:

RULE.—*Multiply the width by one fourth of the length; or, half the length by half the width.*

737. For any lumber 2, 3, 4, or 6 inches thick:

RULE.—*Multiply the width by the same part of the length that the thickness is of* 12.

738. For square timber:

RULE.—*Multiply the length by one twelfth of the area of the end in inches.*

NOTES.—1. In the above rules, understand the width to be in inches, and the length in feet.

2. When the area of the end of a stick of timber is 12 square inches, the length is its contents in square feet, board measure.

3. If the area of the end be 144 square inches, the contents will be 12 times the length, or one square foot for every inch in length.

EXAMPLES.

1. Find the contents, in board measure, of a plank 16 feet long, 8 inches wide, and 2 inches thick.

$16 + 5\frac{1}{3}$ ($\frac{1}{3}$ of 16) $= 21\frac{1}{3}$

or, $8 \times 2\frac{2}{3}$ ($\frac{1}{6}$ of 16) $= 21\frac{1}{3}$. *Ans.* $21\frac{1}{3}$ feet.

2. Find the contents of a plank 14 feet long, 10 inches wide, and 3 inches thick.

$7 \times 5 = 35$. *Ans.* 35 feet.

3. Required the contents of 10 pieces of scantling, each 14 feet long, 3 inches wide, and 2 inches thick.

$10 \times 7 = 70$. *Ans.* 70.

4. How many feet are there in 12 pieces of scantling, $3\frac{1}{2}$ by $2\frac{1}{2}$, and 16 feet long?

$\frac{7}{\cancel{2}} \times \frac{5}{\cancel{2}} \times \frac{\cancel{16}^{4}}{\cancel{12}} \times \frac{\cancel{12}}{1} = 140$. *Ans.* 140.

5. Find the contents, in board measure, of a stick of square timber, 12×8, and 18 feet long.

$96 \times 1\frac{1}{2} = 144$; or $18 \times 8 = 144$. *Ans.* 144 feet.

6. Required the contents of a stick of timber, 8×4, and 15 feet long.

8×5 ($\frac{1}{3}$ the length) $= 40$. *Ans.* 40 feet.

Find the contents of the following:

7. 10 pieces scantling 2 × 3, 16.

8. 20 " joists, 4 × 5, 10.

9 8 planks, 2 × 10, 16.

10. 6 planks, 3 × 9, 14.
11. 1 piece timber, 4 × 9, 14.
12. 1 " " 6 × 14, 30.
13. 18 pieces scantling, 2½ × 3½, 15.

NOTE.—The above examples are given in the form of the items of a Bill. The sign of multiplication is placed between the side dimensions, and omitted before the length, as is customary in practice.

739. To find the cubical contents of square timber.

RULE.—*Multiply the area of one end in inches by the length in feet, and divide the product by* 144.

EXAMPLES.

1. Find the cubical contents of a stick of square timber 24 feet long, and 16 inches square.

SOLUTION. $\frac{16 \times 16 \times 24}{144} = 42\frac{2}{3}$ feet, *Ans.*

2. Required the cubical contents of a stick of timber 26 feet long, and 17 by 15, side dimensions.

3. What are the cubical contents of a stick of timber 14 inches square, and 20 feet long?

4. Find the cubical contents of a stick of timber, 16 × 18, and 28 feet long.

ROUND TIMBER.

740. Round Timber is now very generally bought and sold by its cubical measurement, when reduced to square timber.

The following rule for finding the contents is extensively used, and is considered practically just to both buyer and seller.

RULE.—*Deduct from the mean diameter in inches one third of itself; multiply the square of the remainder by the length in feet, and divide the product by* 144.

NOTE.—The mean diameter is found by taking one half of the sum of the diameters of the two ends.

EXAMPLES.

1. Find the cubical contents, in square timber, of a stick of round timber 14 inches in diameter at the smaller end, 20 inches at the larger, and 26 feet long.

SOLUTION. $20 + 14 = 34 \div 2 = 17$, mean diameter.
$17 - 5\frac{2}{3} = 11\frac{1}{3}$; and $11\frac{1}{3} \times 11\frac{1}{3} = 128\frac{4}{9}$.
$128\frac{4}{9} \times 26 \div 144 = 23\frac{62}{324}$. *Ans.* 23 + feet.

NOTE.—A fraction of ½ a foot or less is disregarded, and a fraction over ½ is called a foot.

2. Find the measurement, as square timber, of a stick of round timber 30 feet long, and of an average diameter of 15 inches.

3. What are the cubical contents, in square timber, of a stick of round timber 25 feet long, and 20 inches in diameter at one end, and 28 inches at the other?

4. Required the cubical contents, as square timber, of a stick of round timber 32 feet long, and averaging 23 inches in diameter.

CUBICAL CONTENTS OF ROUND TIMBER AND SPARS.

741. Round Timber was formerly, and is at present, to some extent, sold by the Ton of 40 cubic feet, computed by the following

RULE.—*Multiply the length in feet by the square of one fourth the mean girt in feet; the result will be the contents in cubic feet.*

NOTE.—If the timber is covered with bark, one inch should be deducted from the mean girt, or more, if the bark is very thick.

The quantity given by this rule as a cubic foot, is in fact a little more than 1¼ cubic feet; so that the Ton is equal to a small fraction over 50 cubic feet. The result is to the true contents, as found by the rule for calculating the solidity of the frustum of a cone, very nearly as 11 to 14. For example, a log 15 feet long, 18 inches in diameter at the larger end and 14 inches at the smaller, contains, on the supposition that it is a perfect frustum of a cone, 21.053 + cubic feet; but, by the above rule, it measures only

16.515 + feet. Now, by multiplying this number by 14 and dividing by 11, or increasing it by $\frac{3}{11}$ of itself, we get 21.019 + feet, which, for all practical purposes, may be taken as the exact cubical contents. Hence, we have the following *practically accurate rule* for finding the cubical contents of round timber, spars, etc.

RULE.—*Multiply the length in feet by the square of one fourth the mean girt in feet, and to the product add* $\frac{3}{11}$ *of itself; or multiply it by* 14 *and divide by* 11; *the result will be the contents in cubic feet.*

NOTES.—1. Take the girt of the middle of the log; or multiply the mean diameter by 3.1416, or $3\frac{1}{7}$.

2. Proper allowances should be made for irregularities and defects.

EXAMPLES.

1. Find the cubical contents of a stick of round timber, 36 feet long, and 18 inches in diameter at one end and 24 inches at the other.

SOLUTION. Mean girt, $5\frac{1}{2}$ feet, nearly.

$$(\tfrac{11}{8})^2 \times 36 = 68 +.$$

$$\frac{68 \times 14}{11} = 86\tfrac{6}{11}.$$

Ans. $86\frac{6}{11}$ feet.

2. Reduce to square timber, cubic measurement, a stick of round timber, 18 feet long, and of an average diameter of 19 inches.

3. What are the contents, reduced to square timber, of a stick of round timber 30 feet long, of which the mean girt is 4 feet?

4. Find the contents, in square timber, of a log 18 feet long, 18 inches in diameter at the smaller end, and 22 inches at the larger end.

LOGS REDUCED TO BOARD MEASURE.

742. The contents of "Saw Logs" are usually estimated by the quantity of square-edged inch boards they will produce.

There is no "accurate" rule known for computing the contents, according to this mode of estimation. Tables, prepared from diagrams, are very frequently used; but the following rule is often adopted in the absence of Tables.

RULE.—*Deduct from the diameter of the smaller end, in inches, one third of itself; multiply the square of the remainder by the length in feet, and divide the product by* 12; *or, multiply the remainder by one half of itself, and divide the product by* 6.

NOTE.—This rule allows too much for waste on large logs; therefore, some dealers deduct 4 inches from the diameter, instead of one third, according to the rule.

EXAMPLES.

1. Required the contents, in board measure, of a log 14 feet long, and 18 inches in diameter at the smaller end.

SOLUTION. $18 - 6 = 12$; $12^2 = 144$.

$$\frac{144 \times 14}{12} = 168. \qquad \textit{Ans.}\ 168\ \text{ft.}$$

2. What are the contents of a log 17 feet long, and 24 inches in diameter at the smaller end?

3. Find the contents of a log 18 feet long, whose diameter is 27 inches.

NOTE.—The author has observed, in practice, one fact which, perhaps, lumber-dealers may take advantage of; namely, that the contents, in feet, of logs upward of 16 or 17 inches in diameter, *are very nearly equal to the square of two thirds of their diameter in inches, for every ten feet of their length.* Hence, *by deducting from the diameter of a log, in inches, one third of itself, and multiplying the square of the remainder by one tenth of the length, the product will be the contents in square feet.*

EXAMPLES.

1. What are the contents, in board measure, making the usual allowance for waste in sawing, of a log 14 feet long, and 24 inches in diameter?

SOLUTION. $24 - 8 = 16$; $16^2 \times 1.4 = 358.4$. *Ans.* 358 ft.

2. Find the contents of a log 15 feet long, and 36 inches in diameter. *Ans.* 864 ft.

PHILOSOPHICAL PROBLEMS.

I. FALLING BODIES.

743. A body descending from a state of rest, under the influence of gravity, falls, the first second, $16\frac{1}{12}$ feet, and during the time acquires a velocity that will carry it through $32\frac{1}{6}$ feet in the next second. During the next second, it falls $16\frac{1}{12} + 32\frac{1}{6} = 48\frac{1}{4}$ feet, and acquires a velocity of $64\frac{1}{3}$ feet; and, uniformly, the velocity increases $32\frac{1}{6}$ feet per second. According to this law,

The velocities of falling bodies are as the TIMES, *and the spaces they describe as the* SQUARES OF THE TIMES.

Hence, to find the velocity acquired by a falling body, in any given time, *multiply* $32\frac{1}{6}$ *feet by the number of seconds it has been falling.*

To find the distance it has fallen, it is convenient to use a formula derived from the following proportion:—

$$1^2 : T^2 :: 16\tfrac{1}{12} : D \text{; or } D = 16\tfrac{1}{12} \times T^2.$$

EXAMPLES.

1. How far will a body, descending freely through space, fall in 8 seconds?

Formula. $D = 16\frac{1}{12} \times 8^2 = 1029\frac{1}{3}$. *Ans.* $1029\frac{1}{3}$ feet.

2. What velocity will be acquired by a body falling, as above, at the end of the time?

SOLUTION.—$32\frac{1}{6} \times 8 = 257\frac{1}{3}$. *Ans.* $257\frac{1}{3}$ feet per second.

3. How long will a body be in falling through $1608\frac{1}{3}$ feet of space?

Formula. $\dfrac{D}{16\frac{1}{12}} = T^2$. Hence, $1608\frac{1}{3} \div 16\frac{1}{12} = 100$; and $\sqrt{100} = 10$. *Ans.* 10 seconds.

4. A man, standing on a precipice, drops a leaden ball into a lake below, and counts 7 seconds before it strikes the water. What is the height of the precipice above the water?

5. With what velocity would an iron ball, dropped from the top of a tower 257$\frac{1}{3}$ feet high, strike the ground at its base? How far would it fall in three seconds?

Note.—To answer the first question, find the time from the formula in Example 3, and multiply it by 32$\frac{1}{6}$.

6. From what height must a body fall, to acquire a velocity of 386 feet per second? *Ans.* 2316 feet.

744. The velocity of a body projected upward, is retarded in the same ratio as the velocity of a falling body is accelerated. Hence, to find the distance it ascends, we have the following

Formula. $D = 16\frac{1}{12} \times \left(\frac{T}{2}\right)^2$

Example 1. A man in a boat on a pond, fires a ball vertically upward from a rifle, and observes that it returns and strikes the water in 20 seconds. How high did it ascend?

Ans. 1608$\frac{1}{3}$ feet.

2. A ball fired from a cannon on a level plain, at an angle of 45 degrees, struck the earth in 8 seconds: what vertical height did it reach?

II. SPECIFIC GRAVITY.

The **Specific Gravity** of a body is the ratio of its weight to the weight of an equal volume of some other body (usually pure distilled water), with which it is compared as a standard.

A cubic inch of distilled water weighs 252.458 grains. Hence, a cubic foot weighs 252.458 × 1728 = 436247.424 grains = 997.1369696 ounces avoirdupois = 62.3210606 pounds; but, for all practical purposes, the weight is reckoned 1000 ounces, or 62$\frac{1}{2}$ pounds.

The specific gravity of the standard is 1.

The specific gravity of a body lighter than the standard is less than 1, and of a body heavier, more than 1.

When a body is immersed in water, it is buoyed up by a force equal to the weight of the water which it displaces. A floating solid displaces its own weight of water.

TABLE,

SHOWING THE SPECIFIC GRAVITIES OF A FEW WELL-KNOWN SUBSTANCES.

Platinum	22.	Ivory	1.822
Gold, cast	19.258	Lignum vitæ	1.333
Mercury, at 32°	13.619	Milk	1.030
Lead, cast	11.344	Sea-water	1.028
Silver, "	10.474	Fresh water	1.
Copper,"	8.788	Hickory wood	.838
Brass; "	8.396	Maple wood	.750
Iron, "	7.271	Poplar wood	.383
Common Stone	2.520	Cork	.240

745. To find the specific gravity of a body heavier than water.

RULE.—*Weigh the substance in air, then in water, and divide its weight in air by its loss of weight in water; the quotient will be its specific gravity.*

EXAMPLES.

1. A piece of copper weighs 2019 grains in air, and loses 230 grains when immersed in water: what is its specific gravity?

SOLUTION. 2019 ÷ 230 = 8.75 —, *Ans.*

2. A piece of lead, weighing in air 272.4 pounds, weighs in water 248.4 pounds: what is its specific gravity?

Ans. 11.35.

3. A mass of gold, weighing 96.3 grains in air, loses, when weighed in water, 5 grains: find its specific gravity.

4. A piece of platinum weighs 44 grains in air, and 42 grains in water: what is its specific gravity?

746. To find the specific gravity of a body lighter than water.

RULE.—*Attach to it somebody heavy enough to sink it; weigh the two together, in air and in water, and, by subtraction, find their loss of weight in water. In like manner find the loss sustained by the heavier body weighed alone. Subtract this loss from the loss of the two, and the difference will be the weight of* *a volume of water equal to the volume of the lighter body. Divide the weight of the body in air by this difference, and the quotient will be its specific gravity.*

EXAMPLES.

1. A piece of maple wood, weighing 12 ounces in air, is fastened to a piece of lead which weighs 15 ounces in air, and 13¾ ounces in water; and both are found to weigh, under water, 9¾ ounces: required the specific gravity of the wood.

SOLUTION. Both lose, in water, 27 − 9¾ = 17¼ ounces.
The lead alone loses 1¼ "

Weight of water displaced by the wood, 16 "
Hence, 12 ÷ 16 = .75, the specific gravity of the wood, *Ans.*

2. A piece of cork weighs in air 4 ounces, and a piece of iron 24 ounces. In water, the two together weigh 6 ounces, and the iron alone weighs 18⅔ ounces: required the specific gravity of the cork. *Ans.* .240.

3. A body weighs, in air, 600 grains; when attached to a piece of copper, the combination weighs 2647 grains in air, and loses, when weighed in water, 834 grains. The copper alone loses 230 grains: required the specific gravity of the body.
Ans. .993.

MISCELLANEOUS EXAMPLES.

1. What number is that which, being increased by ¾ of itself, is equal to ¾?

2. A pole stands ⅛ of its length in the mud, 3 feet in the water, and ½ out of the water: what is its length?

3. John spent ½ of his money for a book, ½ of the remainder for a slate, ⅘ of the remainder for writing paper, and gave the balance to a beggar. He paid 5 cents more for the slate than for the paper: how much money had he at first, and how much did he give to the beggar?

4. Mr. Smith is worth $6000 more than Mr. Jones, and Mr.

Jones is worth just 4/7 as much as Mr. Smith: how much is each of them worth?

5. If an article had cost 20 per cent. less, the gain would have been 30 per cent. more: what was the gain per cent.?

6. An Indian and a white man undertook, for a wager, to run a race over a six-miles course. The speed of the Indian was to that of the white man as 4 to 3, till the former reached the half-way point, when the latter abandoned the contest. The Indian continued on at half his former speed, and made the whole distance in 48 minutes: how many rods per minute did each run at first? *Ans.* The Indian, 60, the white man, 45.

7. If $3½ will buy a turkey and a half, and a turkey is worth three and a half geese, how many geese can be bought for $5.33⅓?

8. The product of two numbers is ⅝, and one of the numbers is ⅞: what is the other?

9. A careless clerk, calculating the cost of a piece of goods, multiplied by .18½, when he should have multiplied by .28½, and obtained $5.73½ for the result: what was the correct result?

10. A can do a piece of work, alone, in 7 days, and B in 9 days: in what time can both do it, if they work together?

11. The sum of two numbers is 3547, and their difference 374: what are the numbers?

12. The product of the sum and difference of two numbers is 1400, and 10 is their difference: what are the numbers?

13. If a man by travelling 30 miles per hour, for 24 hours each day, can accomplish a journey of 2520 miles in 3½ days, how long will it take him to travel twice that distance, at the rate of 25 miles per hour, for 18 hours each day?

14. A merchant bought 75 barrels of flour at $9 per barrel, and sold ⅓ of it at a certain gain per cent., ⅓ at twice that gain, and the remainder at a net profit of $25. If he had sold the last lot for $8.75 more, he would have gained 10 per cent. on the whole: what was his gain per cent. on the first two lots, and his whole net gain?

15. If the minute hand of a clock be 5 inches long, what space in length does the extreme point pass over in 35 minutes?

16. In an arithmetical series, the extremes are 2 and 26, and the number of terms 7: what is the sum of all the terms?

17. A man agrees to labor for one year, at the rate of 1 cent for the first month, 5 cents for the second, 25 cents for the the third, and so on, in a fivefold ratio: what was the amount of his year's wages?

18. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, makes a cord. Required the dimensions of a pile of a similar form, which will contain 91⅛ cords.

19. The gable ends of a house are each 32 feet wide, and the perpendicular height of the ridge-pole above the eaves is 10 feet: how many feet of boards will it take to cover both gable ends?

20. A horse is tied in a meadow to a post, by a rope long enough to allow him to graze over a circular half acre of ground: what is the length of the rope?

21. How much carpeting that is ¾ of a yard wide will cover a floor 15 feet square?

22. A certain amount of money is to be divided between two persons, in such a manner that the one shall receive ⅗ as much as the other: required their respective shares, their difference being $550.

23. A train of cars, which ran at the speed of 25 miles per hour, left New York on the Hudson River Railroad, at 7½ o'clock A.M., for Albany, a distance of 150 miles. A second train, whose speed was to that of the first as 6 to 5, starting from the same place, arrived at Albany 40 minutes before the first: at what time did the second train leave New York?

24. In what time will any sum of money double itself at 6 per cent. compound interest?

25. Required the amount of $100 in 4 years and 10 months, at 6 per cent. compound interest.

26. In what time will any sum of money double itself at 6% simple interest?

27. What is the present value of an annuity of $500 per annum for 20 years, at 6% compound interest?

28. A merchant occupies a store, for the rent of which he is paying $1800 per annum. The building is situated on leased land, and will revert to the owner of the fee when the lease expires, in 20 years; but the ground-rent is paid in full to the end of the term. The owner of the store offers to sell it for $10000 cash, and the tenant concludes to purchase, thinking that in the end he will save money by the operation. Suppose it costs him annually for taxes, insurance, and repairs, $500, what will he save, or lose, in 20 years, reckoning compound interest, at 6%, on the investment, on the annual outlay of $500, and on the $1800 rent, for 20 years? *Ans.* Gain, $15749.91.

29. A miner offered a trader an ounce of gold, at a certain price, with the privilege of having it weighed upon a jeweler's or a grocer's scale. The trader chose the latter. Did he get more or less gold than he would have received, if he had chosen the jeweler's scale, and how much?

30. If five dogs can kill 5 rats in five minutes, how many dogs will it take to kill 100 rats in one hundred minutes?

31. A and B can do a certain piece of work in 6 days, A and C in 9 days, and B and C in 12 days: in how many days can they all, working together, do it?

32. A liquor agent of a New England town held the office for one year, at the close of which he gave the following statement of his accounts:—

Amount of cash received upon assuming the office	$32.17
Value of liquor received at the same time	57.54
Cash received for sales of liquor during the year	102.97
Amount paid for liquor bought during the year	59.91
Amount of agent's salary	25.00
Value of liquor on hand at the end of the year	31.37

Did the agent owe the town, at the end of the year, or did the town owe the agent, and how much? Did the town gain or lose, and how much?

Ans. The agent owed the town $50.23.
The town lost 8.11.

33. A has $10; B has as much as A and half as much as C; and C has as much as A and B together: what sum has each?

34. Required the area of a right-angled triangle, the hypothenuse being 64 feet, and one side 28 feet.

35. A sphere is 16 inches in diameter: what is the diameter of one $\frac{3}{4}$ as large?

36. A certain field is bounded as follows:—By a line running due east, 20.5 chains; thence in a southeasterly direction, 20 chains; thence due west, 32.5 chains; thence north, 16 chains, to the point of beginning: how many acres does the field contain?

37. What is the diameter of the driving-wheel of a locomotive, which, making 152 revolutions per minute, runs 30 miles per hour?

38. At 12 o'clock the hour and minute hands of a clock are together: at what time between 3 and 4 o'clock will they be together?

39. Required the area of an elliptical table whose transverse diameter is 4½ feet, and conjugate diameter 3 feet.

40. Broadway, in New York, is 85 feet wide: suppose a million bales of cotton, each 5 feet long, 4 feet wide, and 3 feet thick, to be piled so as to fill the street, between the buildings, to a height of 45 feet, what distance will they extend?

41. A person, being asked the hour of the day, replied, that the time past noon was equal to $\frac{9}{16}$ of the time to midnight: what was the time?

42. A gentleman has two horses and a carriage worth $1000. The first horse and the carriage are worth three times as much as the second horse, and the second horse and the carriage are worth seven times as much as the first horse: required the value of each horse.

43. If a staff 5 feet 6 inches long cast a shadow of 7 feet, what is the height of a steeple whose shadow is 140 feet?

44. What is the least common multiple of $6\frac{1}{2}$ and $7\frac{3}{4}$?

45. What is the greatest common divisor of $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{3}{16}$?

46. A, B, and C, start together from the same point, to walk around a lake 72 miles in circumference. A walks 9 miles, and B 12 miles a day, in the same direction, and C walks 18 miles in the opposite direction: in how many days will they all meet at the starting-point?

47. Six men are engaged in building a house, which they agreed to finish in 90 days; but, after working 70 days, they find that it will be impossible to complete it in less than 50 days more: how many additional men must be employed, to finish the work according to the contract?

48. The sum of $1240 is to be divided among three persons, in the proportion of 1, $\frac{8}{11}$, and $\frac{7}{9}$, respectively: how much will each receive?

49. A boy wants 12 pieces of silver, in change for a dollar, all dimes and half-dimes: how many of each must he have?

Ans. 8 dimes, and 4 half-dimes.

50. A hare starts 30 yards before a greyhound, but is not seen by him till she has been running 40 seconds. The hare runs at the rate of 8 miles per hour, and the hound pursues her at the rate of 10 miles per hour: how long will the chase continue, and how far must the hound run?

51. If gold be quoted at 110 for currency, how much below par is the currency, gold being the standard?

52. A cistern, of the capacity of 126 gallons, has a supply-pipe, which will fill it in 20 minutes, and two discharge-pipes, one of which will empty it, when full, in 40 minutes, and the other in 80 minutes: how long will it take to fill it with water flowing through all the pipes at the same time?

53. How many yards of thread will be required to wind spirally, from end to end, round a cylinder 6 feet long, and 10

inches in diameter, at a uniform pitch of three inches to one circumference? *Ans.* 21.039 + yds.

54. A ball, falling from the height of 10 feet, rebounds 5 feet, and again falling, rebounds $2\frac{1}{2}$ feet; and so on, it continues, falling, and rebounding one half as high as it fell the last time: through what space will it pass, in the successive falls, before it comes to a state of rest?

55. The diameter of a circle is 4 feet: what is the diameter of a circle whose area is 4 times as large?

56. A market-woman bought eggs at 18 cents a dozen, and sold them at the rate of 3 for 5 cents: what was her gain per cent.?

57. A note for $413.50, dated September 1, 1869, bears the following indorsements:—Received October 1, 1869, $50; Jan. 10, 1870, $100; March 11, 1870, $75: how much is due June 1, 1870, the rate of interest being 7 per cent.?

58. When gold sells at 110 for currency, what rate of interest on his investment, in currency, will a person receive, who buys U. S. 5–20 Bonds at 105?

59. What must be the face of a note payable in 90 days, that, when discounted at a bank, at 7%, will yield $1000?

60. Sold $2543.50 worth of goods, at a profit of 5%, on a note at 60 days, and had the note discounted the same day, at 6% per annum, at a Philadelphia bank: required the net profit on the goods.

61. A and B purchased a watermelon for 8 cents, A paying 3 cents, and B, 5 cents. A third person comes along, and offers 8 cents for a third interest in it, and the offer is accepted: how shall A and B divide the money, so that all three shall have equal shares of the melon?

Ans. B should take 7 cents,
A " " 1 cent.

62. At marriage the wife's age was $\frac{4}{7}$ of the husband's, and in 32 years it was $\frac{4}{5}$: how old were they at marriage?

Ans. 16 and 28.

63. A grain-dealer has a quantity of wheat which he purchased at a certain price. If he sells it at $1.50 per bushel, he will lose $37.50; but if he sells it at $1.90 per bushel, he will gain $22.50: how many bushels of wheat has he?

64. Two young men, A and B, went a hunting for deer, but failing to kill any, they bought one for $8. As B was short of funds, A paid for the deer; but B paid the expressage home, $1.50, and agreed to pay the balance of his share after they reached home. Upon their return, they sold the skin for $1.50, and A received the money. Now, how much ought B to pay to A for his share of the game? *Ans.* $2.50.

65. How many square feet of plank, 2 inches thick, will it take to make a box 4 feet long, 3 feet wide, and 2 feet deep?

66. A person spent one half of all the money he had and half a dollar more; then half of the remainder and half a dollar more; then half of the remainder and half a dollar more, when he had one dollar left: how much had he at first? *Ans.* $15.

67. A flash of lightning was seen 10 seconds before the thunder was heard: what distance from the observer was the cloud from which it came, the velocity of sound being 1090 feet per second?

68. A butcher killed a hog, and wished to ascertain his weight, but he had no scales at hand. He, therefore, "set his wits to work," and recollecting something about arithmetic which he studied when a boy, and knowing his own weight to be 160 pounds, he took a bar of wood, placed it on a support, and suspended the hog on the short arm, 18 inches from the fulcrum, and then found that he could, by his own weight, just balance him at a distance of 2 feet 3 inches from the fulcrum, on the long arm: what did the hog weigh? *Ans.* 240 lbs.

69. A weight of 150 pounds is suspended on the short arm of a lever, 15 inches from the fulcrum; at the end of the long arm, 5 feet from the fulcrum, there is an iron ball, weighing 16 pounds: at what distance from the fulcrum, on the long arm, must another ball, weighing twice as much as the small one, be

placed, so that the two balls will just counterbalance the weight of 150 pounds on the short arm? *Ans.* $40\frac{5}{16}$ inches.

70. A tree, 108 feet high, standing on a horizontal plane, was broken by the wind, in such a manner that the top fell and struck the ground 36 feet from the trunk, the other end of the part broken off resting upon the top of the part left standing: at what point was the tree broken?

NOTE.—Solve this problem by the principle that the product of the sum and difference of two numbers is equal to the difference of their squares.

71. Three men, A, B, and C, bought a grindstone three feet in diameter, paying equally; and they agreed that A should grind off from the circumference his share first; then B his share; then C his share: what part of the radius should each grind off?

Ans. A, 3.304 + inches.
B, 4.303 + "
C, 10.392 + "

72. The lessee of certain premises sub-lets the same to another person, at a rent of $500 more than he pays: what is his lease worth, if it is to continue 5 years, reckoning compound interest at 7 per cent.? *Ans.* $2050.09.

73. A man, at the age of 35 years, takes a Life Policy of Insurance for $1000, for an annual premium of $26.38. His "Expectation of Life" is 31 years; should he live, and continue to pay the premium for this term, what will his payments amount to at 6 per cent. compound interest?

74. A certain flag-pole, in the form of a complete cone, is 2 feet in diameter at the surface of the ground where it stands, and 100 feet in slant height: how many yards of inch ribbon will it require to wind round it, flatwise and spirally, from bottom to top, in such a manner as to leave a continuous space three inches wide between the parts, or convolutions of the spiral? *Ans.* 314.16 yards.

75. What is the area of a square tract of land which contains as many acres as there are rails in the fence inclosing it, each rail being 11 feet long, and the fence 4 rails high?

Ans. 92160 acres.

CENTURY CALENDAR,

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Years from 1800 to 1900, Inclusive.		JAN.	FEB.	MAR.	APR.	MAY.	JUNE.	JULY.	AUG.	SEP.	OCT.	NOV.	DEC.
1809, '15, '26, '37, '43, '54, '65, '82, '93, '99,	1871	SUN.	WED.	WED.	SAT.	MON.	THU.	SAT.	TUE.	FRI.	SUN.	WED.	FRI.
Leap Years, 1816, 1844,	1872	MON.	THU.	FRI.	MON.	WED.	SAT.	MON.	THU.	SUN.	TUE.	FRI.	SUN.
1800, '06, '17, '23, '34, '45, '51, '62, '79, '90,	1873	WED.	SAT.	SAT.	TUE.	THU.	SUN.	TUE.	FRI.	MON.	WED.	SAT.	MON.
1801, '07, '18, '29, '35, '46, '57, '63, '85, '91,	1874	THU.	SUN.	SUN.	WED.	FRI.	MON.	WED.	SAT.	TUE.	THU.	SUN.	TUE.
1802, '13, '19, '30, '41, '47, '58, '69, '86, '97,	1875	FRI.	MON.	MON.	THU.	SAT.	TUE.	THU.	SUN.	WED.	FRI.	MON.	WED.
Leap Years, 1820, 1848,	1876	SAT.	TUE.	WED.	SAT.	MON.	THU	SAT.	TUE.	FRI.	SUN.	WED.	FRI.
1810, '21, '27, '38, '49, '55, '66, '83, '94, 1900,	1877	MON.	THU.	THU.	SUN.	TUE.	FRI.	SUN.	WED.	SAT.	MON.	THU.	SAT.
1805, '11, '22, '33, '39, '50, '61, '67, '89, '95,	1878	TUE.	FRI.	FRI.	MON.	WED.	SAT.	MON.	THU.	SUN.	TUE.	FRI.	SUN.
Leap Years, 1824, 1852,	1880	THU.	SUN.	MON.	THU.	SAT.	TUE.	THU.	SUN.	WED.	FRI.	MON.	WED.
1803, '14, '25, '31, '42, '53, '59, '70, '87, '98,	1881	SAT.	TUE.	TUE.	FRI.	SUN.	WED.	FRI.	MON.	THU.	SAT.	TUE.	THU.
Leap Years, 1828, 1856,	1884	TUE.	FRI.	SAT.	TUE.	THU.	SUN.	TUE.	FRI.	MON.	WED.	SAT.	MON.
" 1804, 1832, 1860,	1888	SUN.	WED.	THU.	SUN.	TUE.	FRI.	SUN.	WED.	SAT.	MON.	THU.	SAT.
" 1808, 1836, 1864,	1892	FRI.	MON.	TUE.	FRI.	SUN.	WED.	FRI.	MON.	THU.	SAT.	TUE.	THU.
" 1812, 1840, 1868,	1896	WED.	SAT.	SUN.	WED.	FRI.	MON.	WED.	SAT.	TUE.	THU.	SUN.	TUE.

DIRECTIONS:—The above Table shows on what day of the week each month of the NINETEENTH CENTURY commenced, or will commence. To find the CALENDAR FOR ANY MONTH, look for the year in the column of years, and on the same line, at the right, under the name of the month, will be found the day of the week on which the month began or will begin; then select from the Monthly Calendars below, the one commencing with the day thus found, and it will be the Calendar for the given month. EXAMPLE.—On what day of the week will the Centennial 4th of July fall in 1876? The month begins *Saturday*, and the Calendar below in which Saturday is the first day of the month, is the Calendar for July, 1876; the 4th day is *Tuesday*.—ANS. On what day of the week will the year 1900 begin? ANS. *Monday*.

The Monthly Calendars are perpetual for all months when they begin on the days of the week over them.

SUNDAY.

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	..	..	..	..
..	..	..	..	..		..

MONDAY.

S	M	T	W	T	F	S
..	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	..	..	..
..	..	..	..	..	..	..

TUESDAY.

S	M	T	W	T	F	S
..	..	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31	..	..
..	..	..	..	..	..	..

WEDNESDAY.

S	M	T	W	T	F	S
..	..	..	1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	..
..	..	..	..	..	..	..

THURSDAY.

S	M	T	W	T	F	S
..	..	..	..	1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
..	.	..	..	..	..	..

FRIDAY.

S	M	T	W	T	F	S
..	..	..	..	..	1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31	..	..	..	..	..	..

SATURDAY.

S	M	T	W	T	F	S
..	..	..	..	..	..	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31	..	..	..	..	..

ANSWERS.

The following are answers to all the problems not given in the body of the book, except those of the first part.

The page is on the left. The numbers of the examples are inclosed by parenthetical marks, and the answers follow.

When there are two or more sets of examples of the same numbers on a page, the second set is indicated thus [2 S.], the third set thus [3 S.], etc.

PAGE EX. ANS. EX. ANS. EX. ANS.

46 || (1) 2, 5, 7, 11, 41. (2) 3, 3, 5, 7, 7, 13. (3) 2, 3, 3, 3, 3, 17.
|| (4) 3, 3, 5, 7, 11, 13. (5) 2, 2, 2, 3, 3, 5, 17. (6) 17, 19; 3,
47 || 11, 13; 3, 587; 3, 7, 7; 7, 19. || **47.** || (1) 5 (2) 15 (3) 13
50 || (4) 19 (5) 6 (6) 23. || **50.** || (1) 6 (2) 19 (3) 3 (4) 46
51 || (5) None. (6) 107. || **51.** || (1) 81 (2) $4\frac{1}{2}$ (3) 15 (4) $\frac{4}{11}$.
52 || (5) 1 (6) 12 (7) 30 (8) 20 (9) $\frac{4}{9}$ (10) $\frac{4}{567}$ (11) $1\frac{5}{19}$ (12) $1\frac{1}{12}$.
56 || (1) 2310 (2) 2394 (3) 9828 (4) 3600 (5) 285 (7) 378
61 || (8) 589050 (9) 918 (10) 156 (11) 6851. || **61.** || (2) $\frac{17}{29}$
|| (3) $\frac{1}{12}$ **[2 S.]** (1) $\frac{3}{4}$ (2) $\frac{7}{12}$ (3) $\frac{1}{4}$ (4) $\frac{1}{3}$ (5) $\frac{1}{3}$ (6) $\frac{164}{261}$ (7) $\frac{43}{47}$
62 || (8) $\frac{19}{37}$ (9) $\frac{23}{24}$ (10) $\frac{277}{381}$ (11) $\frac{1}{2}$ (12) $\frac{17}{81}$. || **62.** || (1) $\frac{35}{46}$ (2) $\frac{27}{36}$.
63 || (1) $\frac{165}{7}$ (2) $\frac{651}{4}$ (3) $\frac{10123}{14}$ (4) $\frac{3153}{25}$ (5) $\frac{3154}{7}$ (7) 70; 235; 660.
|| **[2 S.]** (1) $\frac{141}{8}$ (2) $\frac{148}{5}$ (3) $\frac{130}{11}$ (4) $\frac{1183}{9}$ (5) $\frac{2375}{13}$ (6) $\frac{5903}{17}$
|| (7) $\frac{892}{9}$ (8) $\frac{667}{3}$ (9) $\frac{1179}{8}$ (10) $\frac{4781}{12}$ (11) $\frac{3597}{5}$ (12) $\frac{810}{31}$.
64 || (3) $127\frac{7}{9}$ (4) 48 (5) 1 (6) \$$27\frac{6}{17}$. **[2 S.]** (1) 3 (2) $17\frac{5}{7}$
|| (3) $2\frac{10}{11}$ (4) 1 (5) 5 (6) 6 (7) $320\frac{7}{13}$ (8) $3\frac{762}{1000}$ (9) $744\frac{4}{5}$
66 || (10) $137\frac{9}{14}$ (11) $1\frac{165}{211}$ (12) $37\frac{127}{195}$. || **66.** || (2) $\frac{448}{1232}$, $\frac{539}{1232}$,
|| $\frac{528}{1232}$. (3) $\frac{800}{2800}$, $\frac{1680}{2800}$, $\frac{2450}{2800}$, $\frac{560}{2800}$. (4) $\frac{294}{1078}$, $\frac{616}{1078}$, $\frac{693}{1078}$.
67 || (5) $\frac{72}{576}$, $\frac{480}{576}$, $\frac{432}{576}$. || **67.** || (3) $\frac{6}{45}$, $\frac{27}{45}$, $\frac{6}{45}$, $\frac{12}{45}$. (4) $\frac{3}{32}$, $\frac{4}{32}$,
|| $\frac{12}{32}$, $\frac{8}{32}$. (5) $\frac{35}{40}$, $\frac{32}{40}$, $\frac{6}{40}$, $\frac{30}{40}$. (6) $\frac{18}{42}$, $\frac{27}{42}$, $\frac{14}{42}$, $\frac{22}{42}$. (7) $\frac{7}{12}$, $\frac{6}{12}$,
|| $\frac{9}{12}$, $\frac{10}{12}$. **[2 S.]** (1) $\frac{315}{360}$, $\frac{200}{360}$, $\frac{60}{360}$, $\frac{288}{360}$. (2) $3\frac{5}{6}$, $14\frac{4}{6}$,
|| $19\frac{3}{6}$. (3) $18\frac{14}{20}$, $200\frac{5}{20}$, $\frac{3}{20}$. (4) $\frac{702}{1638}$, $\frac{728}{1638}$, $\frac{1365}{1638}$, $\frac{126}{1638}$.
|| (5) $\frac{36}{90}$, $\frac{80}{90}$, $\frac{75}{90}$, $\frac{63}{90}$. (6) $\frac{455}{1001}$, $\frac{231}{1001}$, $\frac{572}{1001}$, $\frac{616}{1001}$. (7) $\frac{20}{24}$, $\frac{42}{24}$,
68 || $\frac{9}{24}$, $\frac{12}{24}$. (8) $\frac{1}{5}$, $\frac{1}{5}$, $\frac{1}{5}$, $\frac{1}{5}$. || **68.** || (1) $\frac{42}{150}$, $\frac{32}{150}$. (2) $\frac{21}{24}$, $\frac{4}{24}$.
70 || (3) $\frac{18}{24}$, $\frac{18}{24}$, $\frac{4}{24}$, $\frac{3}{24}$. (4) $\frac{3}{8}$, $\frac{2}{8}$, $\frac{1}{8}$. || **70.** || (1) 2 (3) $160\frac{45}{56}$
|| (4) $74\frac{577}{840}$ (5) $1\frac{407}{468}$ (6) $7\frac{3}{4}$. **[2 S.]** (1) $3\frac{1}{8}$ (2) $17\frac{7}{12}$ (3) $236\frac{2}{9}$

70 || (4) $5\frac{461}{546}$ (5) $98\frac{5}{21}$ (6) $816\frac{613}{1190}$ (7) $92\frac{9}{16}$ (8) $2\frac{19}{28}$ (9) $\frac{757}{1012}$

|| (10) $3\frac{1}{32}$ (11) $31\frac{2}{3}$ (12) $2\frac{2}{9}$ (13) $38\frac{1}{8}$ (14) $25\frac{37}{342}$ (15) $1\frac{1}{4}$

|| (16) $2207\frac{1}{2}$ (17) $1275\frac{31}{60}$ (18) $5804\frac{199}{240}$ (19) $428\frac{7}{8}$ (20) $320\frac{15}{34}$

|| (21) $26807\frac{1}{4}$ (22) $224\frac{11}{16}$ (23) $23\frac{37}{66}$ (24) $26\frac{2}{10}$ (25) $560\frac{1}{18}$

71 || (26) $101\frac{17}{56}$ (27) $22\frac{145}{156}$. || **71.** || (1) $\frac{13}{40}$ (2) $\frac{13}{42}$ (3) $\frac{1}{12}$

|| (4) $\frac{23}{132}$ (5) $\frac{17}{52}$ (6) $\frac{16}{39}$ (7) $\frac{17}{72}$ (8) $\frac{17}{60}$. [**2 S.**] (1) $\frac{48}{63}$ (2) $1\frac{1}{15}$

73 || (3) $1\frac{7}{88}$ (4) $1\frac{1}{6}$ (5) $1\frac{47}{72}$ (6) $1\frac{5}{22}$ (7) $1\frac{1}{132}$(8) 1. || **73.** || (1) $\frac{1}{2}$

|| (2) $\frac{7}{9}$ (3) $\frac{5}{16}$ (4) $\frac{21}{148}$ (5) $\frac{9}{91}$ (6) $\frac{1}{5}$ (7) $\frac{5}{99}$ (8) $\frac{1}{34}$.

|| (**2 S.**] (1) $\frac{39}{56}$ (2) $\frac{5}{24}$ (3) $\frac{241}{280}$ (4) $6\frac{19}{24}$ (5) $\frac{935}{1000}$ (6) $1\frac{467}{1248}$

74 || (7) $1\frac{1}{12}$ (8) $\frac{41}{64}$. || **74.** || (1) $\frac{1}{8}$ (2) $\frac{1}{272}$ (3) $\frac{1}{156}$ (4) $\frac{1}{20}$ (5) $\frac{1}{110}$

|| (6) $\frac{1}{10100}$. [**2 S.**] (1) $\frac{7}{24}$ (2) $\frac{4}{15}$ (3) $\frac{8}{35}$ (4) $\frac{4}{65}$.

75 || (1) $255\frac{16}{19}$ (2) $241\frac{4}{13}$ (3) $562\frac{12}{27}$ (4) $99\frac{98}{99}$ (5) $74\frac{12}{37}$ (6) $85\frac{4}{5}$

76 || (7) $131\frac{1}{4}$ (8) $\frac{7}{8}$. || **76.** || (1) $170\frac{10}{12}$ (2) $98\frac{3}{8}$ (3) $134\frac{5}{16}$ (4) $888\frac{37}{39}$

|| (5) $4\frac{11}{20}$ (6) $\frac{319}{357}$ (7) $15\frac{1}{5}$ (8) $158\frac{15}{46}$ (9) $19\frac{3}{4}$ (10) $1\frac{67}{72}$ (11) $18\frac{7}{8}$

|| (12) $\frac{1}{7}$ (13) $81\frac{19}{36}$ (14) $814\frac{1}{12}$ (15) 19 (16) $186\frac{1}{40}$ (17) $\frac{5}{8}$

|| (18) $418\frac{81}{88}$ (19) $271\frac{13339}{13806}$ (20) $7\frac{25}{36}$ (21) $\frac{1}{4}$ (22) $89\frac{10}{171}$.

77 || (1) $2\frac{6}{17}$ (2) $9\frac{27}{43}$ (3) 9 (4) $\frac{8}{9}$ (5) $7\frac{7}{15}$ (6) $16\frac{853}{867}$ (7) $6\frac{7}{9}$ (8) $12\frac{1}{3}$

78 || (9) $\$9\frac{3}{8}$ (10) $\$58\frac{1}{2}$. || **78** || (1) 36 (2) $247\frac{1}{3}$ (3) 190 (4) 9 (5) $206\frac{1}{2}$

|| (6) $281\frac{1}{4}$ (7) 2891 (8) $180\frac{5}{6}$ (9) 9, $80\frac{8}{9}$, $10\frac{1}{5}$, $6\frac{10}{11}$. (10) None.

80 || (11) $20\frac{1}{4}$. || **80.** || (1) $\frac{28}{45}$ (2) $\frac{22}{198}$ (3) $\frac{19}{49}$ (4) $\frac{182}{425}$ (5) $\frac{189}{253}$

|| (6) $\frac{129}{335}$ (7) $\frac{12}{31}$ (8) $\frac{1813}{2000}$ (9) $\frac{1}{24}$ (10) $\frac{2}{77}$ (11) $\frac{35}{192}$.

81 || (1) $787\frac{1}{2}$ (2) $5901\frac{1}{8}$ (3) $14019\frac{9}{13}$ (4) 15200 (5) $214\frac{1}{2}$

|| (6) 220 (7) 2486 (8) 1886 (9) $\$74.39$ (10) $\$3186$ (11) $\$362$

82 || (12) $\$2073\frac{3}{4}$. || **82.** || (1) $15\frac{17}{24}$ (2) $227\frac{11}{24}$ (3) $1448\frac{1}{10}$

|| (4) $140\frac{5}{8}$ (5) 260 (6) $834\frac{23}{28}$ (7) $328\frac{13}{20}$ (8) $287\frac{41}{78}$ (9) $\$5.36\frac{5}{32}$

83 || (10) $\$222\frac{1}{16}$. || **83.** || (1) $\frac{1}{45}$ (2) $\frac{7}{391}$ (3) $\frac{2}{37}$ (4) $\frac{97}{1562}$ (5) $\frac{118}{1071}$

84 || (6) $\frac{11}{184}$ (7) $\frac{1}{2}$ (8) $\frac{2}{57}$. || **84.** || (1) $9\frac{9}{11}$ (2) $28\frac{27}{32}$ (3) $46\frac{3}{14}$

|| (4) 12 (5) $26\frac{1}{14}$ (6) $13\frac{1}{2}$ (7) 16 (8) 7 (9) $2442\frac{6}{13}$ (10) $3332\frac{4}{7}$

|| (11) 17083. [**2 S.**] (1) $\frac{2}{7}$ (2) $1\frac{7}{48}$ (3) $1\frac{1}{27}$ (4) 7 (5) $1\frac{5}{12}$

85 || (6) 8 (7) $1\frac{137}{240}$ (8) 1. || **85.** || (9) $18\frac{9}{52}$ (10) $5\frac{2}{15}$ (11) $\frac{406}{243}$.

|| [**2 S.**] (1) $7\frac{1}{7}$ (2) $3\frac{6}{11}$ (3) $3\frac{25}{84}$ (4) $4\frac{1}{44}$ (5) $249\frac{2}{21}$ (6) $16\frac{31}{216}$

|| (7) $62\frac{11}{25}$ (8) $\frac{1}{2}$ (9) $20\frac{7}{51}$ (10) $1\frac{163}{324}$ (11) $118\frac{11}{16}$ (12) $3\frac{1}{12}$.

86 || (1) $288\frac{32}{111}$ (2) $5\frac{1}{76}$ (3) $15\frac{3}{7}$ (4) $40\frac{29}{47}$ (5) $192\frac{26}{107}$

|| (6) $13\frac{235}{401}$ (7) $82\frac{2}{5}$ (8) $6\frac{31}{44}$ (9) $5\frac{3}{5}$ (10) $34\frac{13}{20}$ (11) $109\frac{37}{79}$

87 || (12) $8\frac{29}{47}$. || **87.** || (1) $5\frac{19}{36}$ (2) $9\frac{28}{129}$ (3) $75\frac{335}{366}$ (4) $3\frac{132}{201}$

88 || (5) $17\frac{1}{5}$ (6) $2\frac{67}{215}$ (7) $10\frac{27}{37}$ (8) $8\frac{6}{113}$. || **88.** || (3) $\frac{2}{11}$ (4) 4

90 || (5) 40. [**2 S.**] (2) $3\frac{11}{20}$ (3) $6\frac{100}{151}$ (5) $\frac{7}{27}$. || **90.** || (3) $\frac{9}{300}$

91 || (4) $\frac{1}{200}$. || **91.** || (2) 2150 (3) 12. || **92.** || (1) $1\frac{13}{84}$.

92 || (2) $247\frac{9}{16}$ (3) $\frac{1}{4}$, $\frac{1}{6}$, $\frac{7}{15}$. (4) $20\frac{68}{435}$ (5) $2\frac{25}{72}$ (6) $\frac{19}{216}$ (7) $\frac{16}{253}$

|| (8) $2\frac{43}{72}$ (9) 42 (10) $7577\frac{3}{4}$ (11) $17770\frac{1}{2}$ (12) $4\frac{3}{32}$ (13) $\frac{648}{685}$

|| (14) $\frac{21}{208}$ (15) $\$222\frac{2}{9}$ (16) $3\frac{15}{32}$ (17) $\$666\frac{2}{3}$ (18) $2112\frac{4}{9}$

|| (19) $1378\frac{2}{3}$ (20) 960 (21) $228\frac{33}{40}$ (22) $30\frac{47}{330}$

93 || (23) $\$2415\frac{55}{224}$ (24) A, 572; B, 264. (25) $\$59\frac{3}{8}$ (26) $\frac{49}{20}$;

|| $2\frac{9}{20}$ ft. (27) $19\frac{367}{747}$ (28) $5\frac{1}{7}$ (29) 96 cts. (30) $1\frac{67}{78}$ (31) $15\frac{5}{7}$

98 || (32) $\frac{23}{64}$ and $\frac{31}{64}$. || **98.** || (2) $\frac{1}{8}$ (3) $\frac{3}{1000}$ (4) $\frac{3}{800}$ (5) $\frac{1}{8}$

99 || (6) $\frac{83}{180}$ (7) $\frac{1}{3}$ (8) $\frac{11}{3}$. || **99.** || (2) .1875 (3) .008

|| (4) .25125 (5) $54.75 (6) .99 (7) .03716+ (8) No;

|| $.185\frac{5}{27}$ (9) .546875, .0475113+, .575, .32, .33333+,

|| .92857+, .005906+, .54117+, .75, .46875, .325, .2663115+,

100 || .625, .13333+. || **100.** || (2) 102.515 (3) 908.425

101 || (4) 27.223125 (6) $2898.01 (7) 1.11111. || **101.** || (8) 2.1493

|| (2) 1.1115 (3) 5674.9 (4) .000999 (5) 6.99993 (6) 43.08997.

102 || (7) 20.41 acres. (8) $148.50 (9) $1.4791\frac{2}{3}$ (10) $0.7475.

|| (11) .0373+ (12) 999999.999999 (13) .22698± (14) 3.35

103 || (15) 999.99999. || **103.** || (1) .64125 (2) 1635.2776

|| (3) .00008 (4) 2431.269 (5) .06200031 (6) .68785

|| (7) 174.95625 (8) .1875 (9) .0075 (10) $0.4375 (11) 9875

|| (12) $798.75 (13) $18537.8125 (14) .012

104 || (1) 287700 (2) .0372 (3) 9 (4) 900 (5) 89802.2185.

106 || (1) .8 (2) 410.8 (4) 1000 (5) $.006\frac{2}{3}$ (7) $801 (8) 100000

|| (9) 10 (10) 5 (11) .0459 (12) $0.875 (13) 12 times.

|| (14) 1500 (15) 1000000.89999991+ (16) 7000 (17) .32

|| (18) 1. (19) 1.6666+ (20) .0000001 (21) .135 (22) .01

|| (23) .00406 (24) 2186 (25) 19.88014+ (26) .00896

|| (27) .00001 (28) .099107+ (29) 225.2432+

109 || (1) $\frac{45}{990}$ (2) $\frac{75}{900}$ (3) $\frac{634}{999}$ (4) $2\frac{559}{990}$ (5) $\frac{7845}{9990}$ (6) $\frac{1}{50}$

116 || (7) $241\frac{723}{999}$. || **116.** || $1418.07 (2) $233.40 (3) $618.75

|| (4) $120.85 (5) $88.89 (6) $25.055 (7) $8.80 (8) $140

|| (9) $2.81+ (10) $47.65 (11) $\$2.06\frac{1}{4}$ (12) 9 (13) $50.45+

118 || (1) $12.50 (2) 20 (3) $9 (4) $43\frac{1}{2}$ || **119.** || (1) $62.50

119 || (2) $62.51 (3) $22.81+. [**2 S.**] (1) $28.60 (2) $4.70+ (1) 7345

142 || (2) 1450 (3) 4029 || **143** || (4) 399300 (5) 1889 (6) 22926

143 || (7) 320488 (8) 240 (9) 3678 (10) 1193 [**2 S.**] (1) 45 A. 3 R.

|| (2) 2 hhd. 1 bbl. 23 gal. 3 qt. (3) 125 bush. 3 pk. 5 qt.

144 || (4) 75 mi. 5 fur. (5) 18 cwt. 3 qr. 14 lb. (6) £45 10s. 6d.
|| (7) 55 lb. 7 oz. 13 pwt. 16 gr. (8) £1. (9) 12 lb. 9 ℥ 2 ʒ
145 || (10) 22 yr. 39 wk. 6 da. || **145.** || (1) 1 qr. 1 lb. 5 oz. $\frac{16}{19}$ dr.
|| (2) 5 fur. 21 rd. 2 yd. 2 ft. $10\frac{8}{13}$ in. (3) 132 da. 17 h. 27 min.
|| $16\frac{4}{11}$ sec. (4) 3 R. 1 sq. rd. 6 sq. yd. 3 sq. ft. 108 sq. in.
|| (5) 1 bbl. 18 gal. 3 qt. $1\frac{1}{5}$ pt. (6) 1 pk. 5 qt. $1\frac{73}{85}$ pt. (7) 19s.
|| (9) 6 fur. (10) 2 R. 20 sq. rd. (11) 2s. 6d. (12) 6 da. 3 h.
|| (13) 1 mi. 3 fur. 8 rd. 4 yd. 1 ft. 2.4 in. (14) 1512 cu. in.;
|| 3 T. 15 cwt.; 2 R.; 4 oz.; 11 oz. 3 dr. 1.2 gr.;
146 || 6 O. 6 f℥. 3 fʒ. 12 m. || **146.** || (1) $\frac{28}{157}$ (2) $\frac{2688}{4289}$
147 || (3) $\frac{11}{144}$. || **147.** || (4) $\frac{3}{1360}$ (5) $9\frac{11}{13}$ (6) $\frac{1}{640}$. [**2 S.**]
148 || (1) $\frac{65}{96}$ (2) $\frac{4598}{5760}$ (3) $\frac{61}{128}$ (4) $\frac{127}{384}$. || **148.** || (5) $\frac{503}{2520}$ (6) $\frac{71}{83}$.
|| [**2 S.**] (1) £12.953125 (2) .065625 (3) $.920\frac{5}{6}$ (4) .3625
|| (5) $2.5000\frac{750}{1089}$ (6) $.04631\frac{31}{99}$ (7) .5703125 (8) $.58509\frac{18}{73}$.
149 || (9) $79\frac{1222}{3024}$ (10) 21.6625 (11) .0625 (12) $4.776\frac{288}{756}$
150 || (13) $.480\frac{85}{108}$ (14) $.000\frac{25}{15768}$. || **150.** || (1) .698 (2) .85
151 || (3) .625. || **151.** || (4) $46.55 (5) .945 (6) .7435 (7) 1.075.
152 || (1) 170 T. 19 cwt. $13\frac{1}{5}$ lb. (2) 94 cd. 1 cd. ft. 8 cu. ft.
|| (3) £611 7s. 10d. (4) 112 A. 3 R. 18 sq. rd. 29 sq. yd. 6 sq. ft.
|| 111 sq. in. (5) 439 lb. 3 oz. $1\frac{1}{2}$ gr. (6) 97 T. 14 cwt. 18 lb.
|| 5 oz. (7) 19 bush. 2 pk. $3\frac{1}{10}$ qt. (8) £304 9s. 2d. (9) 379 A.
|| 2 R. 16 rd. 61 sq. ft. 76 sq. in. (10) 32 S. 10° 59′ 28″
|| (11) $49.01+ (12) 4 fur. 23 rd. 3 yd. 1 ft. $\frac{3}{14}$ in.
153 || (13) 5s. 9d. || **153.** || (1) 2 rd. 13 ft. 4 in. (2) 14 T. 6 cwt.
|| 2 lb. (3) 11 cd. 5 cd. ft. (4) $139\frac{1}{2}$ doz. (5) 290 bush. 3 pk.
154 || (6) 424 A. 4 rd. (7) 16° 14′ 10″. || **154.** || (8) 10° 48′ 17″
|| (9) 9 cwt. $12\frac{1}{2}$ lbs. (10) 26 da. 23 h. 12 min.
156 || (1) 3 yr. 11 mo. 28 da. || **157.** || (2) 1 yr. 2 mo. 15 da.;
157 || 7 yr. 9 mo. 1 da. (3) June 11-14; June 9–12 (4) 38
158 || (6) 6 yr. 24 da. (7) 20. || **158.** || (1) 166 mi. 3 fur. 4 rd. 9 ft.
|| (2) 60 cd. 123 cu. ft. (3) 427 lbs. 3 oz. 4 pwt. (4) 2535 A.
159 || 2 R. 14 P. $1\frac{1}{2}$ sq. yd. || **159.** || (6) 153 rd. 3 yd. $2\frac{1}{2}$ ft.
|| (8) £24 8s. 2d. (9) 311 T. 5 cwt. 3 qr. 34 lb. (10) 65 lb. 1 ℥.
|| 2 ℈ (11) 12 C. 5 S. 20° 10′ (12) 121 lb. 6 oz. 6 pwt. 16 gr.
161 || (1) 4 T. 1425 lb. $1\frac{5}{8}$ oz. (2) 1 lb. 5 ʒ 1 ℈ $7\frac{8}{17}$ gr. (3) 3 A.
|| 2 R. 14 rd. 15 yd. 6 ft. 18 in. (4) 4 da. 23 h. 19 min.
|| $22\frac{6}{7}$ sec. (5) 253 A. 3 R. 17 sq. rd. 4 sq. yd. 2 sq. ft.
|| $128\frac{4}{7}$ sq. in. (6) 5 lb. 10 oz. 12 pwt. $3\frac{9}{28}$ gr. (7) $4\frac{7}{8}$ (8) 645 cu.

161 || ft. 592 cu. in. (9) 96 da. 23 h. 20 sec. **[2 S.]** (1) 5280
|| (2) 24900.48 (4) 136 A. 5 sq. ch. 8 P. 460 sq. l. (5) 21.
162 || (6) 625 (7) 4 cwt. (8) 40 da. 10 min. (9) 3 S. 8′ $53\frac{1}{3}$″
|| (13) 49.4 lb. (14) .675 (15) $.63\frac{1}{18}$ (16) 58 da. 9 h. 36 min.
169 || (19) $340\frac{20}{31}$ (20) 2 mi. 190 rd. 9 ft. || **169.** || (5) 79° 52′
170 || (6) 4 h. 36 m. $34\frac{14}{15}$ s. A.M. || **170.** || (2) 46 ft. 11′ 1″ 8‴.
171 || (3) 5 ft. 9″ 10‴ (4) 15 ft. 10′ 8″ (5) 53 yds. 11′ 10″
172 || (6) 14 ft. 2′ 9″. || **172.** || (1) 332 ft. 5′ 6″ (2) 15 sq. ft. 9′
|| 10″ 5‴ 3⁗ (3) 145 (4) $31.11+ (5) 101 sq. yd. 5 sq. ft.
173 || 7′ 6″ (6) 7 cd. 2 cd. ft. 7 cu. ft. || **173.** || (1) 32 ft. 4″
174 || (2) 115 ft. 8′ 5″ (3) $29\frac{1}{6}$ cu. ft. || **174.** || (1) 7 ft. 3′ 6″
|| (2) 2 ft. 3′ (3) 14 ft. 8′ 6″ (4) 14 ft. 4′ 6″; rem. 1″ 3‴
|| (5) 25 ft. 6′ 3″ (6) 11664 (7) 25 sq. yd. 4′ 2″ 8‴; or
|| $25\frac{19}{54}$ sq. yd. (9) $2.20+ (10) 2 ft. 5′ 9″+ (11) 2 ft. 7′
|| 5″ 0‴ $5\frac{1}{7}$⁗ (12) 102 cu. ft. 7′ $1\frac{1}{3}$″; or $102\frac{16}{27}$ cu. ft.
178 || (13) 3 cd. 6 cu. ft. (14) $36.85. || **178.** || (2) 500 men
|| (4) 140 bush. (5) $653.33\frac{1}{3}$ men (6) £18.75 (7) $0.21875;
|| $0.109375; $0.65625 (8) $1.8975 (9) .12 (10) $\frac{3}{1600}$;
179 || $\frac{7}{4800}$; .0009375. || **179.** || (12) $525 (13) $900
|| (14) 431.25 (15) $3515 (16) $825 (17) $1619.90
|| (18) $7.025 (19) $6250. **[2 S.]** (2) $375 (3) 600.
180 || (4) 200 (5) 800 (6) $750 (7) $325 (8) 94600 (9) $1000
|| (10) 125 (11) $3000 (12) $12380.95+ (13) $675 (14) $95
181 || (15) 1.12. || **181.** || (1) 5% (2) 20% (3) 75% (4) $16\frac{2}{3}$%
|| (5) $7\frac{1}{2}$% (6) 1% (7) $12\frac{1}{2}$% (8) $11\frac{19}{21}$% (9) $4\frac{1}{2}$% (10) $16\frac{2}{3}$%
183 || (11) $\frac{3}{80}$ (12) $\frac{5}{1000}$%. || **183.** || (1) $100; $21; $4.375
|| (2) £22 16s. 5.7d. (3) $9.4375; $230 (4) £98 15s.; £200
|| (5) 325 Tons (6) $75.91. **[Mis. Ex.]** (1) 250 (2) 500
|| (3) $31.50 (4) 16% (5) 480 (6) 60 (7) 200 (8) Rent,
|| $900; gen. ex., $1800; 60% of the whole (10) 75%
184 || (12) 3500. || **184.** || (15) $5000 (16) $9500 (19) 100.
186 || (2) $71.25 (3) $54.814+ (4) $28.86.
187 || (5) $3997.188 (6) $909.828 (7) $2130.22+.
|| **[2 S.]** (3) $92.895+ (4) $1.448+ (5) $3558.305.
188 || (6) $284.551+ (7) $7 (8) $1035 (9) $6.641+ (10) $6.09
189 || (11) $10.808+. || **189.** || (3) $3.20 (4) $14.137+
|| (5) $0.614+ (6) $3.986+ (7) $0.25 (8) $0.35 (9) $0.375+
|| (10) $1.707+ (11) $1.562+ (12) $2.598+ (13) $0.065+

189 || (14) \$0.166 + (15) \$2.408 (16) \$0.594 + (17) \$2.249 +
190 || (18) \$0.858+(19) \$1.30+(20) \$0.462+ || **190.** || (2) \$70.184
|| (3) \$189.583+ (4) \$528.185+ (5) \$365.40+ (6) \$136.575
192 || (7) \$0.213 + (9) \$48.533 + || **193.** || (10) \$36.458 +
193 || (11) \$29.224+(12) \$6.921+(13) \$1123.50 (14) \$154.958+
|| (15) \$154.20 (16) \$8.391 + (17) \$3.75 (18) \$0.487 +
|| (19) \$1.608 + (20) \$1.982 + (21) \$0.252 + (22) By days,
|| \$10316.666 +; by mos. & ds. \$10311.111 + (23) \$376.875
|| (24) \$1.023 + (25) \$7 (26) \$2.416 + (27) \$16.666 +
195 || (1) \$83.693 + (2) \$6.937 + (3) \$10.625 + (4) \$24.375
196 || (5) \$1586.041 + || **196.** || (2) \$216.98 (3) \$69.04 +
198 || (4) \$3.08 + [**2 S.**] (2) \$49.392 || **198.** || (2) £18 15*s.*
199 || (3)£10 1*s.* $4\frac{3}{4}$*d.*+ || **199.** || (4)£1 2*s.*$7\frac{3}{4}$*d.*+(5)£613 0*s.*$4\frac{3}{4}$*d.*+
|| (6) £469 7*s.* $10\frac{1}{2}$*d.*+ [**2 S.**] (3) \$252.285 +
200 || (4) \$1172.164 + (5) \$1652.40 || **200.** || (2) \$3719.287+
201 || (3) \$2515.723 + (4) \$4166.66 + || **201.** || (2) 3yr.
|| 6mo. 6da. (3) 2 yr. 8mo. (4) $16\frac{2}{3}$ yrs.; $14\frac{2}{7}$ yrs.; $12\frac{1}{2}$ yrs
204 || [**2 S.**] (2) 12% || **204.** || (1) \$179.0848 (2) \$134.36 +
205 || (3) \$2674.463+ || **205.** || (4) \$440.44; \$1190.44 (5) \$743.40
207 || (6) \$832,105 + || **207.** || (2) \$870.065 + (3) \$902.753 +
208 || (4) \$710.998 + || **212.** || (3) \$736.086 + (5) \$1635.926+
212 || (6) \$10.512 + (7) \$612.50; \$3750 || **215.** || (3) \$0.109 +
215 || (4) \$2526.95 (6) \$11.87; \$1503.88 (7) \$1251.05
216 || (8) \$410.10 || **217.** || (1) \$91.268 + (3) \$1227.187 +
217 || (4) \$2955.665 + (5) \$12500 (6) \$3691.07+
218 || (7) \$28.359+ (8) \$2357.08 (10) \$12.50 (11) \$25
219 || (1) $12\frac{1}{2}$% (2) \$1250 (3) \$6.90 (4) \$200 (6) \$11 (7) \$5
220 || (9) \$3.64 (10) \$90; $11\frac{1}{9}$% (11) Neither (12) $12\frac{1}{2}$%
|| (13) 5% (14) Lost \$100 (15) Neither (16) $22\frac{2}{9}$%
|| (17) $4\frac{1}{2}$% (18) $26\frac{1}{2}$%; A, \$523.37; B, \$1126.25;
224 || C, \$445.73; D, \$302.10 || **224.** || (2) \$176.75 (3) \$668.41+
225 || (4) \$3005.75 (6) \$8.10 || **225.** || (1) \$91.25 || **226.** || (3) \$11.93
228 || (2) \$140.48 (3) \$381.70 (4) \$319.44 || **230.** || (2) $7\frac{81}{86}$ mills
230 || nearly; A's, \$14.89 + (3) $17\frac{61}{225}$% (4) Amt. \$7794.87;
240 || 5 mills +; \$12.30 || **240.** || (3) 3 (4) 3 (6) 12 (7) $\frac{1}{3}$ (8) $\frac{2}{3}$
243 || (9) 24 (10) 12 (11) $\frac{3}{40}$ || **243.** || (1) 6 (2) 6 (3) 14 (4) 9
|| (5) $12\frac{1}{4}$ (6) 11.25 (7) $4\frac{2}{3}$ (8) 9lbs. (9) 8 (10) 84 : 300 :: 28 : 100
245 || (11) $9\frac{1}{3}$ || **245.** || (1) $337\frac{1}{2}$ miles (2) \$110.50 (3) \$32

|| (4) 111$\frac{3}{7}$ (5) 400 (6) 56 (7) 11 h. 27 min. 30 sec.
246 || (8) 34ft. 8in. (9) 19$\frac{13}{17}$ds. (11) 5h. 54min. 56$\frac{1}{4}$sec. (12) 20gal.
|| (13) 15 (14) $117 (15) 15$\frac{5}{18}$ds. (16) 225 rods
|| (17) 1st, $240; 2d, $672; 3d, $630 (18) 66$\frac{2}{3}$
249 || (19) One, 24.33\frac{1}{3}$, the other, 12.16\frac{2}{3}$ || **249.** || (1) 48ds.
|| (2) 1$\frac{11}{25}$mos. (3) 336bush. (4) 31$\frac{31}{32}$ds (5) 24oz. (8) 9h. 32min.
250 || (9) 32$\frac{1}{2}$ (10) 135$\frac{45}{58}$ bush. || **250.** || (12) 281$\frac{1}{4}$mi. (13) 213$\frac{8}{9}$
251 || (14) 2$\frac{7}{9}$ds. || **251.** || (3) 6$\frac{3}{4}$mos. (4) $375 (5) $2.812 +
|| (6) At 5 min. 27$\frac{3}{11}$ sec. past 1 o'clock, p.m. (10) 10
257 || (11) $\frac{3}{32}$; $\frac{9}{32}$ (12) $500 || **257.** || (3) $5312.50 (4) $4962.50
258 || (3) 10$\frac{30}{77}$% || **259.** || (3) 96 [**2 S.**] (2) Penna. 6's, 5$\frac{185}{203}$%;
276 || 10–40's, 4$\frac{208}{223}$% || **276.** || (2) W's loss, $10,153.59;
277 || G's, $8,122.87; S's, $4,061.44 || **277.** || (3) T's gain, $1,496.33
|| R's, $1,330.07; B's, $831.30. T's cap., $5,196.33;
|| R's, $4,630.07; B's, $2,831.30 (4) G. W's net ins., $6,522.98;
278 || H. W's, $3,313.78; C. S. W's, $2,209.19 || **278.** || (5) T's,
|| $66,978.73; B's, $40,187.24; H's, $34,987.33
279 || (7) H's, $36.11; J's, $27.08; W's, $18.06 (8) H's net ins.,
|| $14,820.13; P's, $10,374.09; W's, $8,892.08. M., R. & Co's
|| share, $8998.68; H. & J's, $13733.48; H. D. & Co's,
|| $23076.52; S. W's, $29922.43; S. & Co's, $15364.89
283 || (1) E's, $5806.81; L's, $5453.61 (2) B's gain, $9380.07;
|| cap., $16,880.07; S's gain, $6486.56; cap., $10,986.56
284 || (3) F., $1441.53; M., $2226.13; L., $1784.99
285 || (4) Bower's Dr. Bal. $110.33; Murphy's Cr. Bal. $399.57;
|| Wilson's Cr. Bal., $6280.47; Brown's Cr. Bal. $8027.59;
|| (5) A, $57.93; B, $38.62; C, 43.45 (6) A gained by int.,
|| $450; B, $405; A's whole gain, $1973.75; B's, $1928.75;
289 || A's cap., $8973.75; B's, $7928.75 || **289** || (3) $8.50
299 || (4) $7.91 + || **299.** || (2) May 12th. (3) 3 days (4) At 6%,
300 || Note at 87 days for $1018.57 || **300.** || (5) Mar. 17, 1871
301 || (6) Sept. 16, 1871 (7) Sept. 9, 1871 || **301.** || (8) Mar. 4, 1871
|| [**2 S.**] (2) 120 days, an extension of 30 days (3) Feb. 11
308 || (3) Mar. 12, 1871 (4) July 21, 1871 (5) Mar. 16, 1871
309 || (6) June 26th (7) Aug. 6th (8) Average date, May 5, 1872;
310 || Cash Bal., at 6%, $2288.98 || **310.** || (9) Averages due,
|| Jan. 26, 1871; Cash value, Jan. 1, 1872, interest at 6%,
325 || $2851.64 || **324.** || (1) $2463 + (2) $646.40 (3) $1485

326 || (4) 1010 [**2 S**] (1) \$230.38 (2) \$1007.55 || **326.** || (3) \$1359.36
330 || (2) £94 11*s.* 7*d.* + || **331.** || (8) £270 4*s.* 10*d.* + (9) £261 12*s.* 3*d.* +
|| (11) \$5273.10 (12) \$1267.72 + (3) £938 3*s.* 5*d.* + (4) \$1149.96
335 || (3) \$110.77 + || **336.** || (3) \$358.12$\frac{1}{2}$ || **337.** || (5) \$428.06$\frac{1}{4}$
|| (7) £48 15*s.* 7*d.* + (12) 20997.37 + marks (13) 1164.93 +
338 || (2) 1945.28 + florins. (3) \$437.40 (4) 1873.26 + florins
|| (5) 2425.53 + fl. || **343** || (1) 64 (2) 125; 625 (3) 9; 216; 1024; $\frac{1}{27}$; $\frac{27}{64}$
343 || (4) 24^9 (5) 49^5 (6) 104.8576 (7) $3^6 = 729$ (8) 150$\frac{1}{16}$
348 || (3) 183 (4) .14 (5) 4.72 (6) 2588.227 + (7) 167.358 +
|| (8) 2222 (9) 1.4142 + (10) 12.222 + (11) (1) 2.7; (2) 9; (3) 87^2
|| (5) 680; (6) .125 (7) 75 (8) 45 (9) 6 (12) 6700 (13) 8164
|| (14) 2.8284 + (15) 361 (16) 908 (17) 605; 2222
349 || (2) 42 (3) 363 (4) 98 [**2 S.**] (1) 6 (2) 49 (3) 2125
351 || (3) $\frac{8}{11}$ [**2 S.**] (2) $\frac{37}{100}$ (3) .9 (4) $\frac{11}{25}$ (5) .7453 || **358.** || (1) 364
|| (2) 319 (3) 2.35 (4) 3.358 + (5) 1.2599 + (6) 3.01
|| (7) 145 (8) 81; 88 (9) 77; 51 (10) 597; 71 (11) 60.389 +
|| (12) 191 (13) 560 (14) 8 (15) 1001 (17) 156 ft.
360 || (2) $\frac{9}{11}$ (3) 1.012 + || **361.** || (2) $\frac{12}{13}$ (3) $\frac{16}{17}$ [**2 S.**] (3) .8
365 || (2) 1$\frac{1}{4}$in. from one end (3) 416$\frac{2}{3}$lbs. || **366.** || (2) 20ft. 10in.
|| (3) 56 inches from the fulcrum (4) One, 72lbs., the
368 || other, 48lbs. || **368.** || (2) 720lbs. (3) $\frac{1}{3}$ ton (5) 2$\frac{1}{4}$in.
370 || (3) 700lbs. || **371.** || (2) 420lbs. (3) 266$\frac{2}{3}$lbs. (4) 155$\frac{5}{9}$lbs.
373 || (3) 1400lbs. (4) 350lbs. || **374.** || (3) 45239.04 (4) 13.262 + lbs.
376 || (1) 10$\frac{1}{2}$ (2) \$5 (3) 2 || **377.** || (1) $\frac{5}{6}$ (2) 10 miles
|| (3) 9, 11$\frac{2}{3}$, 14$\frac{1}{3}$, 17, 19$\frac{2}{3}$, 22$\frac{1}{3}$, 25, 27$\frac{2}{3}$, 30$\frac{1}{3}$, 33
378 || (1) 840 (2) 72 (3) 2550 yds. || **379.** || (2) 6
380 || (3) 9841$\frac{1}{2}$ (4) \$179.0848 + [**2 S.**] (2) 7, 14, 28, 56 (3) $\frac{1}{2}$
381 || (1) 1550 (3) 53$\frac{10907}{32768}$ (4) 5$\frac{21845}{65536}$ || **382.** || (2) 78124
386 || [**2 S.**] (2) 1$\frac{1}{2}$ (3) $.\dot{2}\dot{5}$ || **386.** || \$6776 (2) \$4050 (3) \$12705
387 || (1) \$4146.401 + (2) \$34913.95 (3) \$4099.549 +
390 || (1) \$15104.9948 (2) \$1589.099550 || **398.** || (2) 65 sq. in.
|| (3) 588 sq.rd. (4) 50 A. 1.875 sq. ch. [**2 S.**] (2) 39.98 + sq.ft.
400 || (3) 15 A. 35.698125 sq. rd. || **400.** || (4) 488.872 + ft.
|| (5) 64.222 + ft. [**2 S.**] (2) 14 A. 10 sq. rd. (3) 18 sq. ft.
401 || (2) 38 A. 1.625 sq. ch. || **402.** || (2) 47.124 ft.
|| (3) 17 rd. 10.108 ft. [**2 S.**] (2) 9 in. (3) 3 ft.
403 || (1) 19.635 sq.ft. (2) 4.90875 sq. ft. (3) 50 A. 3 R. 28.992 sq.rd.
|| (4) 98.175 sq. ft. (5) 6 ft. 6$\frac{27}{50}$ in. [**2 S.**] 203.717 + rods.

403 || (3) 72 ft. (4) d. 25 in.; c. 78.54 in. (5) 17.32+in.
404 || (1) 79.758 ft. (2) 42.315 rd. (3) B's contains 4.37248
405 || sq. rds. most. || **405.** || (1) 37.6992 sq. ft. (2) 150.7968 sq. ft.
|| **[2 S.]** (1) 89.5356 sq. in. (2) 69.1152 sq. ft.
406 || (1) 48 cu. ft. (2) 3 cd., 3 cd. ft. **[3 S.]** (2) 37$\frac{1}{2}$ sq. ft.
407 || (3) 162 sq. ft. || **407.** || (1) 129.9039 sq. ft. (2) 254$\frac{1}{27}$ cu. ft.
|| (3) 1$\frac{1}{6}$ cu. ft. **[2 S.]** (1) convex, 14.1372 sq. ft.; entire,
|| 17.6715 sq. ft. (2) 188.496 cu. ft. (3) 14.1372 cu. ft.
408 || (4) 2150.4252 || **408.** || (1) 5250 sq. ft. (2) 797.488 cu. ft.
|| (3) 133.31395+cu.ft. **[2 S.]** (1) 640 sq. ft. (2) 407.67008 sq. ft.
409 || (1) 746$\frac{2}{3}$ cu. ft. (8) 8.248+ cu. ft. **[2 S.]** (1) 7 sq. ft.
410 || 9.8784 sq. in. || **410.** || (1) 1 cu. ft. 1325.6352 cu. in.
|| (2) 4.0906$\frac{22}{96}$ cu. ft. (3) 259333411782.8608 (4) 10.902+in.
|| **[2 S.]** (1) 3.417+ ft. (2) 62.832 sq. in. (3) 192 sq. ft.
412 || (4) 114 sq. ft.; 57 sq. ft. (5) 1152 cu. in. || **412.** || (2) 10 ft.
|| (3) 7$\frac{1}{2}$ ft. (4) (1) 144 ft. (2) 116$\frac{2}{3}$ ft. (3) 270 ft. (4) 33$\frac{1}{3}$ ft.
|| (5) 40 ft. (6) 35 ft. (7) 70 ft. (8) 89$\frac{5}{6}$ ft. (9) 160 ft. (10) 169 ft.
413 || (7) 80 ft. (8) 333$\frac{1}{3}$ ft. (9) 213$\frac{1}{3}$ ft. || **414.** || (10) 189 ft. (11) 42 ft.
414 || (12) 210 ft. (13) 196$\frac{7}{8}$ ft. **[2 S.]** (2) 46$\frac{1}{24}$ cu. ft. (3) 27$\frac{2}{9}$ cu. ft.
415 || (4) 56 cu. ft. || **415.** || (2) 20$\frac{5}{6}$ cu. ft. (3) 44$\frac{43}{72}$ cu. ft.
416 || (4) 117$\frac{5}{9}$ cu. ft. || **416.** || (2) 35.42+ cu. ft. (3) 38$\frac{2}{11}$ cu. ft.
417 || (4) 39.25+ cu. ft. || **417.** || (2) 362+ ft. (3) 486 ft.
418 || (4) 788$\frac{1}{12}$ || **419.** || (5) V, 128$\frac{2}{3}$; D, in 3 sec., 144$\frac{3}{4}$ ft.
|| **[2 S.]** (2) 257$\frac{1}{3}$ ft. || **420.** || (3) 19.26 (4) 22
421 || **[Mis. Ex.]** (1) $\frac{3}{7}$ (2) 8 ft. (3) \$1; 5 cts. (4) Smith, \$24000;
422 || Jones, \$18000 || **422.** || (5) 20% (7) 8 (8) $\frac{5}{7}$ (9) \$8.83$\frac{1}{2}$
|| (10) 3$\frac{15}{16}$ ds. (11) 1960$\frac{1}{2}$ and 1586$\frac{1}{2}$ (12) 65 and 75
|| (13) 11$\frac{1}{5}$ ds. (14) First lot, 5%; Second lot, 10%; whole
423 || net gain, \$58.75 || **423.** || (15) 18.326 in. (16) 98
|| (17) \$610351.56 (18) 36 ft. long, 18 ft. wide, 18 ft. high
|| (19) 320 sq. ft. (20) 83.263+ft. (21) 33$\frac{1}{3}$ (22) \$825; \$1375
|| (23) 7 o'clock, 50 min. (24) 11 yrs. 325+ds. (25) \$132.56+
424 || (26) 16$\frac{2}{3}$ yrs. (27) \$5734.7105 (29 Less, 42$\frac{1}{2}$ grains (30) 5
425 || (31) 5$\frac{7}{13}$ ds. || **425.** || (33) A, \$10; B, \$30; C, \$40;
|| (34) 805.686+ sq. ft. (35) 14.536+in. (36) 42 A. 2 sq. ch.
|| (37) 5.528+ ft. (38) At 16 min. 21$\frac{9}{11}$ sec. past 3 o'clock
|| (39) 10.6029+sq. ft. (40) 2 mi. 310 rd. 11$\frac{14}{51}$ ft. (41) Half-
|| past 4 o'clock p.m. (42) 1st horse, \$125; 2d horse, \$250;

426 || Carriage, $625 || **426.** || (43) 110 ft. (44) 201$\frac{1}{2}$ (45) $\frac{1}{48}$
|| (46) 24 days (47) 9 (48) $495; $360; $385 (50) 3 min.
|| 10$\frac{14}{33}$ sec.; 932$\frac{8}{9}$ yds. (51) 9.091% + (52) 80 min.
427 || (54) 30 feet (55) 8 feet (56) 11$\frac{1}{9}$% (57) $204.36+
428 || (58) 6$\frac{4}{7}$% (59) $1015.74 (60) $94.41 || **428.** || (63) 150
429 || (65) 36 ft. 5′ 4″ (67) 2 mi. 340 ft. || **429.** || (70) 60 feet
|| from the top (73) $2237.06+.